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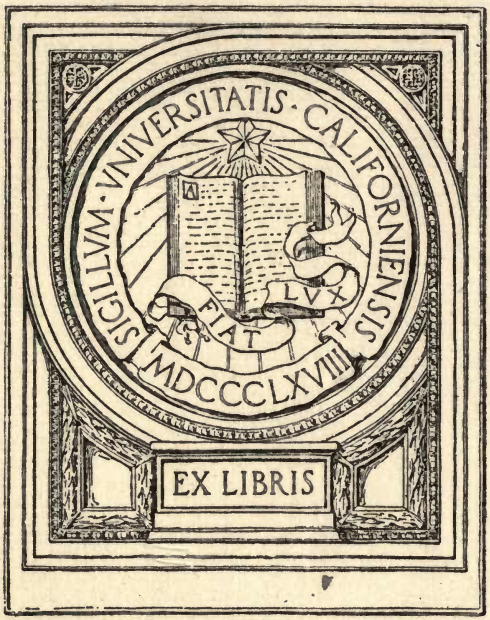

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A TREATISE

ON

THE SUN'S RADIATION

AND

OTHER SOLAR PHENOMENA

IN CONTINUATION OF THE METEOROLOGICAL TREATISE ON

ATMOSPHERIC CIRCULATION AND RADIATION, 1915

BY

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AND IN THE ARGENTINE METEOROLOGICAL OFFICE SINCE 1910

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PREFACE

THERE are two fundamental problems in solar physics that need an immediate solution: (1) the nature of the solar radiation, whether it is black or gray, that is, whether it has full or imperfect efficiency, together with its amount at the sun, and in the different layers of the earth's atmosphere; (2) the physical conditions at which this radiation generates, and the pressures, temperatures, densities, and gas efficiencies that are concerned with the observed phenomena in the solar spectra. It has been found that the system of non-adiabatic meteorology, which was explained in the Author's Treatise on Atmospheric Circulation and Radiation becomes equally applicable in the solar atmospheres by means of a simple transformation. These terms have been computed for several monatomic gases, by the method of trials, and the results are not only instructive in many directions, but they are in complete harmony with the observations made with the spectro-heliograph and the spectro-bolometer. The radiation is black at 5.85 gram calories per square centimeter per minute, when reduced to the equivalent at the distance of the earth; it loses 1.87 calories in the hemispherical non-adiabatic shell of the sun, and is effective on the outside of the earth's atmosphere at 3.98 calories; the course of its depletion is measured by the bolometer to 2.47 calories at the sea level. This conforms with the thermodynamic conditions of the two atmospheres. The pyrheliometer fails to record three important depletions, and it, therefore, cannot account for more than 1.94 calories by the Bouguer formula. The following Treatise is a continuation of the earlier one on Meteorology.

FRANK H. BIGELOW.

SOLAR AND MAGNETIC OBSERVATORY,
PILAR-CORDOBA ARGENTINA.
December, 1917.

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A TREATISE ON THE SUN'S RADIATION AND OTHER SOLAR PHENOMENA

CHAPTER I

The Thermodynamic Processes in the Solar Atmosphere

Introduction

THIS Treatise on Solar Physics is the direct continuation of the author's Meteorological Treatise on the Circulation and Radiation in the Atmospheres of the Earth and the Sun, 1915. In the earlier work the fundamental principles and the formulas were sufficiently explained, so that it is unnecessary to repeat them here. In this research there are several important points essential to the discussion that should be kept in mind by the reader.

(1) In the Boyle-Gay Lussac Law, $P = \rho R T$, all the terms, including the gas efficiency R, are variable. This is the general, non-adiabatic case, which alone is applicable to free atmospheres, the adiabatic case being only occasionally developed. This change of R_a = constant to R = variable is very revolutionary, because it carries with it the so-called constants, k, h, c_1, c_2, in the Wien-Planck spectrum formula, and α, σ, in the general Stefan formula for black radiation.

(2) The transition from terrestrial values to initial solar values of $Pv = R T$ is effected by,

$$\gamma = \frac{G_0}{g_0} = \frac{\text{gravity acceleration at the sun}}{\text{gravity acceleration at the earth}} = 28.028,$$

so that $\gamma P \,.\, \gamma v = \gamma R \,.\, \gamma T$, at one point on the sun.

From this point, by the method of trials with the dynamic and the gravity equations, the complete thermodynamic system has been worked out for several gases, from hydrogen, monatomic, $H_1 = 1.00$ and diatomic, $H_2 = 2.00$, to mercury $Hg = 198$. Each gas develops independently of all the others, and possesses

a low level adiabatic base below the solar photosphere, a middle isothermal region containing the photosphere, and a non-adiabatic region extending to the vanishing planes at different heights, H_2 at 25,000 kilometers above the photosphere, He at 11,000, carbon (assumed monatomic) at 4,000, calcium at 1,000, zinc at 675, cadmium at 380, mercury at 210 kilometers. The sharp disk of the sun is not an optical effect, but the vanishing level of the heavy metallic vapors under the operation of the prevailing thermodynamic conditions. The computed system is in admirable harmony with that which is indicated by the spectro-heliographic observations.

(3) The equation of condition for radiation is derived directly from the first law of thermodynamics, $dQ = dW + dU$, by substituting $dW = P\,dv$,

$$\frac{Q_1 - Q_0}{v_1 - v_0} - P_{10} = \frac{U_1 - U_0}{v_1 - v_0} = K_{10} = c\,T^{\alpha}.$$

The dimension of each term is easily reduced to $\left[\frac{M\,L^2}{T^2} \cdot \frac{1}{L^3} = \frac{M}{L\,T^2}\right]$, the mean kinetic energy per unit volume, and it is so described by Boltzmann, Heaviside, Lorentz, W. Wien, Planck, A. L. Day and C. E. van Ostrand, Richardson, and others generally. When the volume energy is transmitted with the velocity c the result is radiant energy or flux, $\left[\frac{M}{L\,T^2} \cdot \frac{L}{T} = \frac{M}{T^3}\right]$. The conversion factor of radiation-density from (M. K. S.) to (C. G. S.) is 10; that of the radiant-flux is 1,000. Applying to $a = \frac{6\,c_1\,\alpha}{c_2^4}$, the dimensions give:

$$\frac{\text{kilog.}}{\text{m. sec.}^2 \text{ deg.}^4} = \frac{\text{Joule}}{\text{m.}^3 \text{ deg.}^4}, \text{ (M. K. S.)}; \quad \frac{\text{gr.}}{\text{cm. sec.}^2 \text{ deg.}^4} = \frac{\text{erg}}{\text{cm.}^3 \text{ deg.}^4}, \text{ (C. G. S.)}$$

Applying to $\sigma = a.\frac{c}{4}$, the dimensions give:

$$\frac{\text{kilog.}}{\text{sec.}^3 \text{ deg.}^4} = \frac{\text{Joule}}{\text{m.}^2 \text{ sec. deg.}^4}, \text{ (M. K. S.)}; \quad \frac{\text{gr.}}{\text{sec.}^3 \text{ deg.}^4} = \frac{\text{erg.}}{\text{cm.}^2 \text{ sec. deg.}^4}, \text{ (C. G. S.)}$$

The earlier treatise was concerned exclusively with the volume density equations of ponderable gases, and not with the flux equation. This treatise passes from the volume density of the kinetic energy of gases to the electromagnetic flux of radiation in the pure æther, and to its source near the bottom of the isothermal layers in the solar envelope. In this process the Poynting Equation for surface-flux and volume density is employed.

(4) In computing the Boltzmann entropy coefficient, $k = \frac{K}{N} = \frac{mR}{N}$, where N = the number of H-atoms per unit mass, it is evident that while in the adiabatic system k, K, N, are all constants, k, K, are certainly variable in the non-adiabatic system. If H is the inner molecular kinetic energy per unit volume, $H = \frac{3}{2}P = U$, in monatomic gases, and E_0 is the intrinsic kinetic energy of each molecule, assumed in the usual kinetic theory of gases to be a universal constant, we have for the number of molecules per unit volume, $n = \frac{H}{E_0} = \frac{3P}{2E_0}$, and thence $N = n\frac{m}{\rho} = \frac{1}{m_H}$, where m_H is the mass of the standard hydrogen molecule. It appears from the computations that N does not remain constant, but diminishes with the height, especially in the very rarefied high-level strata. We, therefore, encounter a dilemma, because either N or E_0 is variable, and they may both be variables. If it is thought that $N = \frac{1}{m_H}$ is necessarily constant, it assumes a certain view regarding mass, which is not yet settled in the analytic discussions. If E_0 is made a variable it modifies the foundations of the kinetic theory of gases. Finally, if E_0 is a variable, diminishing with the height, the law is unknown, and k, h, c_1, c_2, become different variables from those herewith computed in the first case. We have, accordingly, retained E_0 = constant and allowed N to become variable, in the preliminary discussion of gaseous radiation.

The system has, also, been given for N = constant and E_0 = variable.

(5) The computations have led to the following important result regarding the value of the solar constant of radiation. In each gas, at its special depth below the photosphere, there is a sudden change in the value of all the variable coefficients entering into the process of radiation. The mean temperature in these layers of radiation is 7655° A. absolute; the mean logarithmic value of σ in the Stefan law, $J_0 = \sigma T^{\alpha}$ is $\log \sigma = -5.74000$. and the exponent $\alpha = 4.00$; so that J_0 at the distance of the earth is equivalent to 5.854 $\frac{\text{gr. cal.}}{\text{cm.}^2 \text{ min.}}$. This agrees with the predicted value, Fig. 54, preceding volume; the terrestrial thermodynamics established the effective radiation reaching the earth as 3.980 calories, equivalent to about 6950° A.; the observed depletion by the bolometer continues from station to station down to Washington, 2.47 calories. The scattering process is apparently equivalent to 1.87 calories in the solar atmosphere, 0.76 calorie in the high levels of the earth's atmosphere, and 0.25 to 0.54 calorie in the low levels; the absorption process in the earth's atmosphere increases to about 0.20 calorie; the free heat at sea level is 1.50 calories by the pyrheliometer. *Abbot's extrapolated value of the solar constant, 1.95 calories, is about one-third of the true solar constant.* He has, therefore, made an error in supposing that the sun, though having high temperatures, radiates with very low efficiency. On the contrary, the data of the computations on solar thermodynamics, and those on the terrestrial thermodynamics, both agree with the results of the bolometer in making the solar radiation of full efficiency.

(6) More specifically, the process of depletion of the solar intensity of radiation from 5.85 gr. cal./cm.^{2} min. to 1.30–1.50 calories at the sea level of the earth's atmosphere proceeds by the following steps.

a. Temperature of origin for all the gases = 7655° absolute A. in a deep isothermal layer.

b. Exponent in the Stefan Law of black radiation = 4.00.

c. Equivalent intensity at the mean distance of the earth = 5.85 calories.

d. The depletion in the hemispherical shell of the sun's atmosphere, including the isothermal and the non-adiabatic layers, is measured by the decrease in brightness of the solar disk between the center and the limb. Abbot's Table 55, Vol. III., Ann. S. I., gives the bolometer intensities for several spectrum lines, in parts of the intensity at the center, for certain points of the radial distance. Factors are computed for each line showing its contribution when referred to an undepleted intensity from the center to the limb. The following summary gives the ordinates computed for black radiation at the temperatures 7655°, 6950°, 5810°, 5456°, and Abbot's observed ordinates as extrapolated by him to the outside of the earth's atmosphere, marked Total, at Mt. Whitney, Mt. Wilson, and Washington. The corresponding spectra are in relative numbers, but they can be reduced to calories by the general factor 20.9:

COURSE OF DEPLETION OF THE SOLAR RADIATION DOWN TO THE SEA LEVEL

Wave Lengths	Solar Thermodynamics				Bolometer Observations				Pyrheliometer	
	T 7655° Black	Factor of Depletion	Effective of Sun to Earth	*T* 6950° Black	Total as Extrapolated	Mt. Whitney	Mt. Wilson	Washington	Total as Extrapolated 5810°	Sea Level Generally 5450°
0.323μ	9.790	0.580	5.678	5.406	1.500	0.828	0.764	....	1.578	0.932
.386	10.552	.589	6.215	6.348	3.690	2.959	2.733	1.728	2.218	1.453
.433	10.294	.633	6.516	6.514	5.472	4.413	4.265	3.370	2.586	1.790
.456	9.884	.661	6.533	6.454	6.051	4.992	4.883	3.918	2.618	1.824
.481	9.411	.679	6.390	6.268	6.056	5.254	5.063	4.118	2.656	1.885
.501	9.031	.690	6.231	6.116	6.052	5.445	5.195	4.268	2.684	1.931
.534	8.337	.709	5.911	5.946	5.768	5.267	5.017	4.202	2.648	1.942
.604	6.893	.738	5.091	4.959	4.994	4.670	4.454	3.833	2.480	1.884
.670	5.721	.762	4.359	4.217	4.070	3.856	3.758	3.294	2.229	1.732
.699	5.188	.770	3.995	3.664	3.664	3.495	3.444	3.063	2.116	1.663
.866	3.191	.806	2.572	2.511	2.403	2.471	2.451	2.208	1.602	1.302
1.031	1.994	.830	1.655	1.603	1.536	1.748	1.744	1.574	1.162	0.970
1.225	1.199	.845	1.013	0.986	0.985	1.065	1.055	1.000	0.732	.626
1.655	0.459	.884	0.406	.389	.466	0.456	0.410	0.440	.256	.226
2.097	.204	.899	.183	.176	.211	.201	.191	.181	.130	.117
Calories	5.85	....	3.98	3.98	3.22	2.96	2.87	2.47	1.94	1.50

The solar radiation arrives at the earth as black and of an equivalent temperature 6950°.

e. The terrestrial thermodynamics reproduces the same coefficient and exponent in the Stefan Law as correspond with black body radiation at 6950° on the levels 60000 to 65000 meters.

f. The summary of the several thermodynamic terms in the earth's atmosphere, from the sea level to the vanishing plane, derived directly from the prevailing temperatures, which is the precise equivalent of the effective radiation, whatever may be the details of the paths of the radiant energy, gives the following results:

1. Summation of the free heat.....	4.08	gr. cal./cm²	min.
2. Summation of the hydrostatic pressure.....	4.08	"	"
3. Summation of the inner energy and work.....	4.08	"	"
4. Summation of the black radiation...	3.94	"	"
5. The free heat in the stratum.....	3.92	"	"
6. The hydrostatic pressure in the stratum.....	4.00	"	"
7. The inner energy in the stratum....	3.92	"	"
8. The external work and gas efficiency.	4.02	"	"
9. The kinetic and potential energies with the absorbed radiation.....	3.90	"	"

g. The energy of the solar radiation in the atmosphere separates itself into the following terms:

I. The kinetic energy for temperature effects.

II. The potential energy = 0.641 kinetic energy.

III. The specific heat change from constant volume to constant pressure, or from the inner energy of the Boyle-Gay Lussac Law to the external energy due to the impressed gravitation.

IV. The energy lost by non-selective scattering.

V. The energy lost by selective absorption.

VI. The energy transformed into ionization products, free electric charges, and magnetic fields.

h. The effective intensity 3.98 calories minus (III + IV + V + VI) gives the bolometer ordinates approximately. The further subtraction of the potential energy II gives the data observed by the pyrheliometer in bulk, as the kinetic energy which produces its temperature. Hence, the proper reduction of the pyrheliometer observations follows:

H = the kinetic energy, observed by the pyrheliometer in the zenith.

J = the potential energy $= 0.641\,H$.

$k\,(H + J) = (100 - p)\;(H + J)$ = the proportional part lost by scattering.

$\Delta\,U_e = 0.012\;(e - e_0)$, a small correction for vapor pressure.

a = the nearly constant line and band absorption in the spectrum.

R = the nearly constant specific heat term at the station.

S = the effective solar constant outside the earth's atmosphere.

$S = (H + J) + k\;(H + J) + \Delta\,U_e + (a + R) =$ 3.98 calories.

These reductions of the pyrheliometer observations agree at many stations in producing 3.98 calories. This refers to La Confianza (4485 m.), Mt. Whitney (4420), La Quiaca (3465), Mt. Wilson (1780), Bassour (1160), Mt. Weather (526), Cordoba (438), Pilar (340), Washington, D. C. (34). The Bouguer formula of depletion is used to compute p and $I_{10} = H$ in the zenith, but it does not employ the objectionable Langley-Abbot method of extrapolation beyond its natural limits. Each station produces 3.98 calories with variations on its own level.

(7) The most interesting and striking result is that the solar radiation originates in a *per saltum* process, resembling a radioactive discharge, due to a readjustment of the electrons and to thermal collisions. In the case of hydrogen it was necessary to treat it as a monatomic element, $H_1 = 1.00$, below the radiation level, but as a diatomic element, $H_2 = 2.00$, above that level, in order to terminate on the observed vanishing plane, namely,

the top of the inner corona at 25000 kilometers. This is direct evidence of dissociation below, and association above the plane where the radiation originates. That plane is just above the true adiabatic strata where the isothermal layer begins, and at the levels where the temperature, the pressure, the density, and the gas coefficient, all suffer very quick changes of gradient. The factor of change in the coefficients of radiation is of the order 10^{-6}, dropping from a very high tension to a very low tension in a short vertical distance. The effect of this phenomenon, as well as the general variability of the coefficients of radiation, upon the theories of radiation now under discussion, cannot possibly be indicated. It has been our purpose to bring forward the main thermodynamic conditions at the sun, under which the radiation originates, and no attempt is made here to interpret them fully.

(8) The sun's atmosphere consists of a gaseous envelope containing a spherical isothermal shell, at the mean temperature of about 7687° C. This phenomenon serves to explain several of the common solar problems. Above the isothermal layer the temperatures decrease rapidly to 0° C., and below it the temperatures increase to very great values. Hence, low-temperature spectra are projected upon a uniform continuous spectrum at a definite temperature of 7687° for a background; the flash spectrum, at 500 kilometers, develops for the light gases above the heavy metallic strata of the photosphere; the granulation is an optical effect of segregation within the isothermal layer; the sun spots have two independent parts: (1) The lower on the under side of the isothermal layer, generating a convectional vortex like the terrestrial hurricanes, and forming the dark umbra; (2) the upper, located above the isothermal layer, and falling into the umbral vortex, as into the depression at the center of a hurricane, thus producing the penumbra; the faculæ, flocculi, and prominences, are convectional effects of rising gases originally at very high pressures and temperatures, discharging through the isothermal layer, and seeking a new equilibrium above it; the general circulation of the sun must conform to the continuous production of this isothermal layer at every point of

the photosphere; the radioactive discharge within the isothermal layer sets free vast numbers of electrons, which make up the surface electric charge of the sun, with coronal dispersion to space under the pressure of solar radiation; the movements of free electrons produce the Zeeman effect in the sun-spot vortices; they cause the coronal polar magnetic fields, and the general polarization throughout the sun.

Historical Remarks

The problems of the physical constitution and the processes in the Sun have engaged the attention of scientists for three centuries. Since the invention of the telescope the statistical data regarding the sun-spots, and the solar rotation in different latitudes, have been studied; with the application of the spectroscope the data regarding the prominences, or hydrogen flames on the disk, have been classified, since 1871; with the development of the spectroheliograph the constitution of the visible spots, faculæ and flocculi, has been closely analyzed, and the pressures in certain levels of a few gases have been approximately determined; with the discovery of the ionization of molecules, and the existence of free electrons, together with the radioactive processes in atoms and molecules, and the structural arrangement of the positive nucleus with the circulating negative electrons in the atoms, including their abrupt redistribution by the association of atoms into molecules, or the dissociation of molecules into atoms, under certain pressures, densities, thermal efficiencies, the physical field has been enlarged to an extraordinary degree; the attack upon the thermal spectrum by means of the bolometer, and the measurement of the amount of radiation received at stations in the lower atmosphere of the earth by the pyrheliometer, have afforded much information regarding the coefficients of emission and absorption of the solar radiant energy in the earth's atmosphere; some progress has been made in regard to the synchronism of the solar variations as observed and the corresponding effects upon the meteorological or climatic conditions prevailing as tem-

peratures, pressures, vapor pressures, circulation, electric and magnetic variations in different terrestrial latitudes and longitudes.

In spite of the extensive literature regarding the material just mentioned, it must be admitted that the scientific knowledge available is really superficial, because there has been no thoroughly comprehensive analysis of the many physical conditions which must exist in order to produce the very complex phenomena that are observed on the sun. All the above methods depend upon visual phenomena, but these are of limited scope in respect of the fundamental conditions in the solar gases which produce them. It has seemed to the writer that thermodynamics constitutes the royal road between the earth and the sun, and that the system of equations which can reproduce the general conditions in the earth's atmosphere should be able to do the same in the sun's atmospheres. With this purpose in view, the terrestrial dynamic and thermodynamic relations have been studied, in order to develop such a system of equations as is effective and at the same time simple enough for practical computations. In certain papers published in the U. S. Monthly Weather Review, 1905, 1906, and in Bulletins No. 3, No. 4, of the Argentine Meteorological Office, 1912, 1914, and in my Meteorological Treatise, 1915, such equations and their applications have been explained for the earth's atmosphere. The present treatise contains their modification so that they become equally valid in the sun's envelope, thereby proving that this thermodynamic system is of universal application, as to any star where the surface gravitation is known from the astronomical masses of binary stellar systems. While the labor of developing this method of computation has been very considerable, it has now become relatively simple in consequence of the discovery of certain laws which regulate the distribution of the fundamental quantities.

Derivation of the Non-adiabatic System of Equations

The change from the adiabatic to the non-adiabatic system of equations consists in making the gas coefficient of thermal

efficiency R a variable in the Boyle-Gay Lussac Law, $P = \rho R T$. In the adiabatic system Ra = constant for each gas, and it is generally appropriate to the conditions of laboratory practise, but these do not conform to those prevailing in free atmospheres. If the conditions in a free atmosphere were adiabatic, then the temperature gradients would be at the fixed rate, $-\frac{T_a - T_0}{z_1 - z_0} = \frac{g}{Cp_a}$; where Cp_a is the specific heat at a constant pressure, $Cp_a = R_a \frac{\kappa}{\kappa - 1}$, and $\kappa = Cp_a/Cv_a$, the ratio of the two specific heats. But, as a matter of fact, the observed temperature gradients are different, such that $-\frac{T_1 - T_0}{z_1 - z_0} = \frac{g}{Cp}$, so that $Cp = R \frac{\kappa}{\kappa - 1}$, and R must be a variable coefficient. The most compact way to arrive at the general equation of condition, Treatise (27), is as follows:

Take all the terms variable in $Pv = R T$, and integrate between two levels $z_1 \,.\, z_0$, within which stratum the temperature gradient remains constant.

(1) $$P\,dv + v\,dP = R\,dT + T\,dR,$$

(2) $P_{10}(v_1 - v_0) + v_{10}(P_1 - P_0) = R_{10}(T_1 - T_0) + T_{10}(R_1 - R_0)$,

We have the term for the external work,

(3) $P_{10}(v_1 - v_0) = (W_1 - W_0) = R_{10}(T_a - T_0) - v_{10}(P_1 - P_0)$, (333), so that by substitution,

$$R_{10}(T_a - T_0) = R_{10}(T_1 - T_0) + T_{10}(R_1 - R_0).$$

The equations (2), (3) give the relations between the work, the hydrostatic pressure per unit density, and the gas efficiency in the stratum, but the gravitation acceleration $g(z_1 - z_0)$ does not yet appear in the equation. In order to introduce any external impressed forces, such as gravitation or circulation, we proceed as follows, by adding and subtracting $Cp_a(T_a - T_0) = -g(z_1 - z_0)$. Hence,

(4) $$(W_1 - W_0) + \frac{P_1 - P_0}{\rho_{10}} = -g(z_1 - z_0) - (Cp_a - R_{10})(T_a - T_0).$$

We employ the following forms in the first equation of thermodynamics, Treatise, pp. 81, 82,

(5) $(Q_1 - Q_0) = (W_1 - W_0) + (U_1 - U_0)$,

(6) $(Q_1 - Q_0) = (Cp_a - Cp_{10})(T_a - T_0)$,

(7) $(W_1 - W_0) = (R_{10} - Cp_{10})(T_a - T_0)$,

(8) $(U_1 - U_0) = (Cp_a - R_{10})(T_a - T_0) = Cv_{10}(T_a - T_0)$,

so that by substitution,

$$(9) \quad + g(z_1 - z_0) = -\frac{P_1 - P_0}{\rho_{10}} - (W_1 - W_0) - (U_1 - U_0),$$

$$(10) \quad + g(z_1 - z_0) = -\frac{P_1 - P_0}{\rho_{10}} - (Q_1 - Q_0).$$

If the kinetic energy of circulation $\frac{1}{2}(q_1^2 - q_0^2)$ is obtained by observing the velocities $(q_1 \,.\, q_0)$ on the levels $(z_1 \,.\, z_0)$ respectively, a secondary value of $-(Q_1 - Q_0) = -(Q_1 - Q_0)^1 - \frac{1}{2}(q_1^2 - q_0^2)$ can be introduced, so that,

$$(11) \quad g(z_1 - z_0) = -\frac{P_1 - P_0}{\rho_{10}} - \tfrac{1}{2}(q_1^2 - q_0^2) - (Q_1 - Q_0)^1,$$

and this is the equation of condition.

In the terrestrial atmosphere, some account of the circulation can be obtained from high-level observations. In the sun's atmosphere, while circulation has been somewhat noted, its conditions as to level are so uncertain that as yet no attempt has been made to incorporate it into this Treatise, although it can be done with further experience.

We may note that the following potentials have been developed:

V for $g(z_1 - z_0)$, the external potential energy,

K for $\frac{1}{2} m (q_1^2 - q_0^2)$, the external kinetic energy of circulation.

$\mathbf{J}_a$ for $(U_1 - U_0)_a$, the internal potential energy of atoms,

$\mathbf{H}_m$ for $(U_1 - U_0)_m$, the internal kinetic energy of molecules.

It appears, therefore, that the total inner energy in the atmosphere $(U_1 - U_0) = (Cp_a - R_{10})(T_a - T_0)$, sustains the external potential energy of gravitation, and a certain thermal efficiency $R_{10}(T_a - T_0)$ which is equal to,

$$(12) \quad + \frac{P_1 - P_0}{\rho_{10}} + (W_1 - W_0) = -g(z_1 - z_0) - (U_1 - U_0) = R_{10}(T_a - T_0),$$

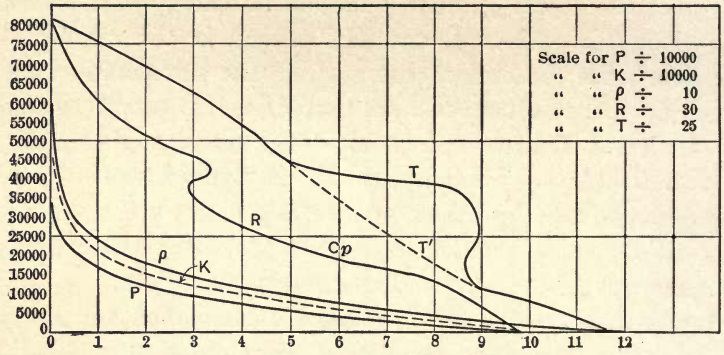
and that these are derived from the state of the inner potential energy of the atoms $\mathbf{J}_a$, and of the kinetic energy of molecular translation $\mathbf{H}_m$. Since $\mathbf{J}_a$ and $\mathbf{H}_m$ depend in the earth's atmosphere upon the radiant energy from the sun that has converged upon the unit volume per unit time, it is proper to compute the terrestrial thermodynamic data, and proceed thence to the radiation data. Similarly, the thermodynamics of the solar atmosphere must afford the corresponding data for the emission of radiation, which, reduced to the distance of the earth, should be equivalent to the terrestrial thermodynamic data.

In the first Treatise on "*Atmospheric Circulation and Radiation*," the terrestrial data have been summarized; in this Treatise on "*Solar Radiation*" the corresponding solar data are briefly given; there still remain certain problems of the electromagnetic radiation and its relations to the electrons, ions, atoms, and molecules, which will require more extended, careful examination. The formulas given above can be illustrated and verified from the Tables 13–27, Bulletin No. 4, O. M. A., 1914.

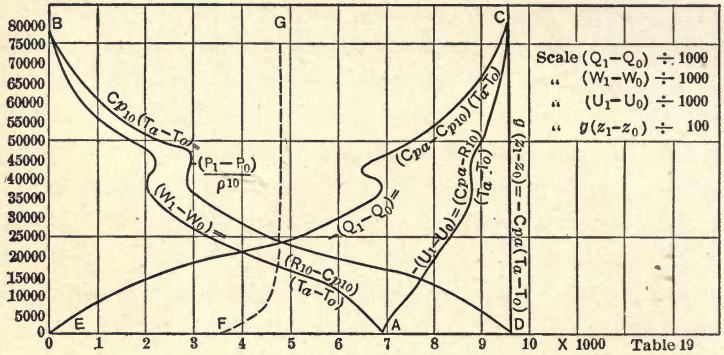
Preliminary Summary of Conditions

Fig. 1 contains a summary of the results computed from the balloon ascension, Uccle, September 13, 1911. The data are reduced to one arbitrary scale by the series of factors indicated, and the lines show the course of development from the sea level to the vanishing plane. In the thermodynamic section it may be noted that the free heat $(Q_1 - Q_0)$ has two equivalent areas, $B\,E\,C = B\,A\,C$. The abscissas give the relative values of each term at the several levels, so that $(Q_1 - Q_0) = 0$ at the sea level, and it is $g\,(z = -z_0)$ on the vanishing plane. The area $A\,C\,D = R_{10}\,(T_a - T_0)$, and it is, also, equal to the area $A\,B\,D$, as given by formula (12). It will be shown how the pyrheliometer at sea level measures a quantity proportional to $E\,F$, while the bolometer measures another quantity proportional to $E\,A$; on the vanishing plane the pyrheliometer would be proportional to $B\,G$ and the bolometer to $B\,C$. We shall be able to indicate the physical significance of the line $G\,F$, and the line

SECTION I. DYNAMIC DATA FOR $P = \rho RT$, (M. K. S.)



SECTION II. THERMODYNAMIC DATA.



SECTION III. KINETIC THEORY DATA.

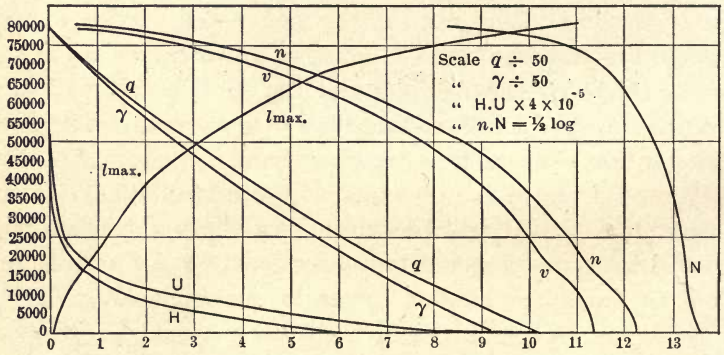


FIG. 1. Summary of the Data for the Atmosphere.
Balloon Ascension, Uccle, September 13, 1911.

$C\,A$, together with their relation to the incoming solar radiation $B\,C$.

Summary of the Data on Section II

Gravity acceleration, $g\,(z_1 - z_0)$... $= D\,E\,B\,C$,
Hydrostatic pressure, $-\frac{P_1 - P_0}{\rho_{10}}$.. $= D\,E\,B$,
Work of expansion, $(W_1 - W_0)$... $= A\,B\,E = A\,C\,E$,
Total inner energy, $(U_1 - U_0)$.... $= A\,E\,B\,C$,
Free heat, $(Q_1 - Q_0)$..... $= E\,B\,C = A\,B\,C = D\,B\,C$,
Gas efficiency, $R_{10}\,(T_a - T_0)$..... $= A\,C\,D = A\,B\,D$,
Pyrheliometer radiation, equivalent to..... $E\,F\,G\,B$,
Bolometer radiation, equivalent to $E\,A\,C\,B$,
Kinetic energy of the Poynting flux $= E\,F\,G\,B$,
Potential energy of the Poynting flux..... $= F\,A\,C\,G$,
Dissipated energy of the Poynting flux..... $= A\,D\,C$,
Kinetic energy of solar radiation.. $= B\,G$,
Potential energy of solar radiation $= G\,C$.

By keeping these conditions in mind the reader will more readily follow the argument of the following pages.

The Working Equations

There are only two important steps which are required in order to transform the adiabatic system of equations into the non-adiabatic system actually applicable at the sun. The first of these is the change of R_a = constant into R = variable in the Boyle-Gay Lussac Law, $P = \rho\,R\,T$; and the second is the multiplication of all the terms in $Pv = R\,T$ by $\gamma = G_0/g_0$, the ratio of the acceleration of gravitation at the sun relatively to that at the earth, so that,

$$(13) \qquad \gamma\,P\,.\,\gamma\,v = \gamma\,R\,.\,\gamma\,T \text{ or } \gamma\,P = \frac{\rho}{\gamma}\,.\,\gamma\,R\,.\,\gamma\,T.$$

If R_a = constant the result is that the temperature gradient

is $\frac{T_a - T_0}{z_1 - z_0} = -\frac{g_0}{Cp_a}$ on the earth, and $\frac{T_a - T_0}{z_1 - z_0} = -\frac{G_0}{Cp_a}$ on the sun, these being fixed gradients for each gas. They exist in the lower levels of the earth's atmosphere in the tropics, and in certain strata below the photosphere of the sun, but above these strata there are in every atmosphere isothermal strata, and above these non-adiabatic strata, where no such temperature gradients can occur. For this reason the adiabatic formulas, which have been universally employed, that is, wherever R_a has been treated as a constant, are inapplicable in two deep strata of every atmosphere. In order to pass from the adiabatic to the non-adiabatic conditions, it is only necessary to put the factor $n = \frac{T_a - T_0}{T_1 - T_0} = \frac{\text{adiabatic gradient}}{\text{observed gradient}}$, into the coefficients of the formulas.

Adiabatic and Non-adiabatic

(14) Pressure. $\log P_1 - \log P_0 = \frac{\kappa}{\kappa - 1}(\log T_1 - \log T_0)$;

$$\log P_1 - \log P_0 = \frac{n\kappa}{\kappa - 1}(\log T_1 - \log T_0).$$

(15) Density. $\log \rho_1 - \log \rho_0 = \frac{1}{\kappa - 1}(\log T_1 - \log T_0)$;

$$\log \rho_1 - \log \rho_0 = \frac{n}{\kappa - 1}(\log T_1 - \log T_0).$$

(16) Thermal Efficiency. $\log R_1 - \log R_0 = 0$;

$$\log R_1 - \log R_0 = (n-1)(\log T_1 - \log T_0).$$

The importance of this step is seen in the fact that the entire thermodynamic and radiation systems, instead of depending upon a set of universal constants, become involved in a series of variable coefficients. The constants apply only within the adiabatic strata, while the variable coefficients are necessary in every non-adiabatic stratum, that is, wherever the natural temperature gradients differ from the fixed adiabatic rate. It is this fact, namely, the adoption of the usual laboratory adiabatic equations for free atmospheres, that has prevented meteorology from entering the region of the exact sciences. Change

R_a from a constant into a variable, and the physics of atmospheres becomes a perfectly simple branch of thermodynamics.

Examining the dimensions of the terms in the Boyle-Gay Lussac Law, we have for the absolute temperature,

$$(17) \qquad T = \frac{P}{\rho R} = \frac{M}{L T^2} \cdot \frac{L^3}{M} \cdot \frac{T^2}{L^2} \text{ degree} = 1 \text{ degree},$$

so that the absolute temperature in degrees has no dimensions, being only the ratio between the pressure and the product of the density with the thermal efficiency. The last is merely the velocity of the molecules per degree, which pertain to the number in the mass ρ when producing a pressure P. It is evident that in the laboratory conditions, the gas enclosed in a vessel of impermeable walls is very different from the same gas in the lower strata of a free atmosphere, which absorbs or emits radiant energy to space. As a matter of fact, the R begins at full adiabatic value in some low-lying strata, and thence diminishes to 0 on the vanishing plane of the gas. If R is variable, so are all the following quantities also variables.

(18) Specific heats, $Cp = \frac{\kappa}{\kappa - 1} R$, $C_v = \frac{1}{\kappa - 1} R$.

Thermal efficiency, $K = m R$, where m = the molecular weight.

Number of molecules per unit mass, $N = m v n = \frac{K}{k}$.

Number of molecules per unit volume, $n = N/mv = \frac{P N}{K T} = \frac{H}{E_0}$.

Boltzmann's entropy coefficient, $k = K/N$.

Planck's Wirkungsquantum, $h = \frac{k^{4/3}}{c}\left(\frac{48 \pi \alpha}{a}\right)^{1/3}$, where

c = the velocity of light, $\alpha = 1.0823$ (Planck),

a = variable.

Wien-Planck coefficients, $c_1 = 8 \pi c h$, $c_2 = c h/k$.

Stefan coefficients, $a = \frac{6 \alpha c_1}{{c_1}^2}$,

$\sigma = \frac{a c}{4}$ in $J_0 = \sigma T^4$.

The thermodynamic equation for the conservation of energy,

(19) $$T\,dS = dQ = dW + dU = P\,dV + dU,$$

is limited by Planck as follows, in certain analyses.

Adiabatic conditions: $dQ = 0 = P\,dV + dU$, so that the work of expansion exactly compensates for the change in the inner energy, and this is correct in the lower adiabatic strata.

The entropy equation, developed for $dV = 0$, so that $T\,dS = dQ = dU$, and the free heat is equal to the inner energy. This is true outside the vanishing plane of the gaseous atmosphere, for the radiation in the æther, but it is not correct within the non-adiabatic regions where $dW = P\,dV$ has real values. The two fundamental developments follow:

Bigelow *Planck*

(20) $$\frac{dS}{dV} = \frac{P}{T} + \frac{1}{T}\frac{dU}{dV}.$$ (21) $$\left(\frac{dS}{dV}\right)_U = \frac{P}{T} = \frac{kN}{V} = \frac{K}{V}.$$

(22) $$\frac{dS}{dU} = \frac{P}{T}\frac{dV}{dU} + \frac{1}{T}.$$ (23) $$\left(\frac{dS}{dU}\right)_V = \frac{1}{T} = \frac{3}{2}\frac{kN}{(U_1 - U_0)} = \frac{3}{2}\frac{K}{(U_1 - U_0)} = \frac{K}{PV}.$$

(24) $$\frac{dU}{dV} = T\frac{dS}{dV} - P.$$ (25) $$\left(\frac{dU}{dV}\right)_S = -P.$$

It is evident that within the non-adiabatic strata, including the isothermal layer, it is not allowable to suppress first one term and then another, as dQ and dV. Hence, the following non-adiabatic monatomic thermodynamic formulas must prevail:

(26) Pressure. $$P = \rho RT = knT = k\frac{NT}{V} = \frac{KT}{V} = \frac{n}{N}KT$$

$$= RTmn m_H = KT\frac{n}{N} = \frac{3KT}{Vu}p. \quad \left[\frac{M}{LT^2}\right]$$

(27) Kinetic Energy. $$PV = KT = mRT = knVT = kNT$$

$$= \frac{n}{N}KTV = RTm m_H N. \quad \left[\frac{L^2}{T^2}\right].$$

(28) Inner Energy. $(U_1 - U_0) = H\,V = \frac{3}{2} P\,V = \frac{P}{2p} U =$

$\frac{3}{2} K\,T = \frac{3}{2} P\,m\,v = \frac{3}{2} k\,N\,T = C_v\,m\,T = C_v\,\rho\,T\,V.$ $\left[\frac{L^2}{T^2}\right]$.

(29) Specific Heat. $Cv = \frac{U_1 - U_0}{m\,T} = \frac{U_1 - U_0}{\rho\,T\,V} = \frac{3}{2}\frac{P\,V}{m\,T} =$

$\frac{3}{2}\frac{K}{m} = \frac{3}{2} R = \frac{3}{2}\frac{P}{\rho\,T} = \frac{3}{2}\frac{k\,N}{m} = \frac{H}{\rho\,T}.$ $\left[\frac{L^2}{T^2\,\text{deg.}}\right]$.

(30) Specific Heat. $Cp = \frac{5}{2} R = \frac{5}{2}\frac{K}{m} = \frac{5}{2} k\frac{N}{m}.$ $Cp - Cv = R.$

$\left[\frac{L^2}{T^2\,\text{deg.}}\right]$.

(31) Volume. $V = m\,v = \frac{m}{\rho} = \frac{N}{n}.$ $\left[\frac{L^3}{M}\right]$.

Energy, $U = V\,u = V\,a\,T^4 = 3\,p\,V.$ $\left[\frac{L^2}{T^2}\right]$.

(32) Work. $W = p\,d\,V.$ $\left[\frac{L^2}{T^2}\right]$.

(33) Entropy. $S = Vs = \frac{4}{3} a\,T^3\,V = \frac{4\,p\,V}{T}.$ $\left[\frac{L^2}{T^2\,\text{deg.}}\right]$.

All these formulas are verified throughout the series of non-adiabatic computations included in this Treatise.

The Initial Constants and Coefficients for Solar Atmospheric Computations

In order to prepare the necessary constants and coefficients which are preliminary to the computations implied in the preceding formulas, and applicable in the gaseous atmospheres of the chemical elements on the sun, the constants which are accepted as standard under the earth's gravitation were transferred to the sun's gravitation by the factor $\gamma = G/g_0 = 274.843/9.806 = 28.028$, as indicated in the four sections of Tables 1, 2. The formulas are given for the several quantities, and these begin with $P = 101323.5$ kilograms per square meter on the

earth, which is one atmosphere under standard conditions; multiply by γ and $P = 2839900$ kilograms per square meter on some solar level of equilibrium, not necessarily the plane of the photosphere, where $P = 6$ atmospheres. The level of 28.028 atmospheres occurs at very different distances below that plane, namely, $-$ 5000 kilometers below for hydrogen (diatomic), $-$ 2500 below for helium, $-$ 420 for carbon (assumed monatomic), $-$ 250 for calcium, $-$ 150 for zinc, $-$ 90 for cadmium, and $-$ 50 for mercury. These differences all depend upon the atomic and the molecular weights. In order to avoid, as far as it is possible to do so, the complicated questions regarding the values of the specific heats Cp, Cv, and their ratio $\kappa = Cp/Cv$ at solar temperatures, the monatomic elements were selected for these computations, wherein $\kappa = 1.666+$ under all temperatures, since the atoms are also molecules. There are two exceptions in this selection. It was found *necessary to treat hydrogen as monatomic H_1 below a certain level, but as diatomic H_2 above* that level, as will be fully explained in connection with the subject of the origin of the solar radiation. For convenience, carbon at $m = 12.00$ was taken as if it were a monatomic element, not only below its plane of transition but above it as well. It is probably monatomic in the adiabatic levels, but not monatomic above its reference plane. However, it is difficult to assign its molecular composition and its specific heats at the high solar temperatures, but it fills a rather wide gap in the series of curves to take carbon as if it were a monatomic element at $m = 12.00$. As our purpose is to discover the primary laws of the distribution of the chemical elements near the surface of the sun, there is no reason why a monatomic element should not be assumed at $m = 12.00$. If the actual heavy molecule of C can be assigned, and its vanishing height above the photosphere established, that part of the computation can be extended to meet these general facts. Similar remarks are applicable to any other chemical element on the sun.

It will be seen from Table 1, section I, that the factor $\gamma = 28.028$ serves to transform the following constants from terrestrial to solar values: pressure P, molecular volume V, thermal

efficiency K, temperature T, molecular kinetic energy per degree $\epsilon = E_0/T$, Boltzmann's entropy constant k, translational kinetic energy of the molecules H, total inner energy U, gravity acceleration G; the factor γ^2 is required for the kinetic energy $P\,V = K\,T$, the kinetic energy of one molecule E_0; the factor γ^{-1} is for the number of molecules per unit volume n; the factor $\gamma^{1/3}$ for the Wien-Planck constants in the energy spectrum c_1, c_2; the factor $\gamma^{4/3}$ for Planck's Wirkungsquantum; and the following require no factor whatever, mechanical equivalent of heat A, the specific heat ratios κ, $\kappa/(\kappa - 1)$, $1/(\kappa - 1)$, the number of molecules per unit mass N, the molecular weight of hydrogen $m_H = \frac{1}{N}$, and the velocity of light c. In adiabatic diatomic conditions for hydrogen the values of the specific heat ratios are given in section II. The adiabatic gradient $\frac{T_a - T_0}{z_1 - z_0} = -\frac{G}{Cp}$ is recorded in section III for eight gases, and they are utilized in the respective computations.

Section IV contains the important thermal coefficients for the same list of gases, which are dependent upon the molecular weight, and are therefore variables from one gas to another.

These are the density $\rho = m\,n\,m_H$, the volume $v = \frac{1}{\rho}$, the thermal efficiency $R = \frac{K}{m}$, the specific heat at constant pressure $Cp = \frac{\kappa}{\kappa - 1} R$, and at constant volume $Cv = \frac{1}{\kappa - 1} R$. The factor γ is required for v, R, Cp, Cv, and γ^{-1} for ρ. The specific values are taken as they are more convenient in the computations. The chemical elements were chosen with m ranging from $H_1 = 1.00$ to $Hg = 198.41$, sufficient in number to bring out clearly the relations which prevail in the families of curves developed with the parameter m, all the other elements being easily interpolated into their proper places. These elementary constants and coefficients apply to certain non-adiabatic conditions of equilibrium, *and they locate one point for each term of the gas, dynamic or thermodynamic, at some unknown level in the solar atmosphere.* There is such a point of equilibrium fulfilling the

TABLE 1

I. CONSTANTS OF GASES AT THE EARTH AND THE SUN IN MONATOMIC AND ADIABATIC CONDITIONS (M. K. S.) SYSTEM OF UNITS

Quantity	Formulas	$\frac{G}{g_0}$	EARTH (M.K.S.)		SUN (M.K.S.)		To C.G.S.
			Number	Log	Number	Log	
Pressure	$P = \rho R T = g_0 \rho_m B_n = k n T = \frac{k N T}{V} = \frac{K T}{V} = K T \frac{n}{N} = R T m n m_H$	γ	101323.5	5.00571	2839900	6.45330	10
Molec. Volume	$V = m v = m/\rho$	γ	22.2235	1.34681	622.871	2.79440	10^3
Thermal Efficiency	$K = m_0 R_0 = m R = k N = \frac{k}{m_H} = \frac{P m}{\rho T}$	γ	8248.2	3.91636	231177	5.36395	10^4
Temperature	T	γ	273	2.43616	7651.6	3.88375	..
Kinetic Energy	$P V = K T = \frac{1}{3} N m u^2 = \frac{2}{3} N E_0$	γ^2	2251740	6.35252	1768880000	9.24770	10^4
Mech. Equiv. Heat	A	..	4185.1	3.62171	4185.1	3.62171	10^4
Therm. Eff. Calories	$K_A = K/A$	γ	1.9708	0.29465	55.2386	1.74224	...
Ratios of the Specific Heats, Cp, Cv	κ (Monatomic gases)	..	1.666+	0.22185	1.666+	0.22185	...
	$\kappa/(\kappa-1)$ " "	..	2.500	0.39794	2.500	0.39794	...
	$1/(\kappa-1)$ " "	..	1.500	0.17609	1.500	0.17609	...
Number Molecules in Unit Mass	$N = V n = m v n = \frac{K}{k} = \frac{1}{m_H}$	..	6.062×10^{26}	26.78262	6.062×10^{26}	26.78262	10^{-3}
in Unit Volume	$n = \frac{N}{m v} = \frac{P N}{K T} = \frac{H}{E_0}$	γ^{-1}	2.7277×10^{25}	25.43581	9.7324×10^{23}	23.98822	10^{-6}
M. K. E. one mol	$E_0 = \frac{2}{3} \frac{P}{n} = \frac{1}{2} m u^2$	γ^2	5.5718×10^{-21}	-21.74599	4.3769×10^{-18}	-18.64117	10^7
Energy per degree	$\epsilon = \frac{E_0}{T}$	γ	2.041×10^{-23}	-23.30983	5.720×10^{-22}	-22.75742	10^7
Boltzmann's Entropy Const	$k = \frac{2}{3} \epsilon = \frac{K}{N} = R m m_H = \frac{R \rho}{n}$	γ	1.3606×10^{-23}	-23.13374	3.8135×10^{-22}	-22.58133	10^7

Planck's Wirkungsquantum	$h = \frac{k^{4/3}}{c}\left(\frac{48\pi\alpha}{a}\right)^{1/3}$	$\alpha = 1.0823$ $a = 7.39\times10^{-15}$	$\gamma^{4/3}$	6.545×10^{-34}	-34.81590	5.5721×10^{-32}	-32.74602	10^{7}
Wien-Planck Constants	$c_1 = 8\pi c h$		$\gamma^{1/3}$	4.935×10^{-24}	-24.69326	4.2013×10^{-22}	-22.62338	10^{9}
	$c_2 = \frac{c h}{k}$		$\gamma^{1/3}$	1.44303×10^{-2}	-2.15928	4.3834×10^{-2}	-2.64181	10^{2}
Molec. Wt. Hyd	$m_H = \frac{1}{N} = \frac{k}{K}$		..	1.650×10^{-27}	-27.21738	1.650×10^{-27}	-27.21738	...
Transl. K. E. of Molecules.	$H = \frac{3}{2}P = \frac{1}{2}\rho N u^2 = n E_0 = \frac{3}{2} n k T$		γ	151986	5.18180	4259800	6.62939	10
Inner Energy	$U = C_v \rho T = H + J.\ J = 0$ (monatomic)		γ	151986	5.18180	4259800	6.62939	10
Velocity of Light	c		..	3×10^{8}	8.47712	3×10^{8}	8.47712	10^{2}
	c^2		..	9×10^{16}	16.95424	9×10^{16}	16.95424	10^{4}
Gravity Acceleration	g_0 and G		γ	9.806	0.99149	274.843	2.43908	10^{2}
Gravity ratio	$\gamma = G/g_0$		..	28.028	1.44759	.....	.....	...

II. In Diatomic and Adiabatic Conditions (M. K. S.), (Hydrogen)

Ratios of the Specific Heats, Cp, Cv	κ	..	1.4063	0.14806	1.4063	0.14806	...
	$\kappa/(\kappa-1)$	..	3.46155	0.53927	3.46155	0.53927	...
	$1/(\kappa-1)$	..	2.46155	0.39121	2.46155	0.39121	...

III. The Adiabatic Temperature Gradient per Kilometer

$$\text{Adiabatic temperature gradient} = \frac{T_a - T_0}{z_1 - z_0} = -\frac{G}{Cp}$$

Hydrogen (1)	Assumed monatomic	..	-0.475544	$-1.67719n$	-0.475544	$-1.67719n$	...
Hydrogen (2)	Diatomic	..	-0.68690	$-1.83689n$	-0.68690	$-1.83689n$	...
Helium (4)	Monatomic	..	-1.90217	$0.27925n$	-1.90217	$0.27925n$	...
Carbon (12)	Assumed monatomic	..	-5.7065	$0.75637n$	-5.7065	$0.75637n$	...
Calcium (40)	Monatomic	..	-19.0217	$1.27925n$	-19.0217	$1.27925n$	...
Zinc (65)	"	..	-30.8393	$1.48910n$	-30.8393	$1.48910n$	...
Cadmium (112)	"	..	-53.0275	$1.72450n$	-53.0275	$1.72450n$	...
Mercury (198)	"	..	-94.3520	$1.97475n$	-94.3520	$1.97475n$	...

TABLE 2

IV. THE ADIABATIC THERMAL COEFFICIENTS IN THE GASES AT THE EARTH AND THE SUN

Element	Formulas		Earth (M.K.S.)		Sun (M.K.S.)		To C.G.S.
			Number	Log	Number	Log	
Molecular Weight	$m = \frac{K}{R} = \frac{\rho}{a} = \frac{\rho}{n\, m_H}$		γ 28.028		1.44759		
Hydrogen	Assumed monatomic	..	1.00	0.00000	1.00	0.00000	...
Hydrogen	Diatomic	..	2.00	0.30103	2.00	0.30103	...
Helium	Monatomic	..	4.00	0.60206	4.00	0.60206	...
Carbon	Assumed monatomic	..	12.00	1.07918	12.00	1.07918	...
Calcium	Monatomic	..	40.00	1.60206	40.00	1.60206	...
Zinc	"	..	64.85	1.81191	64.85	1.81191	...
Cadmium	"	..	111.51	2.04731	111.51	2.04731	...
Mercury	"	..	198.41	2.29756	198.41	2.29756	...
Density	$\rho = m\, a = m\, n\, m_H = \frac{m\, n}{N}$						
Hydrogen (1)	Assumed monatomic	γ^{-1}	0.0449978	−2.65319	0.00160545	−3.20560	10^{-3}
Hydrogen (2)	Diatomic	"	0.089996	−2.95422	0.0032109	−3.50663	"
Helium	Monatomic	"	0.179992	−1.25525	0.0064218	−3.80766	"
Carbon (12)	Assumed monatomic	"	0.539975	−1.73237	0.0192655	−2.28478	"
Calcium	Monatomic	"	1.79992	0.25525	0.064219	−2.80766	"
Zinc	"	"	2.91807	0.46510	0.14114	−1.01751	"
Cadmium	"	"	5.0177	0.70050	0.179024	−1.25291	"
Mercury	"	"	8.92800	0.95075	0.31854	−1.50316	"

Volume $v = \frac{1}{\rho} = \frac{1}{m\ n\ m_H} = \frac{N}{m\ n}$

Hydrogen (1)	Assumed monatomic	γ	22.22350	1.34681	622.872	2.79440	10^{-3}
Hydrogen (2)	Diatomic	"	11.11175	1.04578	311.436	2.49337	"
Helium	Monatomic	"	5.55588	0.74475	155.718	2.19234	"
Carbon (12)	Assumed monatomic	"	1.85196	0.26763	51.906	1.71522	"
Calcium	Monatomic	"	0.55559	−1.74475	15.5718	1.19234	"
Zinc	"	"	0.34269	−1.53490	9.6048	0.98249	"
Cadmium	"	"	0.199295	−1.29950	5.58588	0.74709	"
Mercury	"	"	0.112008	−1.04925	3.13936	0.49684	"

Thermal Efficiency $R = \frac{K}{m}$

Hydrogen (1)	Assumed monatomic	γ	8248.2	3.91636	231179.0	5.36395	10^4
Hydrogen (2)	Diatomic	"	4124.1	3.61533	115589.5	5.06292	"
Helium	Monatomic	"	2062.05	3.31430	57795.0	4.76189	"
Carbon (12)	Assumed monatomic	"	687.35	2.83718	19265.0	4.28477	"
Calcium	Monatomic	"	206.205	2.31430	5779.5	3.76189	"
Zinc	"	"	127.188	2.10445	3564.83	3.55204	"
Cadmium	"	"	73.968	1.86905	2073.20	3.31664	"
Mercury	"	"	41.572	1.61880	1165.16	3.06639	"

TABLE 2.—*Continued*

Element	Formulas		Earth (M. K. S.)		Sun (M. K. S.)		To C. G. S.
			Number	Log	Number	Log	
Specific Heat	$Cp = \frac{\kappa}{\kappa - 1} R = \frac{\kappa}{\kappa - 1} \cdot \frac{K}{m}$						
Hydrogen (1)	Assumed monatomic	γ	20620.5	4.31430	577950	5.76189	10^4
Hydrogen (2)	Diatomic	"	14275.8	4.15460	400118	5.60219	"
Helium	Monatomic	"	5155.13	3.71224	144487	5.15983	"
Carbon (12)	Assumed monatomic	"	1718.39	3.23512	48162.2	4.68271	"
Calcium	Monatomic	"	515.513	2.71224	14448.7	4.15983	"
Zinc	"	"	317.971	2.50239	8912.0	3.94998	"
Cadmium	"	"	184.922	2.26699	5183.0	3.71458	"
Mercury	"	"	103.929	2.01674	2912.93	3.46433	"
Specific Heat	$Cv = \frac{1}{\kappa - 1} R = \frac{1}{\kappa - 1} \cdot \frac{K}{m}$						
Hydrogen (1)	Assumed monatomic	γ	12372.3	4.09245	346769	5.54004	10^4
Hydrogen (2)	Diatomic	"	10151.7	4.00654	284533	5.45413	"
Helium	Monatomic	"	3093.07	3.49039	86692	4.93798	"
Carbon (12)	Assumed monatomic	"	1031.03	3.01327	28897.3	4.46086	"
Calcium	Monatomic	"	309.307	2.49039	8669.2	3.93798	"
Zinc	"	"	190.783	2.28054	5347.25	3.72813	"
Cadmium	"	"	110.954	2.04514	3109.80	3.49273	"
Mercury	"	"	62.357	1.79489	1747.76	3.24248	"

Boyle-Gay Lussac Law, γ ($P = \rho R T$), and from this point, *by the method of trials*, it has been found to be practical to proceed upwards along certain curves to the vanishing plane, or downwards to considerable depths, where this gaseous law begins to give way to the liquid and viscous conditions at great pressures, densities, thermal efficiencies, and temperatures. The following chapter presents such computed values for the thermodynamic and radiation data as are indicated in the formulas. In all the adiabatic levels there are certain constant values, but generally the solar atmospheres are in equilibrium only by means of a series of variable coefficients which will be quite fully examined. It may be stated that *the computed results are in admirable agreement with the data obtained by means of observations made with the spectroheliograph, but that they far outrun them in efficiency, as to a multitude of details which are of great value in all questions of solar physics, regarding the origin of the circulation, radiation, and variations in the visual and the thermal spectra.*

The most convenient formulas for computing the terms in the first law of thermodynamics are as follows:

(34) Free heat $$(Q_1 - Q_0) = (Cp_a - Cp_{10})\ (T_a - T_0) = (S_1 - S_0)\ T_{10},$$

(35) Work $$(W_1 - W_0) = (R_{10} - Cp_{10})\ (T_a - T_0) = P_{10}\ (v_1 - v_0),$$

(36) Inner energy $$(U_1 - U_0) = (Cp_a - R_{10})\ \ (T_a - T_0) = (Q_1 - Q_0) - (W_1 - W_0),$$

and for the second law of thermodynamics,

(37) Entropy $$(S_1 - S_0) = \frac{Q_1 - Q_0}{T_{10}} = (Cp_a - Cp_{10})\ \left(\frac{T_a - T_0}{T_{10}}\right).$$

From these the potential of radiation is

(38) $$K_{10} = \frac{U_1 - U_0}{v_1 - v_0} = \frac{Q_1 - Q_0}{v_1 - v_0} - P_{10} = c\ T^{\alpha}.$$

The coefficient c and exponent α are computed by the methods of Bigelow's Meteorological Treatise.

From the trial equation

(39) $$\log C = \log K_{10} - A \log T_{10},$$

we have, by using successive values of K_{10} and T_{10},

$$\log A = \frac{\log (\log K_1 - \log K_0)}{\log (\log T_1 - \log T_0)}, \tag{40}$$

which gives certain values of A that can be combined with the corresponding log C in Equation (39). These values of (A, log C) are plotted on a diagram, and the mean line is drawn, from which a table for (α, log c) is made. Trial values of α_1 are assumed, and log c is computed from

$$\log c = \log K_{10} - \alpha \log T_{10}. \tag{41}$$

In this table the corresponding value of α is interpolated, and if $\alpha_1 = \alpha$ the trial value is correct. Two trial values (α_1, α_2) are generally enough to produce the pair values (α, log c) on the same horizontal line of the table of the gas. Examples of the results of these computations will be given, and they are unusually interesting in determining the source of the solar radiation in the different gases and their relation to σ in the Stefan Law,

$$J_0 = \sigma T^4. \tag{42}$$

Notation and Fundamental Formulas for the Electromagnetic Field

	Electric Field	*Magnetic Field*
(43) Field strength.	$E_1 \quad (E_x . E_y . E_z)_1$	$H_1 \quad (H_x . H_z . H_y)_1$
Impressed forces.....	e_0	h_0
Motion of the medium.....	e	h
Total force....	$E = \frac{D}{\kappa} = E_1 + e_0 + e$	$H = \frac{B}{\mu} = H_1 + h_0 + h$
(44) Displacement current.....	$\frac{\partial D}{\partial t} = \dot{D} \, (\dot{D}_x . \dot{D}_y . \dot{D}_z)$	$\frac{\partial B}{\partial t} = \dot{B} \, (\dot{B}_x . \dot{B}_y . \dot{B}_z)$
Conduction current.....	$4 \pi C = 4 \pi . c E$	.....
Convection current.....	$4 \pi \rho v$	.....
Motion of the medium.....	$4 \pi j$	$4 \pi g$
Total current..	$4 \pi J_0 = \dot{D} + 4 \pi (C + \rho v + j)$	$4 \pi G_0 = B + 4 \pi g$

The impressed forces and the motion of the medium will not be required in the problems concerning the radiation.

(45) Maxwell's Laws of Circuitation

$$\text{I} \quad 4\pi J_0 = \dot{D} + 4\pi (C + \rho v + j) = \text{curl}\,(H - h_0)$$
$$4\pi J = \dot{D} + 4\pi (C + \rho v) = \text{curl}\, H_1$$
$$4\pi \Gamma_1 = \dot{D} + 4\pi C = \text{curl}\, H$$

(46) $$\text{II} \quad 4\pi G_0 = \dot{B} + 4\pi g = -\,\text{curl}\,(E - e_0)$$
$$4\pi G = \dot{B} = -\,\text{curl}\, E_1$$
$$4\pi \Gamma_2 = \dot{B} = -\,\text{curl}\, E$$

(47) Potential Energy

$$\mathbf{J} = \frac{1}{8\pi}\int\int\int (E_x D_x + E_y D_y + E_z D_z)\, d\tau = \frac{1}{8\pi} E D = \frac{1}{8\pi}\kappa E^2.$$

(48) Kinetic Energy

$$\mathbf{K} = \frac{1}{8\pi}\int\int\int (H_x B_x + H_y B_y + H_z B_z) d\tau = \frac{1}{8\pi} H B = \frac{1}{8\pi}\mu H^2.$$

(49) Joule's Heat

$$\mathbf{Q} = \int\int\int (E_x C_x + E_y C_y + E_z C_z)\, d\tau = \tfrac{1}{2} E C = \tfrac{1}{2} c E^2.$$

(50) Activity

$$A_1 = E\dot{C}. \qquad A_2 = E\dot{D}. \qquad A_3 = H\dot{B}.$$

(51) Scalar product

$$(A\,B) = A\,B\cos A\,B = A_x B_x + A_y B_y + A_z B_z.$$

(52) Vector product

$$[A\,B] = (A_y B_z - A_z B_y) + (A_z B_x - A_x B_z) + (A_x B_y - A_y B_x) = V.\,A\,B.$$

(53) Divergence $\text{div}\, A = \dfrac{\partial A_x}{\partial x} + \dfrac{\partial A_y}{\partial y} + \dfrac{\partial A_z}{\partial z}.$

(54) Curl-rotation $\text{curl}\, A_x = \dfrac{\partial A_z}{\partial y} + \dfrac{\partial A_y}{\partial z}$; $\text{curl}\, A_y = \dfrac{\partial A_x}{\partial z} - \dfrac{\partial A_z}{\partial x}$; $\text{curl}\, A_y = \dfrac{\partial A_y}{\partial x} - \dfrac{\partial A_x}{\partial y}.$

The five fundamental equations are

		Usual Units	*Lorentz' Units*
(55)	1. Divergence	$\operatorname{div} D = \rho$	$\operatorname{div} D = \rho$
(56)	2. "	$\operatorname{div} B = o$	$\operatorname{div} B = o$
(57)	3. Curl	$\operatorname{curl} D = -\dot{H}$	$\operatorname{curl} D = -\frac{1}{c}\dot{H}$
(58)	4. "	$\operatorname{curl} H = D + 4\pi\rho v$	$\operatorname{curl} H = \frac{1}{c}(\dot{D} + \rho v)$
(59)	5. Mechanical force	$F = E + [v.H]$	$F = D + \frac{1}{c}[v.H]$

Poynting's Equation for the Flux of Radiation

The following demonstration of the Poynting equation is according to Richardson's exposition, Electron Theory of Matter, page 202.

This theorem proves that the rate at which work is done on the electric charges within a given volume $d\tau$ has two components, first, the rate at which energy flows into the volume $d\tau$ through the surface, and second, the rate of loss of the energy by the electromagnetic field within the volume. Take the mechanical force acting upon ρ electrons in $d\tau$.

$$F\rho = \rho\,(E + [v\,.\,H])\,d\tau, \tag{60}$$

E and H represent the total electric and magnetic forces.

The activity, or rate of working of the forces in the volume, is,

$$A = \iiint v\rho\,(E + [c\,.\,H])\,d\tau. \tag{61}$$

c = the velocity of propagation in the medium.

The second term is always zero, so that the activity is

$$A = \iiint (\rho v.\,E)\,d\tau + o = \iiint \frac{1}{4\pi}\left\{(\operatorname{rot} H.\,E) - \kappa E\frac{\partial E}{\partial t}\right\} d\tau. \tag{62}$$

Expand by the formulas for curl,

$$A = \iiint \frac{1}{4\pi}\left\{E_x\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) + E_y\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) + E_z\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) - \kappa E\frac{\partial E}{\partial t}\right\} d\tau. \tag{63}$$

Collect the terms and integrate by parts, rearranging them,

$$(64)\quad A = \frac{1}{4\pi}\int\int (E_y H_z - E_z H_y)\, d\,y\, d\,z + (E_z H_x - E_x H_z)\, d\,z\, d\,x + (E_x H_y - E_y H_x)\, d\,x\, d\,y$$
$$- \int\int\int \left\{ \frac{1}{4\pi}\left[H_x\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + H_y\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) + H_z\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) + E\frac{\partial E}{\partial t}\right\} d\tau.$$

Substitute $d\,y\,d\,z = l\,d\,S$, $d\,z\,d\,x = m\,d\,S$, $d\,x\,d\,y = n\,d\,S$, surface integral.

From $H^2 = H_x^2 + H_y^2 + H_z^2$, we have $2\,H\frac{\partial H}{\partial t} = 2\,H_x\frac{\partial H_x}{\partial t} + 2\,H_y\frac{\partial H_y}{\partial t} + 2\,H_z\frac{\partial H_z}{\partial t}$.

Since rot $E = -\frac{\partial H}{\partial t}$, we can employ the components; substitute them,

$$(65)\quad A = \frac{1}{4\pi}\int\int [E\,H]_n\, d\,S - \frac{1}{4\pi}\int\int\int \left\{ \left(H_x \cdot \frac{\partial H_x}{\partial t} + H_y\frac{\partial H_y}{\partial t} + H_z\frac{\partial H_z}{\partial t}\right) + E\frac{\partial E}{\partial t}\right\} d\,\tau.$$

$$(66)\quad A = \frac{1}{4\pi}\int\int [E\,H]_n\, d\,S - \frac{1}{4\pi}\int\int\int \frac{\partial}{\partial t}\left(\tfrac{1}{2}H^2 + \tfrac{1}{2}E^2\right) d\,\tau.$$

Change the general $E\,.\,H$ to the specific $\kappa\,E_0$, $\mu\,H_0$.

$$(67)\quad A = \frac{1}{4\pi}[E\,H] - \frac{\partial}{\partial t}\left(\frac{\kappa}{8\pi}H_0^2 + \frac{\mu}{8\pi}E_0^2\right) = \frac{1}{4\pi}\,V.E\,H - \dot{\mathbf{H}} - \dot{\mathbf{j}}.$$

E and H are perpendicular to each other, and the propagation is perpendicular to both. The internal energies are equal,

$$(68)\qquad \mathbf{J} = \frac{\kappa}{8\pi}E_0^2 = \frac{\mu}{8\pi}H_0^2 = \mathbf{H} = \frac{1}{2}u.$$

In a beam of parallel plane polarized light the electric and magnetic vectors move in equal periodic waves,

$$E = E_0 \cos(\omega t - x), \quad H = H_0 \cos(\omega t - x).$$

The resultant flow of energy perpendicular to $E\,H$ is

$$E^2 = E_0^2 \cos^2(\omega t - x) = H_0^2 \cos^2(\omega t - x) = H^2.$$

Its average value over a long interval of time is

$$c\,E^2 = c\,E_0^2 \frac{1}{t}\int_0^t \cos^2(\omega t - x)\,dt = c.\,\tfrac{1}{2}\,E_0^2 = c\,\tfrac{1}{2}\,H_0^2. \tag{69}$$

The energy flows along with the velocity c, as that of light.

Pressure of Radiation

If E = the average value and E_0 the maximum value, it is proved that the pressure due to radiation is

$$p = \frac{1}{t}\int E^2\,dt = \frac{1}{t}\int E_0^2 \cos^2\omega t\,dt = \tfrac{1}{2}\,E_0^2. \tag{70}$$

The energy per unit volume is

$$u = \frac{\kappa}{8\pi}E_0^2 + \frac{\mu}{8\pi}H_0^2 = \frac{E^2}{4\pi} = \frac{H^2}{4\pi}. \tag{71}$$

Flux of light transported on the average over the unit area in the unit of time is the intensity of light,

$$\mathbf{K} = \frac{1}{t}\int_0^t c\frac{E^2}{4\pi}\,dt = \tfrac{1}{2}\frac{c\,E_0^2}{4\pi} = \frac{c\,u}{4\pi}. \tag{72}$$

Hence, the radiation pressure on a normal surface is equal to the intensity of light divided by the velocity of the propagation.

If the light falls upon a perfectly reflecting surface the value of the radiation pressure and the intensity will be double the amount of that which it exerts when falling upon a perfectly absorbing surface.

Nichols and Hull find that the pressure of the radiation of sunlight is equal to

$$p = 7.01 \pm .02 \frac{\text{degrees}}{\text{cm.}^2}, \quad \frac{\text{erg}}{\text{cm.}^3}, \quad \frac{\text{gr.}}{\text{cm. sec.}^2}.$$

The following formulas define many of the relations that

exist between the radiation terms and the thermodynamic terms in gaseous atmospheres. (Heaviside and others.)

The Mean Flux of Energy

The volume density of radiation $= u = \mathbf{J} + \mathbf{H} = \frac{\kappa}{8\pi} E_0^2 + \frac{\mu H_0^2}{8\pi}$. In a perfectly reflecting enclosure the total energy remains a constant. Plane waves would run back and forth between two parallel plane walls without losing the form of perfect periodic waves, $H = H_0 \cos(\omega t - x)$, but if the walls are irregular in form, the train of waves takes on irregularities which become very complex. There is an average irregularity which has a regular effect, and this is constant as long as the total inner energy remains constant. Insert a plane area into the enclosure at any place. If the energy u moves in one way with the velocity c the flux would be $W = u\,c$; but this would produce an accumulation, so that an equal amount $\frac{1}{2}\,c\,u$ moves in each direction through the area A.

Since the rays in the enclosure take all directions of θ to the normal, it follows that the integral of $u \cos\theta$ over a hemisphere gives the flux W.

$$(73)\quad \pi \mathbf{K} = \frac{c}{2}\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \frac{u\cos\theta}{4\pi} \cdot \sin\theta\, d\theta\, d\varphi = \tfrac{1}{4}\,c.u$$

4π = the area of a sphere of unit radius, and $\sin\theta\, d\theta\, d\varphi$ is the area of an element of the surface.

The specific intensity of the flux over the unit area in the unit time $= \frac{1}{4}\,u \times c$.

The energy per unit volume $u = \frac{\mu H^2}{4\pi}$ or $\frac{\kappa E^2}{4\pi} = \frac{\mu H_0^2}{8\pi}$ or $\frac{\kappa E_0^2}{8\pi}$

if $H \,.\, E$ are average values and $H_0\, E_0$ maximum values in the plane wave front. The energy that crosses the unit surface perpendicular to the line of propagation is equal to the amount contained in a volume having the unit area for base and the distance $c\,t$ for height.

The Mean Pressure of Radiation

The electric and magnetic stresses unite to form a pressure along the ray which is equal to the total inner energy u, with no pressures perpendicular to the line of propagation, that is, in the plane of EH. The ray in the enclosure takes on all directions, and the *radiant pressure becomes like a hydrostatic pressure.* If the energy in the ray u goes in one direction through the area A the pressure is $p = u$; if it makes the angle θ with the normal u becomes $u \cos\theta$, and p becomes $u \cos^2\theta$ on A. For the average pressure in all directions, we have, over the whole sphere,

$$(74) \quad p = \int_0^{2\pi} \int_0^{\pi} \frac{u \cos^2\theta}{4\pi} \cdot \sin\theta \, d\theta \, d\varphi = \frac{1}{3} u.$$

Emission and Temperature, Stefan's Law

When the boundary of the enclosure is not perfectly reflecting (black), but partly absorbing (gray), the temperature and the second law of thermodynamics enter into consideration.

Take a piston with one head A fixed and another B movable.

1. Put B upon A so that the space is nil and A is maintained steadily at the temperature T in a cyclic process.

2. Move B to the distance h, and we have, with $u\,p$,

$(W_1 - W_0) = p\,h =$ the work of expansion.

$(U_1 - U_0) = u\,h =$ the change in the inner energy.

$(Q_1 - Q_0) = (W_1 - W_0) + (U_1 - U_0) = (p + u)\,h =$ the heat lost by A to the enclosure $= \left(\frac{1}{3} u + u\right) = \frac{4}{3} u\,h.$

3. Fix the piston B and let the cylinder cool down to 0°. All the inner energy $(U_1 - U_0) = u\,h$ goes out through A.

4. Push B back to A and then raise A to the temperature T. Integrate by the second law of thermodynamics,

$$\frac{dQ}{T} = \int_0^T \frac{1}{T} \frac{dQ}{dT} \cdot dT.$$

Substitute $Q = \frac{4}{3} u$,

Omit p and h.

$$\frac{\frac{4}{3}u}{T} = \int_0^T \frac{1}{T}\frac{d\,u}{d\,T}\,d\,T. \qquad \text{Differentiate and multiply by } 3\,T.$$

$$\frac{1}{T}\frac{d\,u}{d\,T} = \frac{4}{3}\frac{1}{T}\frac{d\,u}{d\,T} - \frac{4}{3}\frac{u}{T^2}. \qquad \frac{4\,u}{T} = \frac{d\,u}{d\,T}. \qquad \frac{d\,u}{u} = 4\frac{d\,T}{T}.$$

(75) $\log u = 4 \log T + \log a, \quad u = a\,T^4. \quad p = \frac{1}{3}\,a\,T^4.$

Summary of the Formulas of Radiation

The following summary of the formulas and the dimensions of the terms in radiation will be found convenient for general reference.

Planck's Radiation Terms

(76) Spatial Density. $u = \frac{4\pi}{c}\mathbf{K} = \frac{8\pi}{c}\tilde{K} = a\,T^4 = \frac{4}{c}\sigma\,T^4 = 3\,p = \frac{3}{4}\,s\,T = \frac{3}{4\pi}I. \quad \left[\frac{M}{L\,T^2}\right].$

(77) Auxiliary. $\mathbf{K} = \tilde{K} + \tilde{K}_1 = 2\,\tilde{K}$ for polarization along two axes.

$$c\,\tilde{K}_\nu = \lambda^2\,E_\lambda, \quad \tilde{K}_\nu\,d\,\nu = E_\lambda\,d\,\lambda,$$

since $\nu = \frac{c}{\lambda}$, $d\,\nu = -\frac{c\,d\,\lambda}{\lambda^2}$.

(78) Specific Intensity. $\mathbf{K} = 2\,\tilde{K} = \frac{c}{4\pi}u = \frac{c}{4\pi}a\,T^4 = \frac{\sigma}{\pi}T^4 = \frac{3\,c}{4\pi}p = \frac{3\,c}{16\,\pi^2}I = \frac{3\,c}{16\,\pi}s\,T = L\,T. \quad \left[\frac{M}{T^3}\right].$

(79) Pressure of Radiation. $p = \frac{1}{3}u = \frac{1}{3}a\,T^4 = \frac{4}{3\,c}\sigma\,T^4 = \frac{s\,T}{4} = \frac{P\,V\,u}{2\,(U_1 - U_0)} = \frac{P\,U}{3\,K\,T} = \frac{4\pi}{3\,c}\mathbf{K} = \frac{I}{4\pi}. \quad \left[\frac{M}{L\,T^2}\right].$

(80) Entropy. $s = \frac{4}{3}\frac{u}{T} = \frac{4}{3}a\,T^3 = \frac{4\,p}{T} = \frac{16\,\pi}{3\,c\,\mathrm{T}}\mathbf{K} = \frac{I}{\pi\,T}. \quad \left[\frac{M}{L\,T^2\,\mathrm{deg.}}\right].$

(81) Entropy Radiation. $L = \frac{c}{4\pi} a T^3 = \frac{3c}{16\pi} s = \mathbf{K}/T = \frac{c}{4\pi}\frac{\rho}{m}\frac{U}{T}$. $\left[\frac{M}{T^3 \text{ deg.}}\right]$.

(82) Radiation Energy. $I = \frac{4\pi}{3} u = \frac{16\pi^2}{3c}\mathbf{K} = \frac{32\pi^2}{3c}\tilde{K} = \frac{cF}{2}$. $\left[\frac{M}{L T^2}\right]$.

(83) Mechanical Force. $F = F_e + F_m = \frac{2\cos.\theta}{c} I$. $\left[\frac{M}{L^2 T}\right]$.

(84) Radiation Intensity. $E_\lambda = \frac{c\tilde{K}_\nu}{\lambda^2} = \frac{c}{8\pi\lambda^2} u_\nu = \frac{3}{32\pi^2}\frac{c}{\lambda^2} I_\nu = \frac{c}{2\lambda^2}\mathbf{K} = \frac{c^2}{\lambda^5} F\left(\frac{\lambda T}{c}\right)$.

(85) Monochromatic *Spectrum* lines. $u_\nu = \frac{1}{V}\varphi\left(\nu^3 V = \nu^3 \varphi(\nu^3 V)\right)$, Wien's Displacement Law.

(86) Temperature. $T = \nu F_1\left(\frac{c^2\tilde{K}}{\nu^3}\right)$. Intensity. $\tilde{K}_\nu = \frac{\nu^3}{c^2} F_2\left(\frac{T}{\nu}\right) = \frac{3c}{32\pi^2} I_\nu$.

(87) Specific Intensity. $\tilde{K}_\nu = \frac{\mathbf{K}_\nu}{2} = \frac{c}{8\pi} u_\nu = \frac{3c}{32\pi^2} I_\nu = \frac{\lambda^2}{c} E_\lambda = \frac{h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}} - 1} = \frac{ch}{\lambda^3}\frac{1}{e^{\frac{hc}{k\lambda T}} - 1}$.

(88) Spatial Density. $u_\nu = \frac{4\pi}{c}\mathbf{K}_\nu = \frac{8\pi}{c}\tilde{K}_\nu = \frac{3}{4\pi} I_\nu = \frac{8\pi\lambda^2}{c} E_\lambda = \frac{8\pi h}{\lambda^3}\cdot\frac{1}{e^{\frac{hc}{k\lambda T}} - 1}$.

(89) Radiation Energy. $$I_\nu = \frac{16\pi^2}{3c}\mathbf{K}_\nu = \frac{32\pi^2}{3c}\tilde{K}_\nu = \frac{4\pi}{3}u_\nu = \frac{32\pi^2}{3}\frac{h}{\lambda^3}\frac{1}{e^{\frac{hc}{k\lambda T}}-1}.$$

(90) Radiation Intensity. $$E_\lambda = \frac{c}{2\lambda^2}\mathbf{K} = \frac{c\tilde{K}_\nu}{\lambda^2} = \frac{c^2}{8\pi\lambda^2}u_\nu = \frac{3}{32\pi^2}\frac{c^2}{\lambda^2}I_\nu = \frac{c^2h}{\lambda^5}\frac{1}{e^{\frac{ch}{k\lambda T}}-1}$$

(91) Unpolarized Spatial Density. $$\frac{8\pi}{c}E_\lambda = \frac{4\pi}{\lambda^2}\mathbf{K}_\nu = \frac{8\pi\tilde{K}_\nu}{\lambda^2} = \frac{c}{\lambda^2}u = \frac{3}{4\pi}\frac{c}{\lambda^2}I_\nu = \frac{8\pi ch}{\lambda^5}\cdot\frac{1}{e^{\frac{ch}{k\lambda T}}-1}.$$

Planck bases all of these formulas upon strictly adiabatic conditions, assuming a cylinder with perfectly reflecting sides or impermeable walls. Hence they do not apply to free atmospheres without modifications.

Tables of Radiation Constituents in the Volume

The following tables give the values of the terms that are concerned with radiation in the different volumes of the sun's and the earth's atmospheres, taking them at rest. When multiplied by the velocity of light they become radiation fluxes. They are valuable in tracing the intrinsic changes from level to level within the atmospheres, due to the many physical processes at work. The electromagnetic plane waves transport the energy that is finally able to escape from the sun, after the original radiation at the levels z_R^* has been depleted by an equivalent of about 1.87 gr. cal./cm.^{2} min. in the higher strata of the solar envelope.

TABLE 3
Definition of the Terms with Dimensions

Formula	Dimension	M. K. S.	F.	C. G. S.	γ
Practical units of work or kinetic energy	$M\frac{L^2}{T^2}$	$\frac{\text{kilog. } m^2}{\text{sec.}^2} = \text{Joule}$	10^7	$\frac{\text{gr. cm.}^2}{\text{sec.}^2} = \text{erg}$	
Pressure $P = \rho R T$	$M\frac{L^2}{T^2}\cdot\frac{1}{L^3}$	$\frac{\text{kilog.}}{m.\ \text{sec.}^2} = \frac{\text{Joule}}{m^3}$	10	$\frac{\text{gr.}}{\text{cm. sec.}^2} = \frac{\text{erg}}{\text{cm.}^3}$	γ
Density ρ	$\frac{M}{L^3}$	$\frac{\text{kilog.}}{m^3}$	10^{-3}	$\frac{\text{gr.}}{\text{cm.}^3}$	γ^{-1}
Volume v $\quad V = m v$	$\frac{L^3}{M}$	$\frac{m^3}{\text{kilog.}}$	10^3	$\frac{\text{cm.}^3}{\text{gr.}}$	γ
Temperature T	.....	deg.		deg.	γ
Specific heat R, Cp, $C v$	$\frac{L^2}{T^2\ \text{deg.}}$	$\frac{m^2}{\text{sec.}^2\ \text{deg.}}$	10^4	$\frac{\text{cm.}^2}{\text{sec.}^2\ \text{deg.}}$	γ
Specific heat $K = m R$	$\frac{L^2}{T^2\ \text{deg.}}$	$\frac{m^2}{\text{sec.}^2\ \text{deg.}}$	10^4	$\frac{\text{cm.}^2}{\text{sec.}^2\ \text{deg.}}$	γ
M.E.H. $A = 426.8$	$\frac{ML^2}{T^2\ \text{cal.}}$	$\frac{\text{kilog. } m^2}{\text{sec.}^2\ \text{cal.}} = \frac{\text{Joule}}{\text{cal.}}$	10^4	$\frac{\text{gr. cm.}^2}{\text{sec.}^2\ \text{cal.}} = \frac{\text{erg}}{\text{cal.}}$	
Number per mass, N	$\frac{1}{M} = \frac{1}{m_H}$	$\frac{1}{\text{mass } H\text{-atom}}$	10^{-3}	$\frac{1}{\text{mass } H\text{-atom}}$	
Number per vol., n	$\frac{1}{L^3}$	$\frac{1}{m^3}$	10^{-6}	$\frac{1}{\text{cm.}^3}$	γ^{-1}
Kinetic energy $P V = K T$	$\frac{L^2}{T^2}$	$\frac{m^2}{\text{sec.}^2}$	10^4	$\frac{\text{cm.}^2}{\text{sec.}^2}$	γ^2
Kinetic energy $P v = R T$	$\frac{L^2}{T^2}$	$\frac{m^2}{\text{sec.}^2}$	10^4	$\frac{\text{cm.}^2}{\text{sec.}^2}$	γ^2

Kinetic energy $u^2 = \frac{3Pv}{N}$	$\frac{ML^2}{T^2}$	$\frac{\text{kilog. } m^2}{\text{sec.}^2} = \text{Joule}$	10^7	$\frac{\text{gr. cm.}^2}{\text{sec.}^2} = \text{erg}$	γ^2
Velocities $q^2 = Nu^2$	$\frac{L^2}{T^2}$	$\frac{m^2}{\text{sec.}^2}$	10^4	$\frac{\text{cm.}^2}{\text{sec.}^2}$	γ^2
Molecular K.E. $E_0 = \frac{3}{2}\frac{P}{n}$	$\frac{ML^2}{T^2}$	$\frac{\text{kilog. } m^2}{\text{sec.}^2} = \text{Joule}$	10^7	$\frac{\text{gr. cm.}^2}{\text{sec.}^2} = \text{erg}$	γ^2
M.K.E. per deg. $\varepsilon = \frac{E_0}{\text{deg.}}$	$\frac{ML^2}{T^2 \text{ deg.}}$	$\frac{\text{kilog. } m^2}{\text{sec.}^2 \text{ deg.}} = \frac{\text{Joule}}{\text{deg.}}$	10^7	$\frac{\text{gr. cm.}^2}{\text{sec.}^2 \text{ deg.}} = \frac{\text{erg}}{\text{deg.}}$	γ
Boltzmann's entropy const. $k = \frac{K}{N}$	$\frac{ML^2}{T^2 \text{ deg.}}$	$\frac{\text{kilog. } m^2}{\text{sec.}^2 \text{ deg.}} = \frac{\text{Joule}}{\text{deg.}}$	10^7	$\frac{\text{gr. cm.}^2}{\text{sec.}^2 \text{ deg.}} = \frac{\text{erg}}{\text{deg.}}$	γ
Planck's Wirkungsquantum h	$\frac{ML^2}{T^2} \cdot T$	$\frac{\text{kilog. } m^2}{\text{sec.}} = \text{Joule sec.}$	10^7	$\frac{\text{gr. cm.}^2}{\text{sec.}} = \text{erg sec.}$	$\gamma^{4/3}$
Wien-Planck $c_1 = 8\pi ch$	$\frac{ML^2}{T^2} \cdot L$	$\frac{\text{kilog. } m^2}{\text{sec.}^2} \cdot m = \text{Joule} \times \text{meter}$	10^9	$\frac{\text{gr. cm.}^2}{\text{sec.}^2} \cdot \text{cm} = \text{erg.} \times \text{cm.}$	$\gamma^{4/3}$
Wien-Planck $c_2 = \frac{ch}{k}$	L deg.	meter deg.	10^2	cm. deg.	$\gamma^{1/3}$
Molecular inner energy $H = \frac{3}{2}P$	$\frac{ML^2}{T^2} \cdot \frac{1}{L^3}$	$\frac{\text{kilog.}}{m. \text{ sec.}^2} = \frac{\text{Joule}}{m^3}$	10	$\frac{\text{gr.}}{\text{cm. sec.}^2} = \frac{\text{erg}}{\text{cm.}^3}$	γ
Total inner energy $U = C v \rho T$	$\frac{ML^2}{T^2} \cdot \frac{1}{L^3}$	$\frac{\text{kilog.}}{m. \text{ sec.}^2} = \frac{\text{Joule}}{m^3}$	10	$\frac{\text{gr.}}{\text{cm. sec.}^2} = \frac{\text{erg}}{\text{cm.}^3}$	γ
Gravity acceleration g	$\frac{L}{T} \cdot \frac{I}{T}$	$\frac{m}{\text{sec.}^2}$	10^2	$\frac{\text{cm.}}{\text{sec.}^2}$	γ
1 large calorie, A_0	$\frac{ML^2}{T^2 \text{ (cal.)}}$	4.1851×10^3 Joule	10^7	4.1851×10^{10} erg	
1 small calorie, A	$\frac{ML^2}{T^2 \text{ cal.}}$	4.1853 Joule	10^7	4.1851×10^7 erg	

TABLE 3—(*Continued*)

THE VOLUME DENSITY OF BLACK RADIATION

Formula	Dimension	M. K. S.	F.	C. G. S.	γ
Vol. coeff. in Stefan Law $a = \frac{6 c_1 \alpha}{c_2^4} \ldots$	$\frac{M L^2}{T^2} \cdot \frac{1}{L^3} \cdot \frac{1}{\text{deg.}^4}$	$\frac{\text{kilog.}}{m.\ \text{sec.}^2\ \text{deg.}^4} = \frac{\text{Joule}}{m.^3\ \text{deg.}^4}$	10	$\frac{\text{gr.}}{\text{cm. sec.}^2\ \text{deg.}^4} = \frac{\text{erg}}{\text{cm.}^3 \text{deg.}^4}$	
Vol. density $u = a T^4 \ldots\ldots\ldots\ldots$	$\frac{M L^2}{T^2} \cdot \frac{1}{L^3}$	$\frac{\text{kilog.}}{m.\ \text{sec.}^2} = \frac{\text{Joule}}{m^3}$	10	$\frac{\text{gr.}}{\text{cm. sec.}^2} = \frac{\text{erg}}{\text{cm.}^3}$	γ^4
Radiation pressure $p = \frac{1}{3} u \ldots\ldots\ldots$	$\frac{M L^2}{T^2} \cdot \frac{1}{L^3}$	$\frac{\text{kilog.}}{m.\ \text{sec.}^2} = \frac{\text{Joule}}{m^3}$	10	$\frac{\text{gr.}}{\text{cm. sec.}^2} = \frac{\text{erg}}{\text{cm.}^3}$	γ^4
Kinetic energy of black radiation $I = \frac{4\pi}{3} u \ldots$	$\frac{M L^2}{T^2} \cdot \frac{1}{L^3}$	$\frac{\text{kilog.}}{m.\ \text{sec.}^2} = \frac{\text{Joule}}{m^3}$	10	$\frac{\text{gr.}}{\text{cm. sec.}^2} = \frac{\text{erg}}{\text{cm.}^3}$	γ^4
Specific entropy of black radiation $s = \frac{4}{3} \frac{u}{T} \ldots\ldots$	$\frac{M L^2}{T^2} \cdot \frac{1}{L^3} \cdot \frac{1}{\text{deg.}}$	$\frac{\text{kilog.}}{m.\ \text{sec.}^2\ \text{deg.}} = \frac{\text{Joule}}{m.^3\ \text{deg.}}$	10	$\frac{\text{gr.}}{\text{cm. sec.}^2\ \text{deg.}} = \frac{\text{erg}}{\text{cm.}^2 \text{deg.}}$	γ^3
Mechanical force of black radiation $F = \frac{8\pi}{3c} u \ldots$	$\frac{M L^2}{T^2} \cdot \frac{1}{L^3}$	$\frac{\text{kilog.}}{m.\ \text{sec.}^2} = \frac{\text{Joule}}{m^3}$	10	$\frac{\text{gr.}}{\text{cm. sec.}^2} = \frac{\text{erg}}{\text{cm.}^3}$	γ^4
THE RATE OF WORKING OR FLUX OF BLACK RADIATION					
Coefficient in the Stefan Law $\sigma = \frac{c}{4} a \ldots\ldots$	$\frac{M L^2}{T^2} \cdot \frac{1}{L^3 \text{deg.}^4} \cdot \frac{L}{T}$	$\frac{\text{kilog.}}{\text{se}.^3\ \text{deg.}^4} = \frac{\text{Joule}}{m.^2\ \text{sec.}\ \text{deg.}^4}$	1000	$\frac{\text{gr.}}{\text{sec.}^3\ \text{deg.}^4} = \frac{\text{erg}}{\text{cm.}^2 \text{sec.}\ \text{deg.}^4}$	
Intensity of radiation $\mathbf{K} = \frac{c}{4\pi} u \ldots$	$\frac{M L^2}{T^2} \cdot \frac{1}{L^3} \cdot \frac{L}{T}$	$\frac{\text{kilog.}}{\text{sec.}^3} = \frac{\text{Joule}}{m.^2\ \text{sec.}}$	1000	$\frac{\text{gr.}}{\text{sec.}^3} = \frac{\text{erg}}{\text{cm.}^2\ \text{sec.}}$	γ^4
Entropy radiation $L = \frac{c}{4\pi} a T^3 \ldots$	$\frac{M L^2}{T^2} \cdot \frac{1}{L^3} \cdot \frac{L}{T} \cdot \frac{1}{\text{deg.}}$	$\frac{\text{kilog.}}{\text{ec.}^3\ \text{deg.}} = \frac{\text{Joule}}{m.^2\ \text{sec.}\ \text{deg.}}$	1000	$\frac{\text{gr.}}{\text{sec.}^3\ \text{deg.}} = \frac{\text{cm.}^2\ \text{sec.}\ \text{deg.}}{\text{e g}}$	γ^3
Kinetic σ in calories $\sigma_A = \frac{\sigma}{A} \ldots\ldots$	$\frac{M \text{ cal.}}{L^2\ T\ \text{deg.}^4} = \frac{M}{T^3\ A}$	$\frac{\text{kilog.}}{A\ \text{sec.}^3\ \text{deg.}^4} = \frac{\text{kilog. cal.}}{m.^2\ \text{sec.}}$	1000 / A	$\frac{\text{gr.}}{A\ \text{sec.}^3\ \text{deg}^4} = \frac{\text{gr. cal.}}{\text{cm.}^2\ \text{sec.}}$	

The surface integral for flux has the dimension $\left[\frac{M L^2}{T^2} \cdot \frac{1}{L^3} \cdot \frac{L}{T}\right] = 1000.$

The volume integral for density has the dimension $\left[\frac{M L^2}{T^2} \cdot \frac{1}{L^3}\right] = 10.$

TABLE 4

TABLE OF ASTRONOMICAL CONSTANTS

Meter-Kilogram-Second (M. K. S.) System

		Number	Logarithm
r	= Mean radius of the earth (Bessel's spheroid).....	6370191 m.	6.8041525
R	= Mean radius of the sun, (Auwer's diam.31'59.26")	694800800 m.	8.8418603
R/r	= Ratio of the radii R/r...	109.071	2.0377078
D	= Distance of the sun to the earth.....	149 340 870 000 m.	11.1741786
R/D	= Ratio R/D.....	0.0046525	7.6676817−10
$(R/D)^2$	= Ratio $\left(\frac{R}{D}\right)^2$	0.000021645	5.3353634−10
p	= Parallax of the sun, p = tan (r/D).....	8".80756	5.6299739−10
p	= Parallax of the sun (Newcomb)	8.7965	
S/S_1	= Ratio of the surfaces, $S/S_1 = (109.071)^2$.....	11896.4	4.0754156
V/V_1	= Ratio of the volumes, $V/V_1 = (109.071)^3$.....	1297548	6.1131234
ρ_a	= Average density of the earth (Harkness).....	5.576 kil./m.^{3}	0.7463230
ρ_s	= Average density of the sun $= \frac{M}{m}\left(\frac{r}{R}\right)^3 \rho_a$.....	1.43287 kil./m.^{3}	0.1562056
m	= Mass of the earth in kilograms.....	6.0377×10^{24} kil.	24.780872
M	= Mass of the sun in kilograms.....	2.0132×10^{30} kil.	30.303878
M/m	= Ratio, mass of the sun to the earth, M/m.....	333431	5.523006
M/m	= Ratio, mass of the sun to the earth (Newcomb).	333432	5.523008
G	= Acceleration per second at surface of the sun.....	274.843 m./sec.	2.4390844
G_0	= Acceleration per second at surface of the earth....	9.8060 m./sec.	0.9914920
γ	= Ratio of the accelerations, $G/g_0 = \frac{M}{m}\frac{r^2}{R^2}$	28.028	1.4475904
k	The gravitation constant, $k = g_0 r^2/m$.....	6.5906×10^{-11}	9.818925−20
v	= Velocity of the earth in its orbit.....	29806.6 m./sec. 18.5212 miles/sec.	4.474313 1.267670

TABLE 4—*Continued*

TABLE OF ASTRONOMICAL CONSTANTS

Meter-Kilogram-Second (M. K. S.) System

	Number	Logarithm
f = Acceleration at the distance of the earth $= \frac{v^2}{D}$.	0.0059491 *m.*/sec.	7.774446−10
Check, $f = \frac{M}{m}\left(\frac{r}{D}\right)^2 g_0$.		7.774446−10
S = Rate at which the earth falls toward the sun $= \frac{1}{2}f$	0.0029746 *m.*/sec.	7.473416−10
1″ (second of arc) = $\frac{\text{radius of sun in kilometers}}{\text{radius of sun in seconds of arc}}$	724.030 kil.	2.8597565

The astronomical constants given in Table 4 will be found useful in reference to the general relations between the earth and the sun, and these data will be employed in this Treatise.

CHAPTER II

Computation of the Thermodynamic Terms

General Remarks

HAVING obtained the data for one point of thermal equilibrium in the gas under consideration, it is necessary to proceed by the method of trials from level to level, both above and below that point. In order to accomplish this purpose, the formulas (1) to (5) are worked through in the non-adiabatic form; then the terms in the gravitation formula are computed:

$$(43)\quad G_0(z_1 - z_0) = -\frac{P_1 - P_0}{\rho_{10}} - (Cp_a - Cp_{10})(T_a - T_0) + \Delta G_0 \cdot (z_1 - z_0).$$

Take a series of values of T, as T_1, T_2, T_3, . . . T_w, and compute the residuals $\Delta G_0 (z_1 - z_0)$. *There is one definite value of T which will make the residual a minimum.* As T_n approaches this value the residuals diminish; as T_n passes the minimum residual these increase. After some practise with the curve of the residuals the value of T may be readily found approximately for each new level, and as the curves for P, ρ, R, T, develop it is easy to project T quite accurately. The labor of working through the first four elements, hydrogen (diatomic), calcium, cadmium, mercury, was very great, but the curves soon revealed the law of the temperature distribution with the height above the photosphere, so that the other elements, hydrogen (monatomic), helium, carbon, zinc, were computed with very little need of extensive trials. The number of trial computations was from six to ten for hydrogen (diatomic), but it diminished to less than two in the last elements that were tried. It is now easy to assign the temperatures, pressures, densities, and thermal efficiencies for any gas quite closely without the use of the complete set of formulas. An examination of the curves on

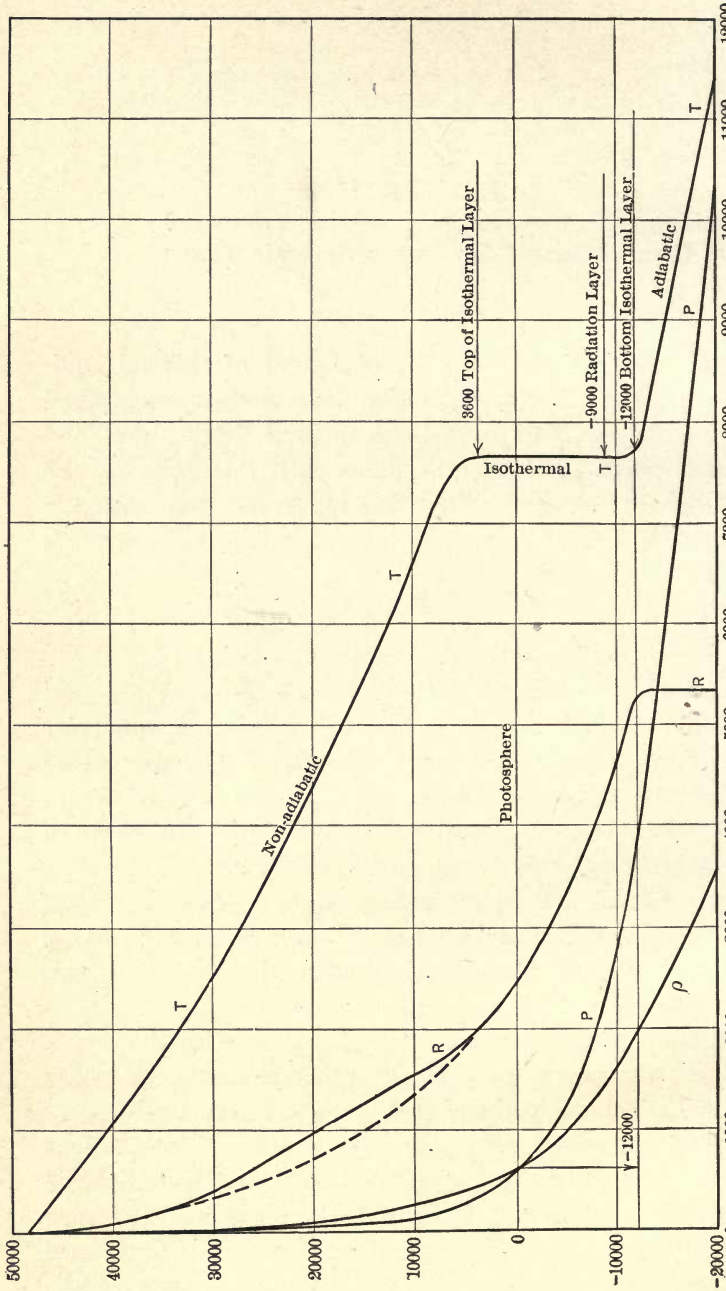


FIG. 2. Hydrogen on the Sun. $m = 1.00$ (Monatomic).

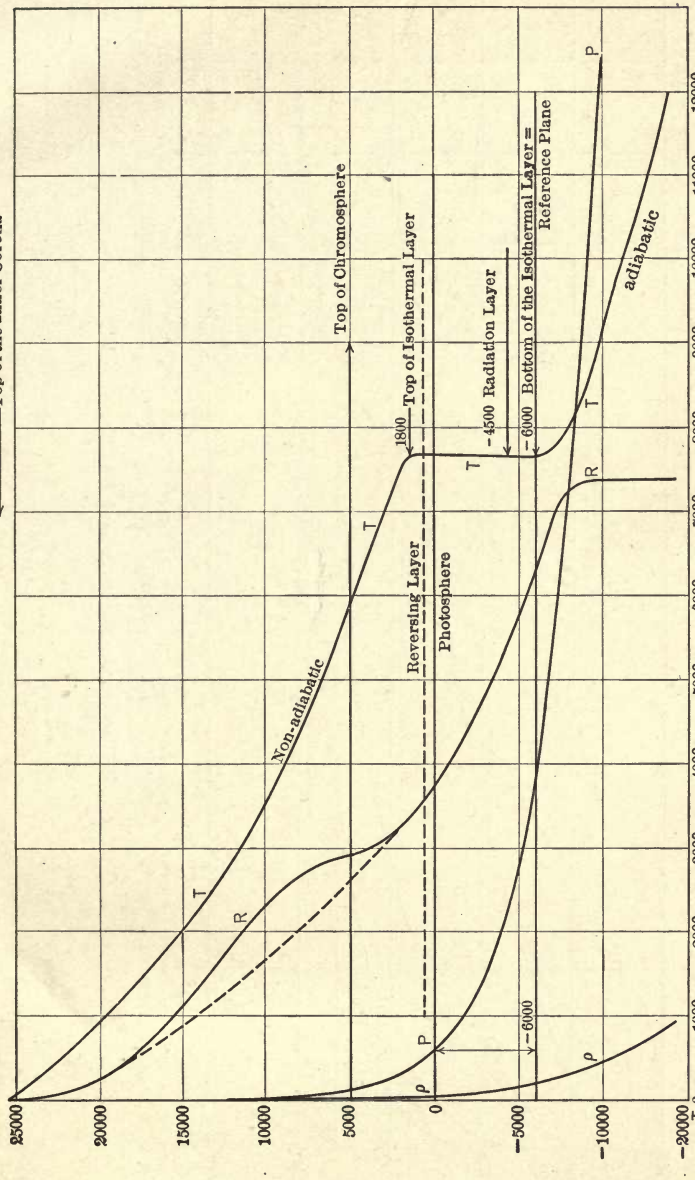


FIG. 3. Hydrogen on the Sun. $m = 2.00$ (Diatomic).

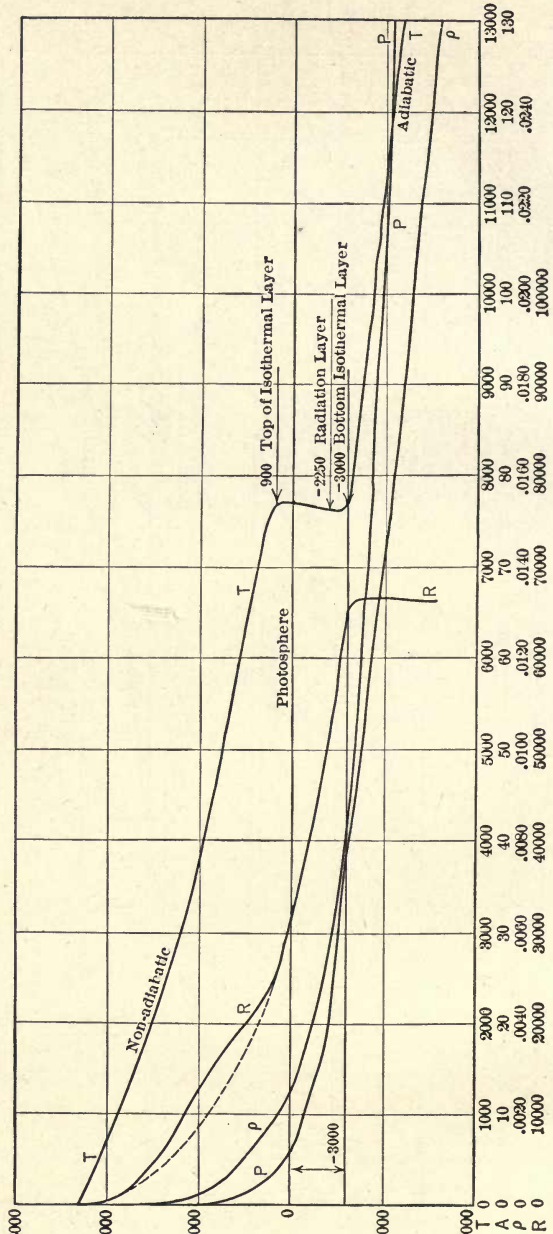


FIG. 4. Helium on the Sun. $m = 4.00$ (Monatomic).

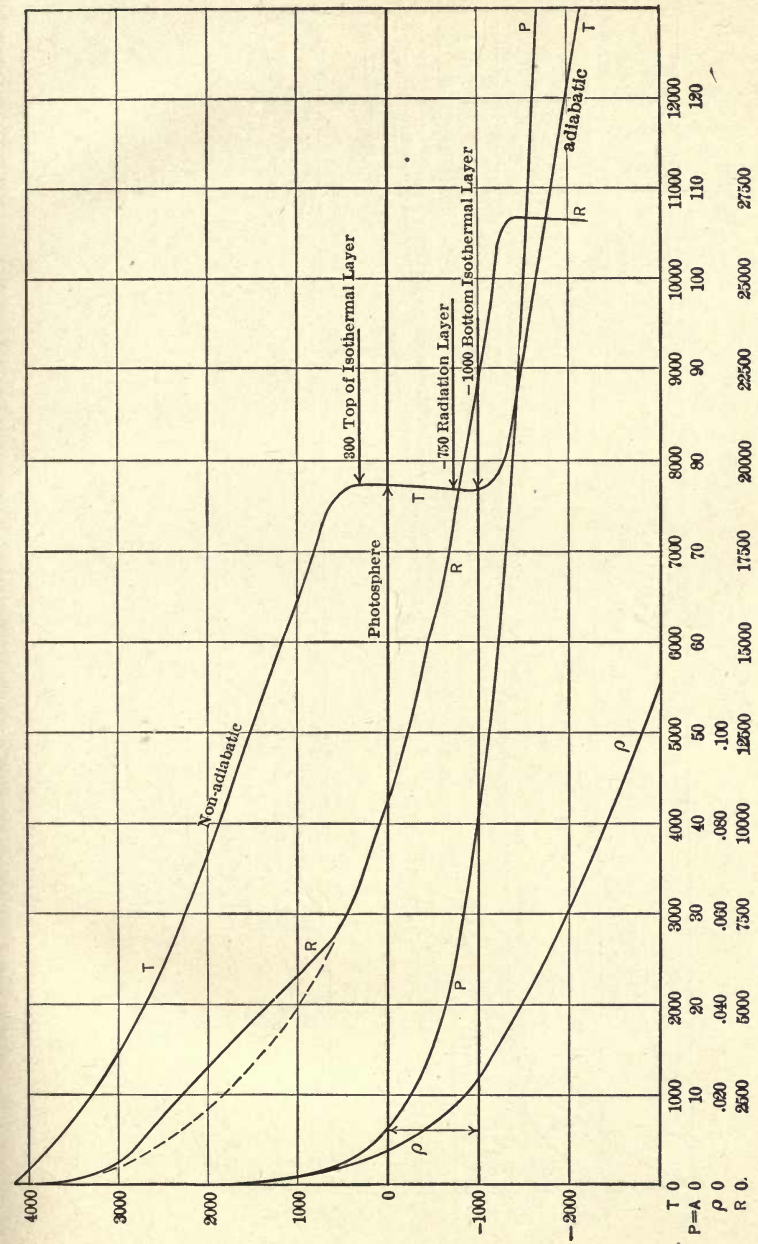


FIG. 5. Carbon Vapor on the Sun. $m = 12.00$ (Assumed Monatomic).

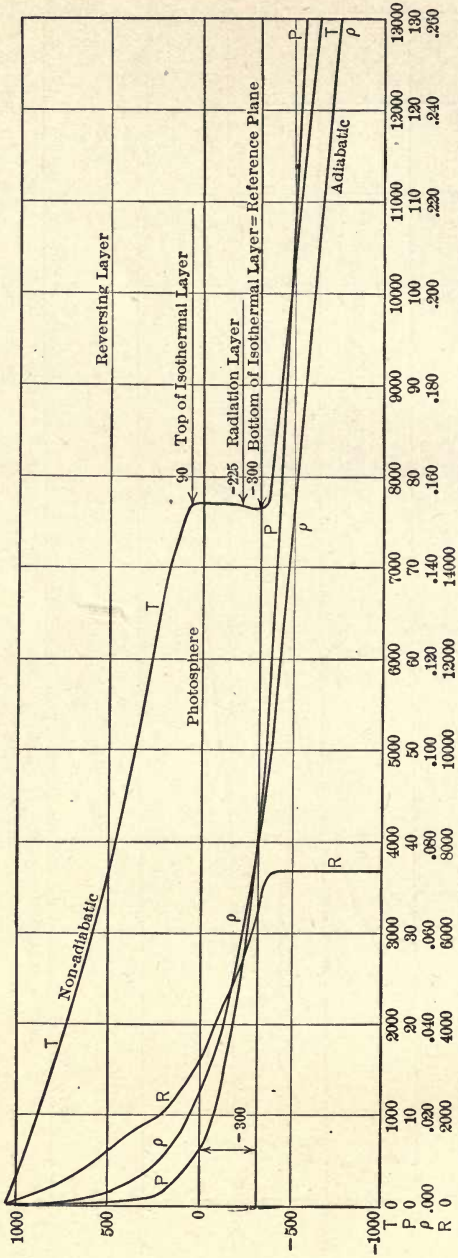


FIG. 6. Calcium Vapor on the Sun. $m = 40.00$ (Monatomic).

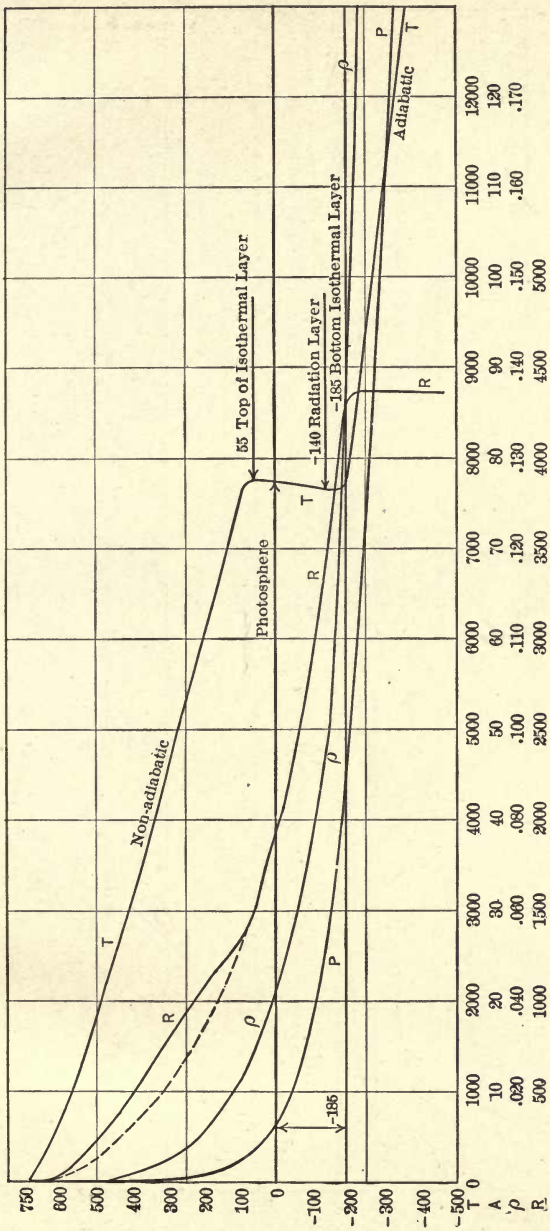


FIG. 7. Zinc on the Sun. $m = 64.85$ (Monatomic).

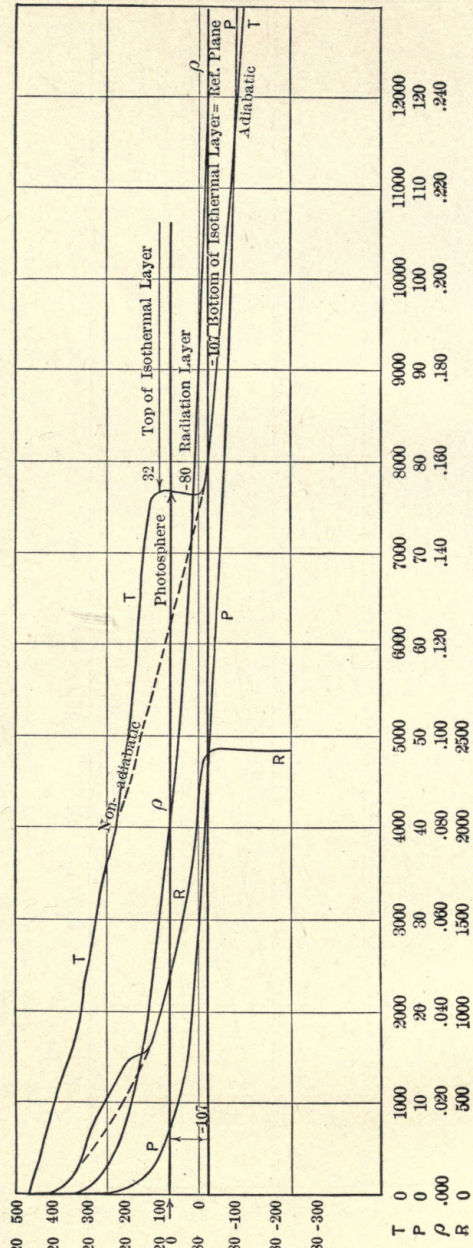


FIG. 8. Cadmium Vapor on the Sun. $m = 111.51$ (Monatomic).

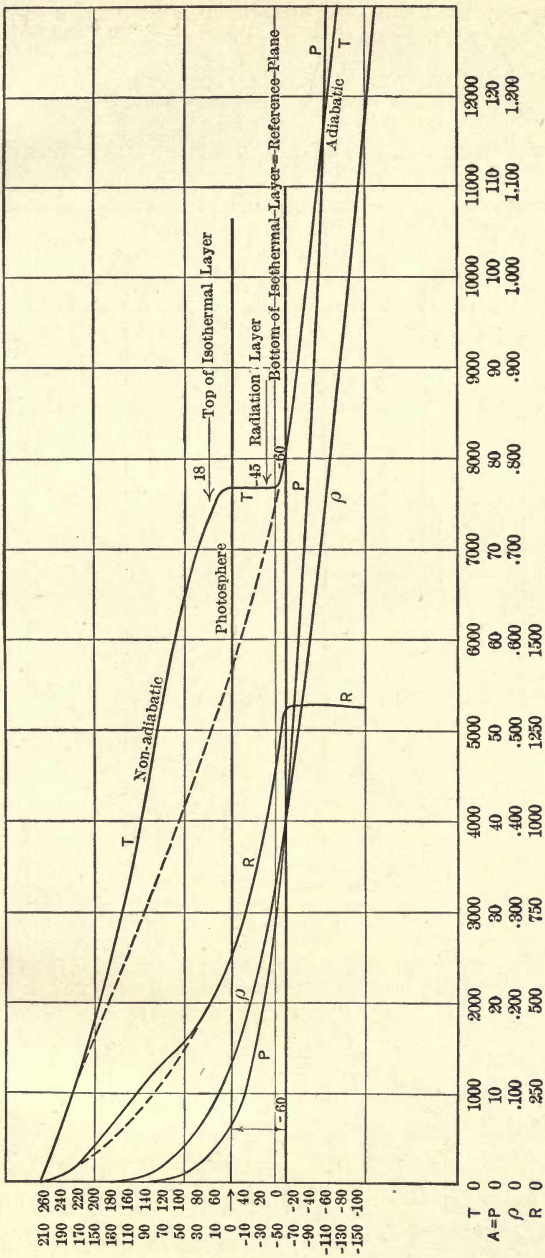


FIG. 9 Mercury on the Sun. $m = 198.41$ (Monatomic).

Figs. 2–9 shows that they are all constructed on the same model, while Figs. 10–13 indicate that they form families of curves which are interrelated by hyperbolic conditions with the parameter m.

The constitution of the atmospheres of the several gases on

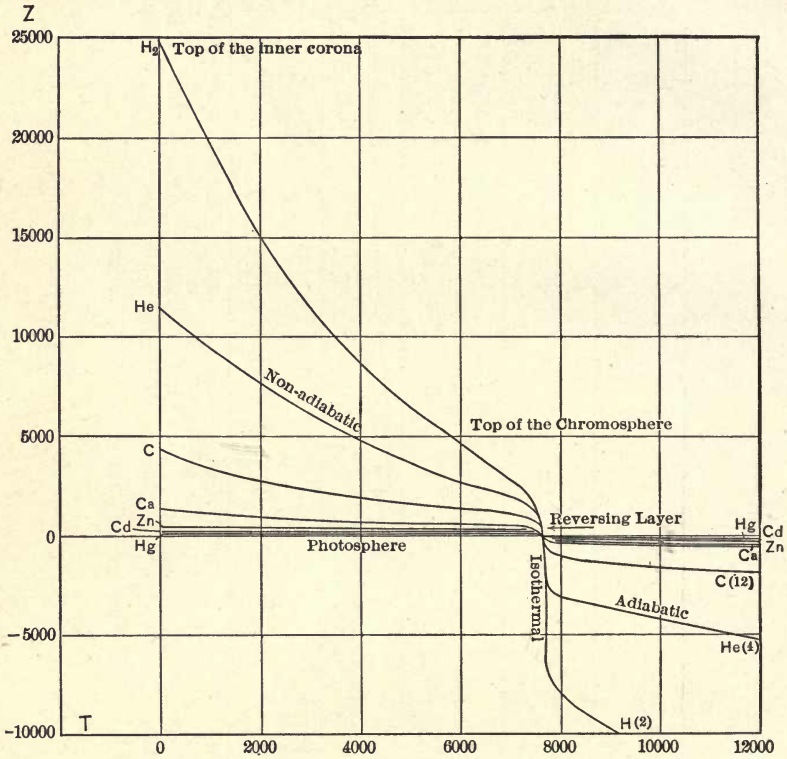


FIG. 10. T. Temperature in Absolute Centigrade Degrees. Z = height in kilometers from the photosphere.

the sun is exactly the same in principle as was found in the atmosphere of the earth. This is seen by comparing the curves of Fig. 1, which summarizes the terrestrial data for the balloon ascension, Uccle, September 13, 1911. Compare Bigelow's Treatise, Tables 96, 97. In the case of the earth's atmosphere, the lower adiabatic section is not very clearly developed at the latitude of Belgium, because the temperature gradients in the

lower levels do not agree with the adiabatic rate. This would, however, be found generally for the balloon ascensions made in the Tropics. The isothermal layers, 12000 to 37000 meters, and the non-adiabatic layers, 38000 to 90000 meters, are clearly determined. The peculiar secondary curvature in the gas

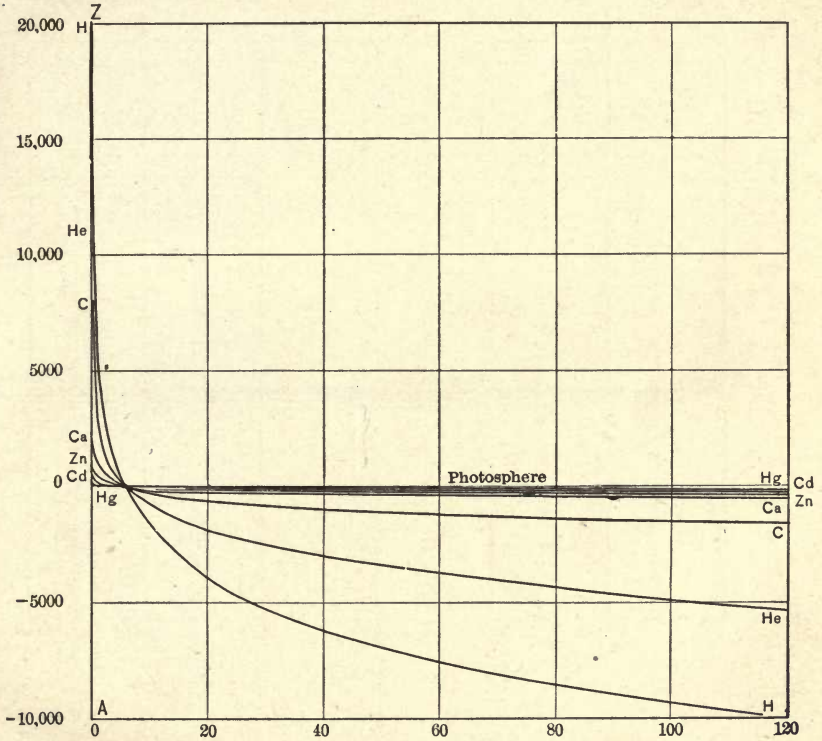


FIG. 11. A. Pressure in Terrestrial Atmospheres.

$$\frac{\text{force}}{\text{meter}^2} = \frac{ML}{T^2} \cdot \frac{1}{L^2}; \quad 1\ \text{Atm.} = 101323.5 \frac{kg}{m^2}$$

efficiency R, which occurs in the levels where the temperature gradient is at its greatest value, is found in all the atmospheres.

In the case of the sun there were from 50 to 70 points determined on each of the curves of P, ρ, R, T, from deep within the adiabatic region to the vanishing plane of the gas. An inspection of Figs. 2–9 shows clearly that the gases H_1, H_2, He, C, Ca, Zn, Cd,

Hg differ from one another chiefly in the value of the vertical scale, or depth, within which the gas develops its distribution. The original computations in logarithms check very closely throughout; these are reserved for another publication. In order to make clear the series of fundamental laws of distribution,

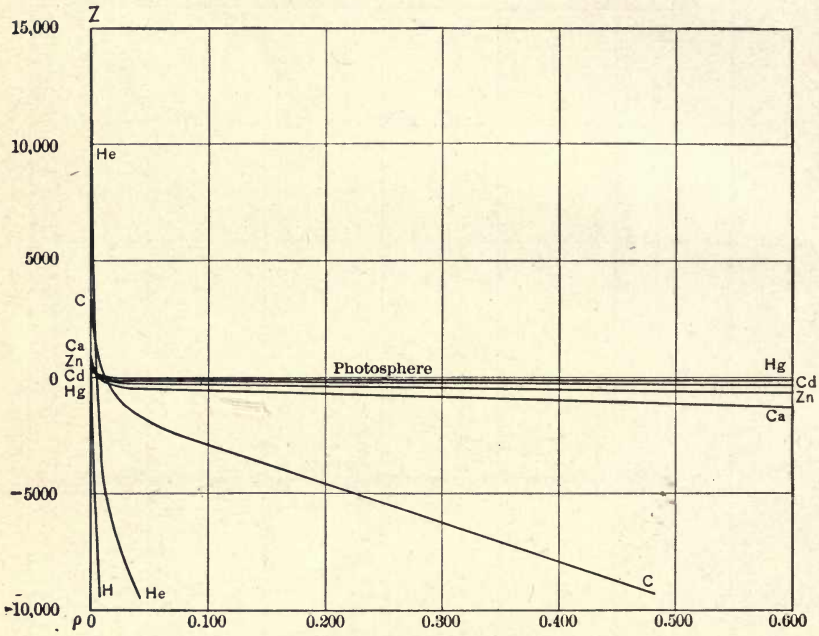


FIG. 12. ρ. Density.

$\frac{\text{kilograms}}{\text{meter}^2}$; $\frac{M}{L^3}$ (M. K. S.)

the values of T, P, A, ρ, R have been interpolated, where necessary, so that the values are here given on the same selected levels for each of the gases. As all of the terms will be studied in detail, it is not now necessary to explain further the Figs. 2–9.

It will be convenient to give at this point Table 5 of the atomic and molecular weights of all the chemical elements, together with the approximate order of brightness of occurrence of those identified upon the sun generally, but in spots and faculæ especially.

a = the atomic weight referred to hydrogen, $H = 1.00$, $(O = 15.88)$.

m = the corresponding molecular weight for some elements.

a_1 = the atomic weight referred to $H = 1.008$ $(O = 16.00)$.

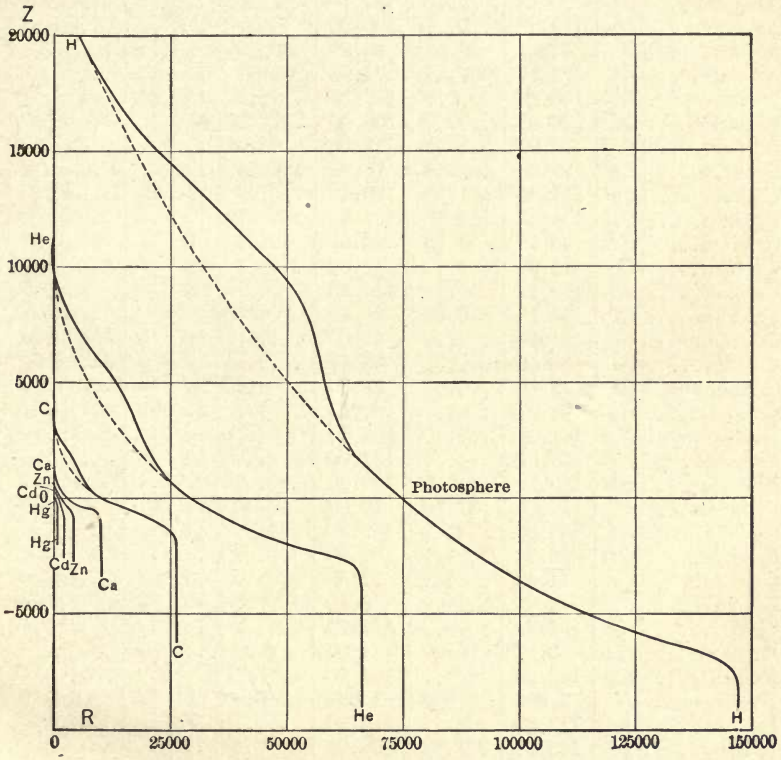


FIG. 13. R. Gas-Coefficient.

$\frac{\text{velocity}^2}{\text{degree}}$; $\frac{L^2}{T^2\text{deg.}}$ (M. K. S.)

m_1 = the corresponding molecular weight.

$\odot$ B = the order of brightness in the solar spectrum (1 to 36).

$\odot$ L = the order of the number of lines in the solar spectrum (1 to 36).

$\odot$ S = the number of lines of the element in sun-spots.

TABLE 5

THE ATOMIC AND MOLECULAR WEIGHTS

The Occurrence of the Chemical Elements on the Sun

Element	S	a $H = 1.00$	m	a_1 $H = 1.008$	m_1	$\odot B$ Order	$\odot L$ Order	$\odot S$ No.	$\odot F$ No.
Aluminum...	*Al*	26.88	.....	27.10	.....	..	..	...	...
Antimony...	*Sb*	119.24	.....	120.20	.....	..	..	...	...
Argon.....	*A*	39.56	39.56	39.88	39.88	9	25	...	...
Arsenic.....	*As*	74.37	297.48	74.96	299.84	..	..	...	...
Barium.....	*Ba*	136.28	.....	137.37	.....	15	24	2	...
Bismuth.....	*Bi*	206.35	.....	208.00	.....	*D*	*D*	...	...
Boron.....	*B*	10.91	.....	11.00	.....	..	..	...	...
Bromine.....	*Br*	79.29	158.58	79.92	159.84	..	..	...	...
Cadmium....	*Cd*	111.51	111.51	112.40	112.40	27	26	...	...
Cæsium.....	*Cs*	131.57	131.57	132.81	132.81	..	..	...	...
Calcium.....	*Ca*	39.75	39.75	40.07	40.07	1	11	60	33
Carbon.....	*C*	11.91	.....	12.00	.....	16	7	1	105
Cerium.....	*Ce*	139.14	.....	140.25	.....	28	10	2	47
Chlorine.....	*Cl*	35.18	70.36	35.46	70.92	..	..	...	...
Chromium...	*Cr*	51.59	.....	52.00	.....	11	5	386	78
Cobalt.....	*Co*	58.50	.....	58.97	.....	7	6	118	66
Columbium..	*Cb*	92.76	.....	93.50	.....	22	16	...	...
Copper.....	*Cu*	63.07	.....	63.57	.....	25	30	...	...
Dysprosium..	*Dy*	161.21	.....	162.50	.....	..	..	...	...
Erbium.....	*Er*	166.37	.....	167.70	.....	35	28	...	...
Europium...	*Eu*	150.79	.....	152.00	.....	..	..	...	...
Fluorine.....	*F*	18.85	37.70	19.00	38.00	..	..	...	...
Gadolinium..	*Gd*	156.05	.....	157.30	.....	..	..	...	...
Gallium.....	*Ga*	69.35	.....	69.90	.....	..	..	...	...
Germanium..	*Ge*	71.92	.....	72.50	.....	30	33	...	...
Glucinum....	*Gl*	9.03	.....	9.10	.....	29	32	4	2
Gold.....	*Au*	195.64	.....	197.20	.....	..	..	...	...
Helium.....	*He*	3.97	3.97	4.00	4.00	..	..	...	3
Holmium....	*Ho*	162.20	.....	163.50	.....	..	..	...	...
Hydrogen....	*H*	**1.000**	2.000	**1.008**	2.016	3	22	...	...
Indium.....	*In*	113.89	.....	114.80	.....	*D*	*D*	...	...
Iodine.....	*I*	125.91	251.82	126.92	253.84	..	..	...	...
Iridium.....	*Ir*	191.57	.....	193.10	.....	*D*	*D*	...	...
Iron.....	*Fe*	55.40	.....	55.84	.....	2	1	1108	327
Krypton.....	*Kr*	82.26	82.26	82.92	82.92	..	..	...	...
Lanthanum..	*La*	137.90	.....	139.00	.....	21	14	...	24
Lead.....	*Pb*	205.56	.....	207.20	.....	34	35	...	...
Lithium.....	*Li*	6.88	6.88	6.94	6.94	..	..	...	...
Lutecium....	*Lu*	173.61	.....	175.00	.....	..	..	...	...
Magnesium..	*Mg*	24.13	.....	24.32	.....	6	19	8	5
Manganese..	*Mn*	54.49	.....	54.93	.....	13	4	167	21
Mercury.....	*Hg*	198.47	198.47	200.06	200.06	*D*	*D*	...	...
Molybdenum	*Mo*	95.24	.....	96.00	.....	20	17	...	...
Neodymium .	*Nd*	143.16	.....	144.30	.....	24	12	3	42

TABLE 5 (*Continued*)

THE ATOMIC AND MOLECULAR WEIGHTS

The Occurrence of the Chemical Elements on the Sun

Element	S	a $H = 1.00$	m	a_1 $H = 1.008$	m_1	$\odot B$ Order	$\odot L$ Order	$\odot S$ No.	$\odot F$ No.
Neon.....	*Ne*	19.86	19.86	20.02	20.02	..	..	...	...
Nickel.....	*Ni*	58.21	.....	58.68	.....	5	2	251	71
Niobium....	*Nb*	92.76	.....	93.50	.....	..	..	...	...
Niton(radium emanation)	*Nt*	220.63	.....	220.40	.....	..	..	...	...
Nitrogen....	*N*	13.90	27.80	14.01	28.02	..	..	...	...
Osmium.....	*Os*	189.39	.....	190.90	.....	*D*	*D*	...	...
Oxygen.....	*O*	**15.88**	31.76	**16.00**	32.00	*D*	*D*	...	...
Palladium...	*Pd*	105.85	.....	106.70	.....	23	18	...	...
Phosphorus..	*P*	30.79	123.16	31.04	124.16	..	..	...	...
Platinum....	*Pt*	193.65	.....	195.20	.....	*D*	*D*	...	...
Potassium...	*K*	38.79	38.79	39.10	39.10	36	36	...	...
Praseodymium.....	*Pr*	139.78	.....	140.90	.....	..	..	...	...
Radium.....	*Ra*	224.21	.....	226.00	.....	..	..	...	...
Rhodium....	*Rh*	102.08	.....	102.90	.....	31	27	...	...
Rubidium...	*Rb*	84.77	84.77	85.45	85.45	..	..	...	...
Ruthenium..	*Ru*	100.89	.....	101.70	.....	*D*	*D*	...	...
Samarium...	*Sa*	149.21	.....	150.40	.....	..	..	...	...
Scandium....	*Sc*	43.75	.....	44.10	.....	17	13	45	34
Selenium....	*Se*	78.57	157.14	79.20	158.40	..	..	...	...
Silicon.....	*Si*	28.08	.....	28.30	.....	..	..	...	4
Silver.....	*Ag*	107.02	.....	107.88	.....	32	31	...	...
Sodium.....	*Na*	22.82	22.82	23.00	23.00	4	20	8	6
Strontium...	*Sr*	86.93	.....	87.63	.....	12	23	2	2
Sulfur.....	*S*	31.81	63.62	32.06	64.12	..	..	...	...
Tantalum....	*Ta*	180.06	.....	181.50	.....	*D*	*D*	...	...
Tellurium...	*Te*	126.49	252.98	127.50	255.00	..	..	...	...
Terbium.....	*Tb*	157.94	.....	159.20	.....	..	..	...	...
Thallium....	*Tl*	202.38	.....	204.00	.....	*D*	*D*	...	...
Thorium.....	*Th*	230.56	.....	232.40	.....	*D*	*D*	...	...
Thulium.....	*Tm*	167.16	.....	168.50	.....	..	..	...	...
Tin.....	*Su*	117.76	.....	118.70	.....	33	34	...	...
Titanium....	*Ti*	47.72	.....	48.10	.....	10	3	432	131
Tungsten....	*W*	183.00	.....	184.00	.....	*D*	*D*	...	...
Uranium....	*U*	236.31	.....	238.20	.....	*D*	*D*	...	...
Vanadium...	*V*	50.59	.....	51.00	.....	14	8	176	42
Xenon.....	*Xe*	129.71	129.71	130.20	130.20	..	..	...	...
Ytterbium...	*Yb*	172.12	.....	173.50	.....	..	..	...	...
Yttrium.....	*Yt*	88.00	.....	88.70	.....	18	15	3	19
Zinc.....	*Zn*	64.85	64.85	65.37	65.37	26	29	3	2
Zirconium...	*Zr*	89.88	.....	90.60	.....	19	9	7	...

$\odot F$ = the number of lines of the element in the flash spectrum.

D = doubtful identification in the solar spectrum.

The order of brightness is taken from Abbot's Sun, page 206.

The number in sun-spots, *Astrophysical Journal*, June, 1913, St. John.

The number in the flash spectrum, *Astrophysical Journal*. March, 1915, Mitchell.

I am indebted to Professor F. W. Clarke for the revised values of the atomic weights, which are those of the International Committee for 1916.

The Distribution of the Temperatures

The computations were conducted in height by taking as 0, or the reference plane, the point of equilibrium that was derived from the terrestrial data through the factor γ, but as this point has no special significance in solar physics it has been found convenient to define two other planes of reference, (1) the surface of the photosphere, and (2) the bottom of the isothermal layer. By means of the spectroheliographic observations made at Mt. Wilson, and by other spectroscopic determinations, the pressure on the photosphere lies between 5.80 and 6.00 standard terrestrial atmospheres, and this has been accepted as the definition of the level of the photosphere. Compare Fig. 11, where all the pressure curves pass through this point, no matter to what heights and depths the several pressures extend in their distribution. This is confirmed as correct by the general fact that the diatomic hydrogen vanishes at 25000 kilometers above the photosphere, at the top of the inner corona as observed in eclipses, that the top of the chromosphere, 5000 kilometers, is at the level where the hydrogen pressure and density become very small, as in Figs. 11, 12. The light gases extend beyond the level of the reversing layer which is at 400–500 kilometers, but the heavy gases do not reach it normally, only in great disturbances, and so do not generally appear in the flash spectrum of the Fraunhofer lines. The vanishing monatomic molec-

ular weights range for the flash spectrum from 60 to 80, so that helium (4), carbon (as 12), calcium (40), zinc (65), appear readily, while cadmium (112), mercury (198), are seen with difficulty. Thus, the flash spectrum by Adams* gives somewhat the following distribution of lines by numbers as counted: Carbon (12), 105; calcium (40), 33; cerium (139), 47; cobalt (59), 66; chromium (52), 78; iron (55), 327; glucinum (9), 2; helium (4), 3; lanthanum (138), 24; magnesium (24), 5; manganese (54), 21; sodium (23), 6; neodymium (143), 42; nickel (58), 71; scandium (44), 34; silicon (28), 4; strontium (87), 2; titanium (48), 131; vanadium (51), 42; yttrium (89), 19; zinc (65), 2; as contained in the tables. The exceptions are cerium (139), lanthanum (138).

Table 6 contains the temperatures, and it shows clearly the distribution of the isothermal temperature, deep for hydrogen and shallow for mercury, with non-adiabatic layers above and adiabatic layers below it. It is seen, from Fig. 11, that the pressures have one point in common in passing the plane of the photosphere at 6 atmospheres, and from Fig. 10, that the temperatures have one nearly vertical line in passing the photosphere, namely, the isothermal layer, having a temperature for each gas of about 7650° at the bottom, and of about 7710° at the top of this characteristic layer. The means refer to the temperatures in the isothermal region to the left of the broken line of separation. The non-adiabatic region is above this line and the adiabatic region is below it. *The mean value of the temperature of the isothermal layer is about 7687° absolute Centigrade.* Compare the Table 6 and Figs. 2 to 9. These figures were constructed on different convenient vertical scales of heights for ordinates, and with various abscissas for P, ρ, R, T, respectively, in order to show the interrelation of these quantities for each gas; the Figs. 10 to 13 bring together on the same vertical scale all the elementary gases, for temperature, pressure, density, gas coefficient, in order to determine the general laws of their distribution. It is easily found that the equilateral hyperbola, $x\,y = c$, is fundamental and since this passes over into the

* *Astroph. Journ.*, March, 1915.

TABLE 6

THE TEMPERATURES OF THE DIFFERENT ELEMENTS ON THE SAME LEVELS

Z Kilometers	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zu 65	Cd 112	Hg 198	Means
25000....	3320°	0°	.....	Top of the Inner Corona					
20000....	4300	955	.....	.....	.....	.....	.....	.....	.....
15000....	5370	2055	0	.....	.....	.....	.....	.....	.....
10000....	6600	3495	740	.....	.....	.....	.....	.....	.....
5000....	7550	5890	3780	0	0	Top of Chromosphere			.....
1000....	7696	7680	7600	6450	300	.....	.....	.....	7688
900....	7710	7679	7650	6700	900	.....	.....	.....	7680
800....	7708	7678	7670	7000	1500	.....	.....	.....	7685
700....	7706	7677	7690	7250	2200	0	.....	.....	7691
600....	7705	7676	7705	7500	2950	640	.....	.....	7695
500....	7704	7675	7715	7615	3750	1790	Reversing layer		7698
400....	7703	7674	7714	7715	4700	3120	0	.....	7702
300....	7702	7673	7713	7710	5750	4480	1200	0	7705
200....	7702	7672	7712	7705	6800	5940	2900	2500	7698
100....	7701	7671	7711	7704	7500	7500	5700	3950	7697
90....	7701	7671	7710	7703	7600	7600	6000	4450	7696
80....	7701	7671	7710	7702	7650	7650	6600	4950	7681
70....	7701	7671	7709	7701	7690	7690	6950	5450	7694
60....	7701	7671	7709	7700	7705	7720	7300	5900	7701
50....	7701	7671	7708	7699	7715	7735	7475	6350	7705
40....	7701	7670	7708	7698	7713	7730	7650	6750	7696
30....	7701	7670	7707	7698	7711	7725	7670	7100	7696
20....	7701	7670	7707	7697	7709	7720	7690	7400	7699
10....	7701	7670	7706	7696	7707	7718	7687	7600	7698
Photos. 0..	7701	7670	7705	7695	7705	7715	7684	7670	7693
− 10....	7701	7670	7705	7694	7703	7711	7680	7667	7691
− 20....	7701	7670	7704	7693	7701	7707	7677	7663	7690
− 30....	7701	7670	7704	7692	7699	7703	7673	7659	7688
− 40....	7701	7670	7703	7691	7697	7699	7669	7655	7686
− 50....	7701	7669	7703	7691	7695	7695	7665	7651	7684
− 60....	7701	7669	7702	7690	7693	7691	7660	7800	7687
− 70....	7701	7669	7702	7689	7691	7687	7656	8550	7685
− 80....	7701	7669	7701	7688	7689	7683	7652	9494	7683
− 90....	7701	7669	7700	7687	7687	7679	7686	10436	7687
− 100....	7700	7669	7699	7686	7685	7675	7720	11381	7690
− 200....	7699	7668	7698	7685	7665	7700	12742	20816	7686
− 300....	7698	7667	7697	7680	7676	10713	18045	30251	7684
− 400....	7697	7666	7696	7675	8651	13797	23348	39686	7684
− 500....	7696	7665	7695	7670	10553	16881	28651	49121	7682
− 600....	7695	7664	7693	7665	12455	19965	33954	58556	7684
− 700....	7694	7663	7691	7658	14357	23049	39257	67991	7677
− 800....	7694	7662	7689	7651	16259	26133	44560	77426	7674
− 900....	7693	7661	7687	7646	18161	29217	49863	86861	7672
− 1000....	7692	7660	7685	7640	20063	32301	55166	96296	7669
− 2000....	7691	7662	7665	11974	39083	63140	108193	190648	7670
− 3000....	7686	7676	7680	17680	58103	93979	161220	285000	7681
− 4000....	7681	7640	9451	23387	77123	124818	214247	379352	7661
− 5000....	7676	7651	11354	29094	96143	155657	267274	473704	7661
− 6000....	7671	7665	13255	34801	115163	186496	320201	568056	7668
− 7000....	7666	7690	15157	40508	134183	217335	373228	662408	7678
− 8000....	7661	8000	17059	46215	153203	248174	426255	756760	7686.7
− 9000....	7656	8500	18961	51922	172223	279013	479282	851112	
−10000....	7651	9187	20868	57629	191243	309852	532309	945470	
−12000....	7690	10561	22765	63336	229283	340691	585336	1039822	
−14000....	8476	11934	24667	69043	267323	371530	638363	1134174	

−47.554 −686.90 −1902.17 −5706.5 −19021.7 −30839.3 −53027.5 −94352.0

Adiabatic gradient per 1000 kilometers.

logarithmic law of decrease or depletion, this distribution conforms to the natural law of physical conditions.

The bottom of the isothermal layer is nearly the same as the top of the adiabatic strata, making some allowance for the transition from the adiabatic to the isothermal conditions; the top of the isothermal layer is nearly coincident with the bottom of the non-adiabatic strata. The adiabatic lines of pressure are nearly parallel to the plane of the photosphere, especially for the heavy gases; the non-adiabatic lines of pressure are nearly vertical to the plane of the photosphere; the isothermal layer of temperature occurs within the layers where the pressure is changing rapidly from very large to very small gradients. The density line for the light gases in the adiabatic layers is nearly vertical, as hydrogen, but horizontal for the heavy gases; the gas coefficient has a very great range for hydrogen, but contracts to a very short range for mercury. It follows that the solar atmospheres constitute a very perfect thermodynamic engine in the adiabatic and isothermal layers, but one of less efficiency in the non-adiabatic region, which can be reduced to corresponding secondary adiabatic and isothermal components.

The relations between the height z, the atomic weight m, and the temperature T, are very interesting and instructive, as can be seen in the following tables:

TABLE 7

BOTTOM, TOP, AND DEPTH OF THE ISOTHERMAL LAYER

Take m = the atomic weight, or molecular weight (monatomic).
z_A = the bottom of the isothermal layer
z_I = the top of the isothermal layer
$z_I - z_A$ = the depth of the isothermal layer

Element	m	z_A	z_I	$z_I - z_A$
Hydrogen H_1	1.00	−12000	+3600	15600
Hydrogen H_2	2.00	− 6000	+1800	7800
Helium He	4.00	− 3000	+ 900	3900
Carbon C	12.00	− 1000	+ 300	1300
Calcium Ca	40.00	− 300	+ 90	390
Zinc Zn	65.00	− 185	+ 55	240
Cadmium Cd	112.00	− 107	+ 32	139
Mercury Hg	198.00	− 60	+ 18	78

Hence, the following general hyperbolic laws prevail:

Bottom of the isothermal layer, $m \cdot z_A = -12000$ kilometers
Top of the isothermal layer, $m \cdot z_I = +3600$ "
Depth of the isothermal layer, $m.(z_I - z_A) = 15600$ "

The corresponding depths for any other gas can be found on the assumption that it is monatomic, but since other molecular combinations may prevail above a certain level within the isothermal layer, the conditions on the sun become so complicated by the association of the atoms into molecules of different configurences and valences that it is necessary to reserve this portion of the subject.

The Heights at which the Same Temperature Occurs for the Different Elements, by the Hyperbolic Law, $m\,z = C_T$.

An inspection of Fig. 10, which gives the temperature lines of the different elements, on the same scales of ordinates and abscissas, suggests that the curves form a family whose parameter is m, connected by the law of the equilateral hyperbola, $m\,z = C_T$, where C_T is a constant for a selected temperature T. To test this theorem, the heights z at which the same temperature T occurs for the different elements m, was interpolated from the original tables, and the results are collected in Table 8 for certain temperatures, $T = 000°, 1000°, 2000°, \ldots 12000°$. The first column contains the element and its atomic weight m, the second the height z at which the temperature T was computed by the thermodynamic formulas, the third the product $m\,z$. It is easily seen that the products tend toward a constant value for the same T, and the mean is the constant C_T, referred to the photosphere. Hence, we obtain the hyperbolic constants from which the height may be computed where the given temperature occurs in the different elements.

$T =$	0°	occurs at the height	45400/m	in kilometers
	1000	" " "	37792	" " "
	2000	" " "	30246	" " "
	3000	" " "	24304	" " "
	4000	" " "	19611	" " "
	5000	" " "	15422	" " "
	6000	" " "	11713	" " "
	7000	" " "	7826	" " "

Isothermal Layer								
8000	occurs	at	the	height	−14036/m	in	kilometers	
9000	"	"	"	"	−16594	"	"	"
10000	"	"	"	"	−18972	"	"	"
11000	"	"	"	"	−21223	"	"	"
12000	"	"	"	"	−23522	"	"	"

The non-adiabatic group can be referred to the bottom of the isothermal layers by adding 12000 to the respective values C_T, as $45400 + 12000 = 57400$ for the temperature $T = 0°$. The first group for $T = 0°$ gives the height of the vanishing plane of the several elements. Hydrogen does not exist in the monatomic state, $m = 1.00$ above the photosphere, but only in the diatomic form of the molecule for $m = 2.00$. It is, however, very convenient to have $H = 1.00$ fully computed, because it is the implied standard of reference for all the chemical elements and the entire scheme is more readily understood by reference to this $H = 1.00$, as if it actually existed. It will be shown that the molecule $H_2 = 2.00$ does not exist below a certain plane in the isothermal layer, but only in the atomic form $H_1 = 1.00$. It was necessary to use $H_1 = 1.00$ in certain levels, and $H_2 = 2.00$ in other levels, and this constitutes an important piece of evidence that at certain values of P, ρ, R, T, there is dissociation of H_2 into two atoms of H_1, or conversely that at this level there is association of two atoms of H_1 into one molecule H_2. An examination of the products $m\,z$ of Table 8 seems to indicate that the variations are quite accidental in their general arrangement. It should be remembered that the data have been obtained by the method of trials applied in succession to eight elements, so that they are entirely independent of one another. The general process of the computations, therefore, resulted in a family of equilateral hyperbolas, which can be readily developed for the fundamental theory of the distribution of the solar elements. Since the Naperian logarithms are based upon the law of the equilateral hyperbola, it follows that an exponential law of decay or depletion must be placed at the foundations of solar gaseous distributions in height under the force of the gravity acceleration there existing. As already stated, it was found that the first four elements

TABLE 8

DISTRIBUTION OF THE TEMPERATURE HEIGHTS BY THE HYPERBOLIC LAW, $mz = (\text{CONST.})_T$

Temp.		$T = 000°$		$T = 1000°$		$T = 2000°$		$T = 3000°$		$T = 4000°$	
m		z	mz	z	mz	z	mz	z	mz	z	mz
H_1	1..	48000	48000	42000	42000	32900	32900	26800	26800	21500	21500
H_2	2..	25500	51000	19740	39480	14710	29200	11460	22920	8720	17440
He	4..	11500	46000	9500	38000	7680	30720	6060	24240	4720	18880
C	12	4000	48000	3280	39360	2690	32280	2040	24480	1660	19920
Ca	40..	1050	42000	880	35200	727	29080	594	23760	472	18880
Zn	65..	675	43900	565	36725	483	31395	408	26520	335	21775
Cd	112..	380	42600	312	34944	247	27664	196	21952	151	16912
Hg	198..	210	**41500**	185	**36630**	144	28512	120	**23760**	109	**21582**
To the Photosphere			45400		37792		30246		24304		19611
			12000		12000		12000		12000		12000
Bottom of Isothermal layer			57400		49792		42246		36304		31611

Temp.		$T = 5000°$		$T = 6000°$		$T = 7000°$	
m		z	mz	z	mz	z	mz
H_1	1..	16600	16600	12400	12400	8400	8400
H_2	2..	6560	13120	4800	9600	2910	5820
He	4..	3670	14680	2720	10080	1770	7080
C	12..	1500	18000	1160	13920	800	9600
Ca	40..	370	14800	277	11080	226	9040
Zn	65..	263	17095	196	12740	131	8515
Cd	112..	120	13440	93	10416	68	7616
Hg	198..	79	**15642**	68	**13464**	33	**6534**
To the Photosphere			15422		11713		7826
Bottom of Isothermal layer			12000		12000		12000
			27422		23713		19826

The temperature of the top of the isothermal layer is 7700; of the bottom 7620; of the photosphere 7687; of the radiation layer 7655.

The constant for the photosphere + 12000 becomes the constant for the bottom of the isothermal layer.

Temp.		$T = 8000°$		$T = 9000°$		$T = 10000°$		$T = 11000°$		$T = 12000°$	
m		z	mz	z	mz	z	mz	z	mz	z	mz
H_1	1..	−13000	−13000	−15062	−15062	−17200	−17200	−19300	−19300	−21450	−21450
H_2	2..	− 8000	−16000	− 9740	−19580	−11207	−22414	−12630	−25260	−14080	−28160
He	4..	− 3220	−12880	− 3772	−15088	− 4200	−16800	− 4815	−19260	− 5345	−21380
C	12..	− 1267	−15204	− 1482	−17784	− 1660	−19820	− 1827	−21924	− 2000	−24000
Ca	40..	− 366	−14640	− 418	−16720	− 472	−18880	− 523	−20920	− 576	−23040
Zn	65..	− 211	−13715	− 245	−15925	− 275	−17875	− 309	−20085	− 342	−22230
Cd	112..	− 107	−11984	− 130	−14560	− 149	−16680	− 167	−18704	− 189	−21168
Hg	198..	− 53	**−10494**	− 75	**−14850**	− 86	**−17028**	− 96	**−19008**	− 107	**−21186**
To the Photosphere			−13490		−16196		−18337		−20558		−22827

pointed to the existence of this elementary law, so that it became easy to locate the temperature curves of *He*, *C*, *Zn*, *Cd*, from which fact the later computations were greatly facilitated.

On Fig. 14, the values of C_T have been plotted, showing that

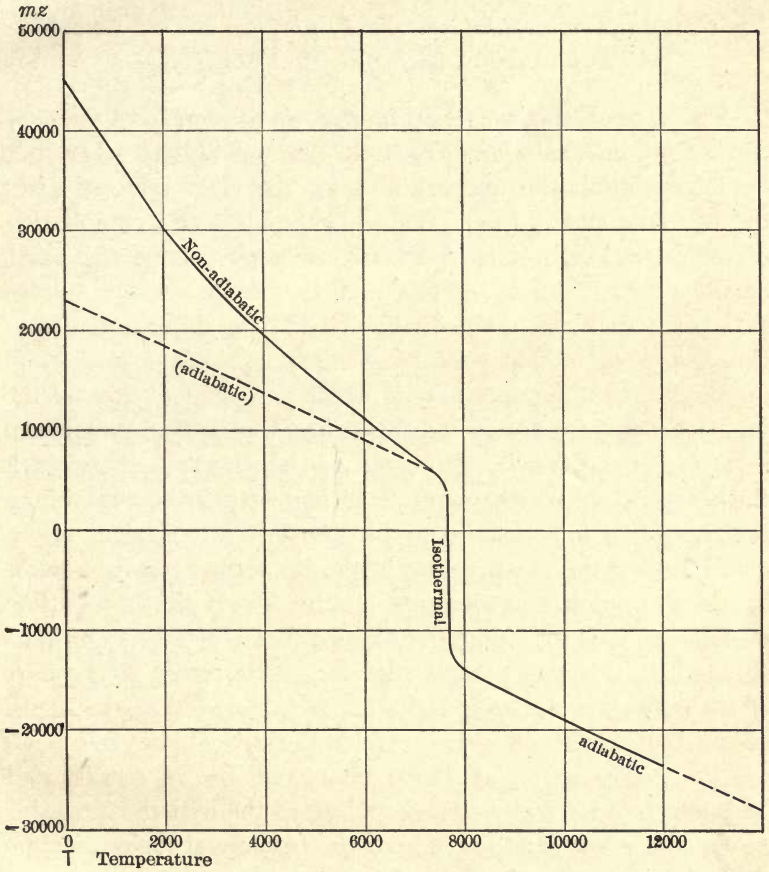


FIG. 14. Distribution of the Temperature Heights by the Hyperbolic Law, $mz = (\text{Const})_T$.

there is a convex curvature downward in the non-adiabatic region, a nearly vertical line in the isothermal region, and a straight line in the adiabatic region. An auxiliary adiabatic line has been drawn in the non-adiabatic region, and by resolution the non-adiabatic curve can be broken up into its adiabatic

and isothermal components. The non-adiabatic branch terminates on the ordinate $T = 0°$, but the adiabatic branch can be extended as far downward as the law of gases prevails.

The Constant Temperature Near the Photosphere and the Variations of Temperature Outside of the Isothermal Region

The temperatures of the different gases were computed at convenient intervals, differing from one gas to the other, but by interpolation the temperatures for the eight selected gases can be found on any level. Table 6 contains such a compilation of solar temperatures from 25,000 kilometers above the plane of the photosphere to − 14000 kilometers below that plane. It is seen that the table divides itself by a broken line into three parts, the middle containing nearly constant or isothermal temperatures; the upper part in which the temperatures on the same level decrease from a maximum for $H_1 = 1.00$ to a minimum or to 0° for the heavier elements; and the lower part in which the temperature on the same level increases from a minimum for $H_1 = 1.00$ to a maximum for the heaviest element $Hg = 198$. Taking the mean values in the isothermal region, according to the number of elements on the left of the broken line of division, it is seen that the temperature is nearly a constant throughout this region, and that *the general mean temperature of the isothermal region is* 7686.°7. It is noted that the depth of the isothermal region is great for the light gases, and small for the heavy gases, as 15600 kilometers for $H_1 = 1.00$ and 78 kilometers for $Hg = 198$, according to the hyperbolic laws.

On the other hand, outside of the isothermal region, on the same level there are enormous changes in temperature from the light to the heavy gases, as 7696° to 0° on the 1000-kilometer level, or 7651° to 945470° on the − 10000-kilometer level. In the adiabatic region the temperatures have been extended below the actual levels of the computations by merely adding the adiabatic gradients. These can be studied for what they may be worth, but it is thought that at such high temperatures, pressures, densities as are implied, the simple gaseous law,

$P = \rho R T$, must become greatly modified in the passing of the gas into its liquid, viscous, or solid state.

It is perhaps superfluous to remark that in the solar envelope there is no constant temperature except in the isothermal layers near the photosphere, and that the popular idea of thinking of the sun as having fixed temperatures on the same level is erroneous. By equation (6) it is seen that temperature has no dimensions, such as pressure, density, and gas efficiency possess, but that it merely represents a sort of thermal equilibrium

$$T = \frac{P}{\rho R}, \left[\frac{M}{L T^2} \cdot \frac{L^3}{M} \cdot \frac{T^2 \deg}{L^2} = \deg.\right]$$

T being merely a ratio, the same value occurs at different heights for the different elements, or else different values of the ratio T occur on the same level for the different elements. Indeed, the temperature ratios are so very complicated in all atmospheres composed of different gases as to become intelligible only after special analyses. The energy of radiation originates as a special effect within the isothermal layers of the different gases, so that it is proper to speak of a constant temperature of radiation, but this ignores the complex phenomena that exist in the non-adiabatic and the adiabatic regions. Similarly, the temperature of a star as determined in the spectrum refers only to the source of radiation in its isothermal layer, and not to the other temperatures above or below that region. It should be noted that on Fig. 10, and in Table 6, the *temperature of the black body solar radiation is somewhat less than* 7687° *Centigrade Absolute, and it will be shown to be* 7655°. *There is nothing characteristic of the solar radiation at the temperature* 5810°, *as deduced from the pyrheliometer data by Mr. Abbot and other observers.* Indeed, it only remains to determine the value of the coefficient σ in,

$$J_0 = \left(\frac{R}{D}\right)^2 \sigma T^4, \tag{92}$$

to obtain the solar constant of radiation at the distance of the earth. We shall show that σ at the sun's isothermal layer agrees

with the Kurlbaum coefficient, and hence that $J_0 = 5.85$ gr. cal./cm.^{2} min. before any depletions occur in the solar and in the terrestrial atmospheres.

The Distribution of the Pressures

The values of P, the pressure in kilograms per meter^2, and the values of A, the pressure in standard terrestrial atmospheres, have been collected in Tables 9 and 10 respectively, for the same values of z, for the eight elements. The original computations contain many more points between $z = 1000$ and $z = 25000$, or the vanishing plane of the gas. It is necessary to use P in all thermodynamic discussions, but A is a practical unit that may be conveniently employed in descriptive explanations, and its meaning is easily understood in the different levels. It is to be noted that P has almost exactly the same value on the plane of the photosphere for each gas, the mean value being,

$$P = 616190 \text{ kil./m.}^2,$$
$$A = 6.0814 \text{ standard atmospheres.}$$

There is no other plane on which this relation holds, all the pressures diminishing in horizontal lines above the photosphere, from H_1 to the heavy gases, and increasing below the photosphere from H_1 to the heavy gases. In the adiabatic region, below the photosphere, these computations can be extended to enormous pressures corresponding with the great temperatures.

On the other hand, in vertical directions, the hyperbolic law $m\,z = (\text{Const})_P$ for a selected pressure holds true as shown by Table 11. The height of the vanishing plane is seen under the first section for $A = 0$, where the product approximates the mean value $m\,z = 45002$, and similarly in the following sections.

TABLE 9

THE PRESSURE OF THE DIFFERENT ELEMENTS ON THE SAME LEVELS
(Kilograms /meter^2), P (M. K. S.)

Z Kilometers	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	Means
25000	2648.2	2.727×10^{-16}		Top of	the Inner	Corona			
20000	12877	0.03778	.....	.....	.....	.....	.....	.....	.....
15000	44710	110.52	.....	.....	.....	.....	.....	.....	.....
10000	120450	8165.6	0.24958	.....	.....	.....	.....	.....	.....
5000	276700	108265	10770.3	.....	.....	Top of	Chromo	sphere	
1000	515610	440880	328086	92408	7.755×10^{-5}	.....	.....	.....	.....
900	523640	454850	348870	114260	0.46894	.....	.....	.....	.....
800	531780	468670	371020	141280	26.881	.....	.....	.....	.....
700	540000	483210	394560	172020	362.57	.....	.....	.....	.....
600	548440	498220	419610	209435	2350.4	0.771869	.....	.....	.....
500	556960	513688	446250	252660	9785.8	83.238	Reversi	ng Layer	
400	565630	529640	474760	304807	30407	2110.5	.....	.....	.....
300	574430	546110	505060	367570	75783	16411	3.7724	$_{-5}$	.....
200	583350	563090	537290	443250	148280	73052	4185.1	3.150×10	.....
100	592410	580600	571590	534560	287830	231670	110313	11380	.....
90	593350	592390	575100	544300	306410	256110	140109	19967	.....
80	594250	584170	578670	554210	326180	283120	169904	32999	.....
70	595170	585960	582260	564320	347230	312970	209405	51958	.....
60	596090	587760	585890	574000	369640	345990	248906	78746	.....
50	597010	589560	589530	585070	393510	382510	301903	115760	.....
40	597930	591370	593190	595740	418850	422470	354900	165960	.....
30	598860	593190	596870	606600	445830	466620	428378	233350	.....
20	599780	595000	600580	617620	474540	515380	501856	323060	.....
10	600700	596830	604310	628860	505110	569230	606612	442456	.....
Photos.	601630	598657	608063	640370	537620	628814	711367	603000	616190
−10	602570	600550	611790	652700	571630	695150	856834	816820	.....
−20	603500	602460	615510	665240	607760	768500	1002300	1121320	.....
−30	604450	604390	619290	678050	646200	849580	1211515	1526210	.....
−40	605390	606300	623060	691080	687070	939200	1420730	**2090450**	2593212
−50	606330	608230	626870	704380	730520	1038290	1713140	2839900	.....
−60	607270	610170	630700	717930	777570	1147800	2005550	3853640	.....
−70	608210	612110	634560	731750	827660	1268900	2422725	5144000	.....
−80	609160	614060	638430	745820	880960	1402800	**2839900**	6682670	.....
−90	610110	616010	642330	760170	937720	1550800	3425725	8468400	.....
−100	611060	617970	646250	774780	998070	**1714440**	4011550	10514000	.....
−200	620670	637910	686820	931240	**2064900**	4700900	15310000	.....	.....
−300	630430	658490	729930	1119300	3871580	10972600	.....	.....	.....
−400	640360	679730	775750	1345370	6955330	.....	.....	.....	.....
−500	650430	701667	824440	1622400	11488400	.....	.....	.....	.....
−600	660670	723480	877240	1956450	17435200	.....	.....	.....	.....
−700	671060	746020	933400	**2357200**	24918300	.....	.....	.....	.....
−800	681620	769250	993160	2839900	34050000	.....	.....	.....	.....
−900	692340	793220	1056760	3423500	44937800	.....	.....	.....	.....
−1000	703230	817920	1124390	4127100	57678750	.....	.....	.....	.....
−2000	821980	1115480	**2082760**	19705500	306112700	.....	.....	.....	.....
−3000	958160	1522400	3873360	52206000	.....	.....	.....	.....	.....
−4000	1116920	**2078830**	**6776430**	105061000	.....	.....	.....	.....	.....
−5000	1305550	2839900	10718300	.....	.....	.....	.....	.....	.....
−6000	1526000	3874000	15786300	.....	.....	.....	.....	.....	.....
−7000	1778800	5287000	22074500	.....	.....	.....	.....	.....	.....
−8000	2073500	7158670	.....	.....	.....	.....	.....	.....	.....
−9000	**2426700**	9555800	.....	.....	.....	.....	.....	.....	.....
−10000	2839900	12498900	.....	.....	.....	.....	.....	.....	.....
−12000	3348700	20247000	.....	.....	.....	.....	.....	.....	.....
−14000	3858900	30917100	.....	.....	.....	.....	.....	.....	.....
Radiation layer Z_R	−9000	−4500	−2250	−750	−225	−140	−80	−45	
Pressure	2426700	2459365	2530410	2598550	2516570	2909024	2839900	2465175	2593212

TABLE 10

THE PRESSURE OF THE DIFFERENT ELEMENTS ON THE SAME LEVELS (TERRESTRIAL ATMOSPHERES).

Z Kilometers	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	Means
25000	0.026136	2.692×10^{-19}		Top of the	Inner Corona				
20000	0.07112	3.728×10^{-7}							
15000	0.43620	0.0011	—			(M.K.S.)			
10000	1.1888	0.0806	2.463×10^{-7}						
5000	2.7315	1.0687	0.1063	—		Top of Chrom	osphere		
1000	5.0888	4.3512	3.2374	0.9120	7.654×10^{-10}				
900	5.1680	4.4861	3.4431	1.1277	4.628×10^{-6}				
800	5.2483	4.6254	3.6617	1.3944	2.653×10^{-4}				
700	5.3299	4.7690	3.8941	1.1277	.003579	—			
600	5.4128	4.9171	4.1413	2.0670	.023197	7.093×10^{-7}			
500	5.4969	5.0698	4.4042	2.4936	.09658	8.215×10^{-4}	Reversing Layer		
400	5.5824	5.2272	4.6856	3.0082	.30009	.020829			
300	5.6691	5.3898	4.9846	3.6277	.74793	.16197	3.723×10^{-5}		
200	5.7573	5.5574	5.3028	4.3746	1.4634	.72097	.04130	3.108×10^{-10}	
100	5.8469	5.7303	5.6411	5.2581	2.8405	2.2864	1.0887	.1123	
90	5.8559	5.7478	5.6759	5.3556	3.0241	2.5277	1.3512	.1971	
80	5.8649	5.7654	5.7111	5.4551	3.2193	2.7943	1.6768	.3257	
70	5.8740	5.7831	5.7466	5.5564	3.4271	3.0891	2.0296	.5128	
60	5.8830	5.8009	5.7823	5.6595	3.6543	3.4149	2.4565	.7772	
50	5.8921	5.8186	5.8183	5.7646	3.8836	3.7751	2.9333	1.1424	
40	5.9011	5.8364	5.8544	5.8716	4.1335	4.1695	3.5027	1.6379	
30	5.9103	5.8544	5.8907	5.9807	4.3997	4.6054	4.1652	2.3030	
20	5.9194	5.8723	5.9274	6.0917	4.6831	5.0869	4.9530	3.1885	
10	5.9286	5.8903	5.9641	6.2049	4.9849	5.6186	5.8970	4.3668	
Photos. 0	5.9377	5.9084	6.0012	6.3200	5.3060	6.2060	7.0207	5.9513	6.0814

−10	5.9468	5.9271	6.0380	6.4397	5.6413	6.8607	8.3188	8.0616
−20	5.9569	5.9460	6.0748	6.5614	5.9980	7.5845	10.007	11.067
−30	5.9654	5.9649	6.1120	6.6854	6.3773	8.3848	11.845	15.028
−40	5.9747	5.9839	6.1493	6.8118	6.7807	9.2694	14.022	**20.631**
−50	5.9841	6.0029	6.1869	6.9405	7.2097	10.248	16.660	28.028
−60	5.9935	6.0220	6.2246	7.0717	7.6738	11.325	19.793	38.033
−70	6.0029	6.0411	6.2629	7.2053	8.1680	12.523	23.554	50.768
−80	6.0123	6.0604	6.3009	7.3414	8.6942	13.844	**28.028**	65.954
−90	6.0217	6.0797	6.3394	7.4802	9.2542	15.304	33.312	83.578
−100	6.0311	6.0990	6.3781	7.6215	9.8502	**16.920**	39.591	103.76
−200	6.1259	6.2958	6.7784	9.1908	**20.380**	46.395	151.10	384.17
−300	6.2223	6.4989	7.2040	11.047	38.210	108.29		
−400	6.3200	6.7086	7.6562	13.278	68.645	203.83		
−500	6.4194	6.9250	8.1368	16.012	113.384			
−600	6.5203	7.1403	8.6378	19.309	172.076			
−700	6.6230	7.3628	9.2122	**23.264**	245.928			
−800	6.7270	7.5920	9.8020	28.028	336.054			
−900	6.8328	7.8285	10.430	33.788	**443.510**			
−1000	6.9402	8.0724	11.085	40.732	569.263			
−2000	8.1124	11.009	**20.556**	194.48	3021.143			
−3000	9.4565	15.025	38.226	503.52				
−4000	11.023	**20.517**	66.879	1036.9				
−5000	12.885	28.028	105.781					
−6000	15.061	38.234	155.80					
−7000	17.556	52.180	217.86					
−8000	20.469	70.652						
−9000	**23.949**	94.260						
−10000	28.028	123.36						
−12000	32.671	199.82						
−14000	38.085	305.14						

The mean pressure for all elements at the levels where black radiation originates is 2593212 kilograms per square meter, or kinetic energy per cubic meter.

TABLE 11

THE DISTRIBUTION OF THE HEIGHTS FOR PRESSURE BY THE HYPERBOLIC LAW, mz = (CONST.)P

log P		—		5.0057		5.3067		5.4828		5.6078	
A		0		1		2		3		4	
	m	z	mz	z	mz	z	mz	z	mz	z	mz
H_1	1...	48000	48000	10910	10910	6940	6940	4400	4400	2550	2550
H_2	2...	25000	50000	5660	11320	3340	6680	2700	5400	1770	3540
He	4...	11000	44000	3210	12840	1750	7000	1120	4480	660	2640
C	12...	4000	48000	957	12440	616	8008	402	5226	247	3211
Ca	40...	1050	42000	253	10120	154	6160	91	3600	50	2000
Zn	65...	675	43875	174	11375	112	7280	73	4745	44	2860
Cd	112...	380	42560	103	11536	71	7952	49	5488	32	3584
Hg	198...	210	41580	63	12474	34	6732	22	4356	13	2574
Means.....			45002		11627		7094		4712		2870

log P		5.7047		5.7839		5.8508		5.9088		5.9600	
A		5		6		7		8		9	
	m	z	mz	z	mz	z	mz	z	mz	z	mz
H_1	1...	1180	1180	−80	−80	−1070	−1070	−1810	−1810	−2680	−2680
H_2	2...	540	1080	−60	−120	−540	−1080	−980	−1960	−1345	−2690
He	4...	250	1000	0	0	−254	−1016	−473	−1892	−663	−2652
C	12...	127	1524	+31	+372	−55	−660	−127	−1651	−189	−2457
Ca	40...	29	1160	−10	−400	−26	−1040	−47	−1880	−66	−2640
Zn	65...	22	1430	+3	+195	−12	−780	−25	−1625	−37	−2405
Cd	112...	12	1344	+1	+112	−7	−784	−16	−1792	−23	−2576
Hg	198...	6	1238	0	0	−6	−1188	−10	−1980	−13	−2574
Means.....			1188		10		−952		−1824		−2582

log P		6.0057		6.3067		6.4828		6.6078		6.7047	
A		10		20		30		40		50	
	m	z	mz	z	mz	z	mz	z	mz	z	mz
H_1	1...	−3350	−3350	−7870	−7870	−10450	−10450	−12300	−12300	−13165	−13165
H_2	2...	−1690	−3380	−3930	−7960	−5220	−10440	−6150	−12300	−6870	−13740
He	4...	−833	−3332	−1963	−7852	−2610	−10440	−3078	−12312	−3435	−13400
C	12...	−246	−3198	−620	−8060	−837	−10881	−990	−12870	−1112	−13344
Ca	40...	−85	−3400	−198	−7920	−261	−10440	−307	−12280	−340	−13600
Zn	65...	−48	−3120	−117	−7605	−157	−10205	−185	−12025	−208	−13520
Cd	112...	−30	−3360	−69	−7728	−94	−10528	−111	−12432	−122	−13664
Hg	198...	−17	−3366	−39	−7722	−52	−10298	−62	−12276	−69	−13662
Means.....			−3313		−7840		−10460		−12349		−13412

log P is the logarithm of the pressure P in kilos./meter^2.

A = the pressure in terrestrial atmospheres.

Unit A = 101323.5 kilos./meter^2 [5.00571].

log P − 5.00571 = log. A.

A was selected at the numbers 0 . 1 . 2 . 3 . . . as given.

From the computed pressures, at arbitrary heights z, the height corresponding to A was interpolated, and the product $m\,z$ for each A is a constant, the variations seen in the table depending upon the inaccuracy of T determined by the method of trials. By working backward from these constants for T and P more accurate computations for the entire thermodynamic system can be made.

A =	0 log P		occurs at the height	45002/m	in kilometers
	1 "	= 5.0057	" " " "	11627 "	" "
	2 "	5.3067	" " " "	7094 "	" "
	3 "	5.4828	" " " "	4712 "	" "
	4 "	5.6078	" " " "	2870 "	" "
	5 "	5.7047	" " " "	1238 "	" "
	6 "	5.7839	" " " "	10 "	" "
	7 "	5.8508	" " " "	— 952 "	" "
	8 "	5.9088	" " " "	— 1824 "	" "
	9 "	5.9600	" " " "	— 2582 "	" "
	10 "	6.0057	" " " "	— 3313 "	" "
	20 "	6.3067	" " " "	— 7840 "	" "
	30 "	6.4828	" " " "	−10460 "	" "
	40 "	6.6078	" " " "	−12349 "	" "
	50 "	6.7047	" " " "	−13512 "	" "

In the region of the adiabatic layers, as 10 to 50 A, it may be stated that the differences in the constants decrease in value, 4527, 2620, 1889, 1163, so that finally the pressure approaches asymptotically to a maximum. The same is true for each gas in the differences (log P_0 − log P_1) in succession, the maximum for each gas being the end of the gaseous stage, or entrance into the viscous or quasisolid stage, the change being very gradual on approaching the asymptote. Further computations are desirable to bring out the maximum for each gas and the depth at which it occurs, these depths being very different for the several gases. The entire independence of the distribution of the gases in the family of curves conforms to the fundamental

(93) Laws of Dalton, $P = P_1 + P_2 + \ldots P_2$.

(94) Clausius, $\frac{1}{2} m q^2 = \frac{1}{2} m_1 q_1^2 = \frac{1}{2} m_2 q_2^2 = \ldots$.

(95) Avogadro, N = constant for $(P \,.\, V \,.\, T)$ constant.

Fig. 15 shows for the pressure (constants)$_P$ their change in height in passing through the non-adiabatic, isothermal, and

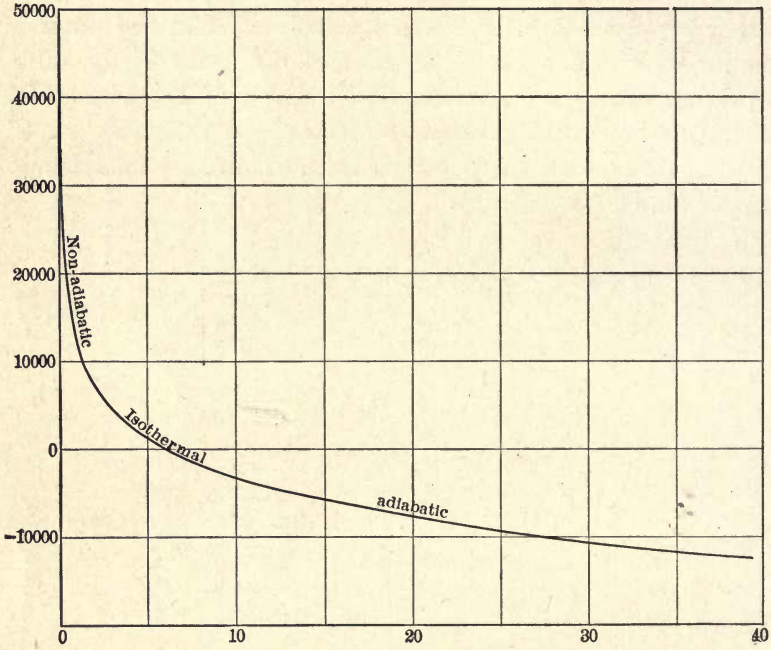


FIG. 15. The Distribution of the Pressure Heights by the Hyperbolic Law. mz = (Const.) p.

adiabatic layers, which is to be continued to the asymptotic maximum.

The Distribution of the Densities

Collecting the data for the densities in Table 12 in the same way as for the temperatures and pressures, we note that there are two important characteristics:

(1) On the plane of the photosphere the densities are related by the hyperbolic law,

$$0.000629 \times m = \rho, \text{ or}$$
$$\rho/m = 0.000629.$$

On no other line, horizontal or vertical, does this law seem to apply.

(2) In the vertical columns the densities decrease by the curves of Fig. 12. On the horizontal lines there is a maximum, that occurs just below the height at which the next heavier element begins to diminish rapidly in its density. Below the photosphere there is steady increase in the density along the line of a given level.

The density of the earth's atmosphere at the sea level is $\rho_0 = 1.29305$ kil./m.^{3} The density of

Hydrogen on the photosphere is $\frac{1}{1218.4}\ \rho_0$ (sea-level-air).

Calcium " " " " $\frac{1}{54.66}$ " " "

Cadmium " " " " $\frac{1}{16.575}$ " " "

Mercury " " " " $\frac{1}{10.286}$ " " "

The extreme rarefaction of the solar gases above the photosphere, where the spectroscopic observations are made, can be readily appreciated.

In the flash spectrum, where the Fraunhofer lines originate, assumed here at 500 kilometers, these ratios become,

Hydrogen, $1/1358.5\ \rho_0$ (sea-level-air).
Helium, 1/ 611.2
Carbon, 1/ 286.6 maximum.
Calcium, 1/ 604.7
Zinc, 1/5037.8
Cadmium, ...
Mercury, ...

At the top of the chromosphere, 5000 kilometers, hydrogen is $1/41102\ \rho_0$ sea-level-air. Similarly, other ratios can be computed from the table. At the depths to which the computations are here quoted only calcium at between -2000 and -3000 kilometers reaches the air density. The densities near the disk of the sun are, therefore, very small.

TABLE 12

THE DENSITY OF THE DIFFERENT ELEMENTS ON THE SAME LEVELS (KILOGRAMS PER METER^3), ρ (M. K. S.)

Z Kilometers	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	Means
25000.....	.0000244	7.074×10^{-9}		Top of the	Inner Corona				
20000.....	630	8.051×10^{-8}							
15000.....	1321	2.348×10^{-6}	—						
10000.....	2411	.00005006	9.419×10^{-8}						
5000.....	4790	.0003146	2.265×10^{-4}	—	—	Top of Chro	mosphere		
1000.....	.0005768	.0008538	.0017589	.0024673	2.948×10^{-8}				
900.....	5822	8726	18251	28024	5.475×10^{-6}				
800.....	5876	8917	18937	31830	6.214×10^{-5}				
700.....	5930	9113	19650	35820	.0002960	—			
600.....	5985	9314	20389	40311	9086	2.881×10^{-6}			
500.....	.0006041	.0009518	.0021156	.0045114	.0021383	1.985×10^{-4}	Reversing	Layer	
400.....	6097	9727	21957	50489	42217	.0013809	—		
300.....	6154	9937	22788	56493	73022	47274	5.334×10^{-5}	—	
200.....	6211	10161	23650	60321	109230	115800	.0035814	8.514×10^{-8}	
100.....	6269	10384	24544	70589	162620	231460	.025499	.011611	
90.....	.0006275	.0010407	24636	.0071373	.016883	.024581	.029026	.016269	
80.....	6281	10430	24728	72165	17569	26105	33042	21993	
70.....	6286	10452	24819	72967	18199	27724	37052	28878	
60.....	6292	10475	24912	73775	18895	29443	41549	37061	
50.....	6298	10498	25004	74593	19617	31269	46221	46699	
40.....	.0006304	.0010521	.0025097	.0075420	.020367	.033192	.051406	.057967	
30.....	6310	10544	25190	76257	21144	35232	57036	71118	
20.....	6315	10567	25282	77103	21951	37398	63283	86450	
10.....	6321	10590	25378	77958	22788	39695	70265	.104400	
Photos. 0.....	.0006327	.0010613	.0025472	.0078821	.023657	.042135	.078017	.12571	$.000629\times m$

−10.....	6333	10637	25564	79711	24543	44740	86470	.15082	
−20.....	6339	10661	25658	80612	25463	47522	95838	.18240	
−30.....	6345	10685	25752	81523	26417	50471	.10641	.21945	
−40.....	6351	10709	25846	82444	27408	53602	.11815	**.26504**	
−50.....	.0006357	.0010733	.0025941	.0083376	.028435	.056928	.13103	.31854	
−60.....	6363	10758	26034	84318	29521	60456	.14530	.38256	
−70.....	6369	10782	26132	85270	30647	64207	.16128	.45494	
−80.....	6375	10806	26228	86234	31816	68190	**.17902**	.53229	
−90.....	6381	10831	26324	87208	33030	72420	.19857	.61356	
−100.....	.0006387	.0010855	.0026421	.0088194	.034290	**.076913**	.22025	.69862	
−200.....	6447	11103	27403	98680	**.053042**	.14088	.49193		
−300.....	6508	11357	28423	110200	77343	.23428	.82440		
−400.....	6569	11616	29479	123050	.109918	.33968			
−500.....	.0006631	.0011881	.0030576	.013768	.148538				
−600.....	6693	12143	31736	15406	.190783				
−700.....	6756	12411	32940	**.017228**	.236374				
−800.....	6820	12684	34189	19266	.285075				
−900.....	6884	12964	35487	21552	.336708				
−1000.....	.0006948	.0013250	.0036832	.024109	.391100				
−2000.....	7630	16521	**.0053316**	.061594	1.064630				
−3000.....	8365	20610	77364	.110520					
−4000.....	9171	**.0025721**	108220	.168130					
−5000.....	10071	32109	142490						
−6000.....	.0011060	.0040044	.017975						
−7000.....	12126	49954	.021979						
−8000.....	13294	61969							
−9000.....	**.0014609**	76070							
−10000.....	16055	92106							
−12000.....	.0017602	.0129703							
−14000.....	.0019297	.0175384							

The normal density of the atmosphere of the earth at sea level = 1.29305

z_R.....	−9000	−4500	−2250	−750	−225	−140	−80	−45	
Density.....	.0014609	.0028915	.0059328	.018247	.059117	.102500	.17902	.29179	
	.0014609	.0014458	.0014832	.0015206	.0014779	.0015769	.0015904	.0014736	.0015047

The density along the radiation layer is equal to $0.0015047 \times m = \rho_R$.

The Gas Efficiency of the Different Elements

The gas efficiency R is the velocity-square per degree of temperature, and hence is the kinetic energy of the gas molecules in their movements of translation, oscillation, and collision, corresponding to a change of one degree of temperature. This is the thermodynamic coefficient and does not refer to the electromagnetic oscillations within the molecules and atoms, or the forces concerned with association, dissociation, and ionization. The distribution of R is given in Table 13 for the same coordinates. We find that there are two places in which the hyperbolic law applies. Along the photosphere,

$$R = 127702/m, \text{ or } R\,m = 127702,$$

and along the line dividing the adiabatic layers from the isothermal layers, the relation is

$$R_a = 281808/m, \text{ or } R_a\,m = 281808,$$

that is, approximately along the broken line, though the transition is not so abrupt in the computations. The relations for R are shown clearly on Fig. 13, with a very large range for hydrogen, contracting to a small range for mercury. Hence, one degree of temperature signifies a large change in the velocity-square of the hydrogen molecule, and only a small change in the kinetic energy of the heavy mercury molecule, in fact, in accordance with Clausius' Law, that kinetic energies are constant for the different elements. The secondary deflection in the curves for R occurs at the height of the greatest gradient in the temperatures, and is the compensation for it. In the lower levels of each gas R becomes a vertical line, and these lines are connected together by the constant $R_a\,m = 281808$. In the case of calcium the values in the adiabatic region seem to require a slight readjustment by a second computation. This example is left in order to illustrate how by another trial such divergences can be removed, that is, by selecting somewhat lower values of the temperature T in this case.

At the levels of the source of the radiation layers the value of $R = \dfrac{221545}{m}$.

The Cause of the Sharp Limb of the Sun

The cause of the sharp visible edge of the sun, that is, the configuration of the disk in the midst of a gaseous envelope, has been a problem of much difficulty to explain. Perhaps the favorite theory is that this layer is composed of a cloudy condensation, as evidenced by the granulation, the appearance of the spots, faculæ, and similar forms. Another view is that of Schmidt, who advocates the application of circular refraction, by which a ray of light originating at great depths in an atmosphere of densities diminishing from the center outward, will at a proper height pursue a circular path. Visible surface phenomena may in this way actually exist at much lower depths than the surface of the disk, so that sun-spots and the other phenomena represent deep-seated optical effects. Julius has applied the principles of anomalous dispersion, or irregular refraction, among neighboring masses which differ greatly in density from one another, to account for the prominences, the Fraunhofer lines, as absorption lines enveloped in dispersion bands, reversals of lines in the chromosphere, some of the sun-spot appearances, and similar effects. Abbot intimates that the effect of scattering upon the outgoing solar radiation is such as to "prevent us from seeing toward the center of the sun, when looking directly at the middle of the solar disk, to more than 5000 miles (8047) kilometers, below the reversing layer." "At the edge of the disk, owing to the oblique line of sight, gaseous scattering will probably extinguish almost all yellow light starting from more than 500 miles below the chromosphere, while an even less thickness suffices for blue or violet light." It may be remarked that all these hypothetical suggestions make of the sun's disk an optical effect of some sort, rather than a material distribution under the thermodynamic laws prevailing in the solar envelope.

An examination of Tables 6 for T, 9 for P, 12 for ρ, and the corresponding figures 10 for T, 11 for A or P, 12 for ρ, makes it evident that the sharp edge of the solar disk is due to the rapid diminution in height of the heavier gases under the solar

TABLE 13

THE GAS EFFICIENCY OF THE DIFFERENT ELEMENTS ON THE SAME LEVELS (Velocity² per degree), R (M. K. S.)

Z Kilometers	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	
25000	32688	4.8187		Top of the	Inner Corona				
20000	47516	4912.8							
15000	62307	22901.							
10000	75712	46669	358.07						
5000	92316	58436	12581			Top of the C	hromosphere		
1000	116170	67237	24539	5806.7	8.7702				
900	116880	67843	25075	6067.9	95.170				
800	117590	68454	25623	6340.7	288.39				
700	118310	69070	26183	6627.7	556.73				
600	119030	69692	26755	6927.3	876.88	38.985			
500	119770	70318	27340	7362.6	1220.4	234.28	Reversing L	ayer	
400	120490	70952	28034	7825.0	1532.5	489.86			
300	121230	71592	28743	8439.0	1804.9	791.12	58.951		
200	121975	72237	29471	9101.2	1996.4	1062.0	402.99	1.4792	
100	122723	72888	30218	9802.6	2360.1	1334.6	759.02	248.12	
90	122797	72954	30293	9876.2	2406.1	1380.7	769.00	275.79	
80	122871	73020	30369	9949.7	2453.2	1428.4	779.13	303.10	
70	122947	73085	30445	10024	2501.2	1477.7	799.63	330.11	
60	123020	73152	30521	10098	2550.1	1528.7	820.66	360.13	
50	123097	73217	30598	10173	2599.8	1581.5	860.62	390.36	
40	123170	73282	30674	10249	2666.3	1646.5	902.50	424.15	
30	123246	73348	30751	10327	2734.4	1714.1	964.76	462.12	
20	123322	73414	30828	10402	2804.3	1784.6	1031.29	505.02	
10	123397	73480	30905	10480	2875.9	1858.0	1106.20	557.65	
Photos. 0	123470	73547	30983	10558	2949.5	1934.4	1186.60	625.39	127704/m
−10	123550	73615	31060	10638	3023.6	2014.6	1271.4	706.43	
−20	123630	73683	31136	10719	3099.4	2098.2	1362.3	802.27	
−30	123710	73753	31213	10800	3177.2	2185.2	1461.5	907.98	
−40	123780	73822	31289	10882	3256.8	2275.8	1568.0	**1030.26**	221545/m

−50	123870	73891	31366	10964	3338.5	2370.2	1680.9	1165.16	
−60	123940	73958	31444	11048	3424.0	2468.5	1801.88	1291.50	281808/m
−70	124020	74028	31529	11131	3511.4	2570.9	1932.8	1322.43	
−80	124100	74097	31598	11216	3601.1	2677.5	2073.2	1322.43	
−90	124170	74167	31676	11301	3693.0	2788.6	2211.6	1322.43	
−100	124250	74235	31754	11386	3787.3	2904.2	2359.2	"	
−200	125040	74928	32545	12280	5078.7	4333.7	2442.4	"	
−300	125830	75630	33356	13226	6521.7	4372.1	2442.4	"	
−400	126630	76336	34188	14245	7359.0	4372.1	2442.4	"	
−500	127430	77050	35040	15363	7359.0	4372.1	"	"	
−600	128230	77747	35931	16569	7359.0	"	"	"	
−700	129050	78448	36843	17866	"	"	"	"	
−800	129860	79155	37779	19265	"	"	"	"	
−900	130680	79870	38738	20776	"	"	"	"	
−1000	131510	80590	39722	22406	"	"	"	"	
−2000	140070	88127	50963	26717	"	"	"	"	
−3000	149020	96230	65193	26717	"	"	"	"	
−4000	158552	105786	66258	26717	"	"	"	"	
−5000	168870	115590	66258	"	"	"	"	"	
−6000	179867	126220	66258	"	"	"	"	"	
−7000	191370	137632	"	"	Adiabatic Region				
−8000	203595	144403	"	"	"	"	"	"	
−9000	216950	147714	"	"	"	"	"	"	
−10000	231177	147714	"	"	"	"	"	"	
−12000	260465	147714	"	"	"	"	"	"	
−14000	266020	147714	"	"	"	"	"	"	
Z_R	−9000	−4500	−2250	−750	−225	−140	−80	−45	
m_R	216950	110688	54519	18566	5439.5	3476.0	2073.2	1097.71	
	216950	221376	218076	222792	217580	225940	232198	217447	221545

gravitation. Thus, by Table 8, calcium (40) vanishes at 1000 kilometers above the photosphere, that is, 1″.4 arc; zinc (65) at 675 kilometers = 0″.9; cadmium (112) at 385 kilometers = 0″.5; mercury (198) at 210 kilometers = 0″.3. Since the radius of the sun is 31′ 59″.26 = 1919″.26, it follows that all the gases whose atomic weight is greater than 40 are so distributed as to vanish physically and materially at one two-thousandth of the radius of the sun from the plane of the photosphere, as defined by the common pressure of six atmospheres for all the elements. That is to say, these heavy gases pass from enormous values of temperature (Table 6), and from enormous values of pressure (Tables 9, 10), to vanishing values in a very narrow vertical stratum. The density changes (Table 12) are by no means so surprising, except that all the densities for the heavy gases are really very small. The sharp disk of the sun is, therefore, a thermodynamic and gravitational effect upon gases of which the monatomic weights are somewhat greater than the 40 for calcium. This element just floats comfortably upon the layers of still heavier gases, and it is easily observed in the spectroheliograph as a surface phenomenon, having many configurations, as in the faculæ and flocculi. The lighter gases, especially helium and hydrogen, rise to great heights, the hydrogen to the top of the inner corona. The gases between helium and calcium, up to bromine, columbium, germanium, krypton, molybdenum, rubidium, selenium, strontium, yttrium, zirconium, ought to just reach the level of the reversing layer, if they exist in the solar envelope. These distributions conform well with the results of the spectroheliograph observations upon the height of the chemical elements in the sun.

CHAPTER III

The Determination of the "Solar Constant" of Radiation in the Isothermal Layer of the Sun

Statement of the Problem

In Bulletins No. 3, 1912; No. 4, 1914, of the Oficina Meteorologica Argentina, and in the Treatise on Circulation and Radiation, 1915, an account was given of an effort to solve the discrepancies in the data of radiation as observed by the pyrheliometer and the bolometer. The criticisms regarding the former consisted in showing that in using the Bouguer formula for pure radiation in the forms

$$I = I_0 e^{-km}, \text{ or } I = I_0 p^m, \tag{96}$$

for absorption and transmission respectively, it is not proper to interpret the original formula,

$$I = I_0 p^{\sec z}, \tag{97}$$

as if $\sec z = \frac{m}{m_0}$ were $\sec z = m$, for $m_0 = 1$, the depth of the atmosphere. The graph (log I . sec z) of observation serves only up to $\sec z = 1$ in the zenith, but it cannot be extended to the non-mathematical $\sec z = 0$ without unwarranted assumptions. These are, that p is constant from the sea level to the vanishing plane of the earth's atmosphere, whereas in fact p is very variable; no account is taken of the potential, specific heat, and ionization energies which are absorbed to balance the kinetic energy, so that the extrapolated value of $I_0 = 1.950$ gr. cal./cm.^{2} min. depends upon a very incomplete treatment of the observations made at stations having different elevations above the sea level; the thermodynamics of the radiation throughout the earth's atmosphere shows that the total amount of black body radiation falling on the outermost strata is about

3.980 gr. cal./cm.^{2} min., and that this suffers a series of depletions which reduce it to about 1.500 gr. cal./cm.^{2} min. at the sea level.

The data of the bolometer spectrum indicate that the surviving long-wave ordinates, $\lambda = 1.50\,\mu$ to $\lambda = 2.50\,\mu$, seem to originate in a radiating solar temperature of 7700° Centigrade absolute, while the middle waves, $\lambda = 0.45\,\mu$ to $\lambda = 1.50\,\mu$, appear to represent a radiation at about 6900°, having lost a large amount from the original source, and the short waves, $\lambda = 0.00\,\mu$ to $\lambda = 0.045\,\mu$, have evidently been excluded entirely by reflection or absorption within the high levels, 40,000 to 70,000 meters, of the earth's atmosphere. The middle ordinates, $\lambda = 0.45\,\mu$ to $\lambda = 1.50\,\mu$, suffer progressive depletion from one high station to another, and arrive at sea level with a value of about 2.47 gr. cal./cm.^{2} min., when fully corrected for band and minor losses in the thermal spectrum. These two types of observations afford such divergent data that it is impossible to reconcile them on the hypothesis that the solar radiation originates in gases whose temperature is about 6900°, but whose efficiency corresponds only to a temperature of about 5800°. This is, of course, equivalent to assuming that the Kurlbaum coefficient of radiation σ in the Stefan Law, at the distance of the earth D,

$$J = \sigma T^4 \left(\frac{R}{D}\right)^2, \tag{92}$$

does not hold true at the sun. Otherwise,

$$J_0 = \sigma_1 (6900)^4 \left(\frac{R}{D}\right)^2 = \sigma (5800)^4 \left(\frac{R}{D}\right)^2, \tag{98}$$

so that there results,

$$\sigma_1 = \sigma \left(\frac{5800}{6900}\right)^4 = 0.4992\,\sigma. \tag{99}$$

If we take as a mean value of the Kurlbaum coefficient $\log \sigma = -5.74103$, then we should have for the defective $\log \sigma_1 = -5.44000$, so that while $\sigma = 5.509 \times 10^{-5}$, the defective or inefficient value would be $\sigma_1 = 2.754 \times 10^{-5}$, half as much as σ.

We have already proved that the mean temperature of the sun's isothermal layer is about 7687°, as in Table 6. It only remains to determine the temperature within the isothermal layer at which the emitted radiation is generated, as well as the value of σ actually prevailing in those solar strata.

The Distribution of the Free Heat $(Q_1 - Q_0)$

The free heat in any stratum $(z_1 - z_0)$ is computed by (34),

$$(Q_1 - Q_0) = (Cp_a - Cp_{10})\ (T_a - T_0),$$

where $(T_a - T_0)$ is the adiabatic fall of temperature in the stratum, Cp_a the adiabatic specific heat at constant pressure for the gas, $Cp_a = R_a \frac{\kappa}{\kappa - 1}$, and $Cp_{10} = \frac{1}{2}\ (Cp_1 + Cp_0) = \frac{1}{2}\ (R_1 + R_0) \frac{\kappa}{\kappa - 1}$, the mean of the non-adiabatic values at the top and the bottom of the stratum. In the computations leading to the adiabatic value R_a there is one fact of interest. At the initial point, determined from the terrestrial values, we have for calcium vapor on the sun, as an example,

$\gamma \times P$ (earth) $= P$ (sun) $= 28.028 \times 101323.5 =$
2839900 (M. K. S.)

$\gamma \times T$ " $= T$ " $= 28.028 \times 273 = 7651.6$ (M. K. S.)

$\gamma^{-1} \times \rho$ " $= \rho$ " $= 1.7999/28.028 =$
0.064219 (M. K. S.)

$\gamma \times R$ " $= R$ " $= 28.028 \times 206.205 =$
5779.5 (M. K. S.)

This value of R is not the adiabatic R_a, but only one point in the gas at some distance above the adiabatic strata. In computing the terms of the gravity equation,

$$g_0 \times 28.028\ (z_1 - z_0) = -\frac{P_1 - P_0}{\rho_{10}} - (Cp_a - Cp_{10})\ (T_a - T_0),$$

this value of $(Cp_a - Cp_{10})\ (T_1 - T_0) = (Q_1 - Q_0)^1$ is not the same as $(Q_1 - Q_0)$ in (34), because it is not referred to the true adiabatic $Cp_a = R_a \frac{\kappa}{\kappa - 1}$. The initial value of R serves as a preliminary in computing R_a. Thus, $(Q_1 - Q_0)^1$ is a positive

term, that is, it has positive values which must be added to the positive value of $-\frac{P_1 - P_0}{\rho_{10}}$ to produce $G(z_1 - z_0)$; below this initial R we find that $(Q_1 - Q_0)^1$ is a negative quantity to be subtracted from $-\frac{P_1 - P_0}{\rho_{10}}$, which has now become greater than $G(z_1 - z_0)$. Proceeding in this way, *R enters at last into a stratum where it becomes a constant, R_a, which has a true adiabatic value.* Now the course of the development of the R-term depends upon the trial temperatures T from level to level, and if these are not exactly correct there will be a corresponding error in $Cp_a = R_a \frac{\kappa}{\kappa - 1}$, and all the subsequent terms derived from it. This explanation is needed to account for the minor irregularities that occur in the tables for eight elements.

In Table 14 the free heat has been reduced to 20-kilometer strata, whatever may have been the intervals in the computation, for the sake of comparison at different heights. Only a small portion of the data in hand is here quoted, but this is full in the isothermal layer and near the level of the photosphere, where it is likely to be of most practical usefulness. A similar arrangement is made for the other tables of this Chapter.

We note that along the lower boundary of the table the value of $(Q_1 - Q_0)$ is a vanishing quantity in the adiabatic region, and that along the upper diagonal a common maximum value of about 6500000 is reached for all the elements whatever the height may be between the adiabatic level and the vanishing plane. The irregularities in this diagonal line depend upon the facts that the vanishing plane does not occur on the exact height quoted in the Table, and that the adiabatic value Cp_a may not have been found with precision by the method of trials in this computation.

In Table 14 the mean value of $(Q_1 - Q_0)$ on the photosphere is $-$ 3443372 for 20 kilometers, or 172.1686 for one meter, this being the mean kinetic energy or velocity square per degree. *It appears that all the chemical elements on the sun have the same kinetic energy at the surface of the photosphere.* Above that level

TABLE 14

DISTRIBUTION OF THE FREE HEAT $(Q_1 - Q_0)$ IN 20-KILOMETER STRATA AT DIFFERENT HEIGHTS

Z Kilometers	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	
25000....	−5475120	−6558160							
20000....	−5121890	−6393430							
15000....	−4776000	−5633800					(M. K. S.)		
10000....	−4463670	−4624120	−6248280						
5000....	−3998000	−4108960	−5023440						
1000....	−3370100	−3742720	−3682240	−5903620	−6976000				
800....	−3333500	−3681000	−3523540	−5730250	−6665320				
600....	−3297540	−3620250	−3371700	−5518500	−6165160	−6651840			
400....	−3262000	−3560330	−3226470	−5508250	−5541680	−6339360	−6473170		
200....	−3226800	−3501470	−3087360	−4818440	−5022280	−5051680	−5302880	−6193930	−6656690
100....	−3203000	−3472410	−3020100	−4586200	−4643240	−4610240	−4436500	−4937720	
80....	−3208000	−3466580	−3006860	−4408400	−4508800	−4387200	−4354800	−4677840	
60....	−3198600	−3460750	−2293640	−4452000	−4378220	−4175000	−4191300	−4393000	
40....	−3196400	−3455000	−2980500	−4496100	−4251500	−3973100	−3912000	−4053250	
20....	−3194200	−3449230	−2967430	−4540700	−4128400	−3780900	−3535420	−3610120	
Photos. 0....	−3192000	−3443600	−2954332	−4365100	−4008840	−3597920	−3096570	−2888610	−3443372
−20....	−3187000	−3436400	−2931800	−4309300	−3822550	−3225530	−2590880	−1935650	
−40....	−3182900	−3429170	−2909300	−4254300	−3644670	−2891670	−2008230	−752050	
−60....	−3178500	−3422000	−2887000	−4200000	−3475150	−2592410	−1338470	−36533	
−80....	−3173900	−3414800	−2865000	−4146406	−3313460	−2324050	−599590	000	0000000
−100....	−3169500	−3407700	−2843100	−4093640	−3159280	−2083440	−110190	"	
−200....	−3147200	−3372230	−2734800	−3839000	−1835440	000	000	"	
−400....	−3103000	−3302540	−2531700	−3227000	000	"	"	"	
−600....	−3059430	−3234310	−2343600	−2510940	"	"	"	"	
−800....	−3016530	−3167430	−2169550	−1678400	"	"	"	"	
−1000....	−2974100	−3102000	−2008400	−771430	"	"	"	"	
−2000....	−2775000	−2640860	−791436	000	"	"	"	"	
−4000....	−2301840	−1760640	000	"	"	"	"	"	
−6000....	−1766320	−750750	"	"	"	"	"	"	
−8000....	−1156240	−78650	"	"	"	"	"	"	
−10000....	−828300	000	"	"	"	"	"	"	
−12000....	−131918	"	"	"	"	"	"	"	
−14000....	000	"	"	"	"	"	"	"	

(34), $(Q_1 - Q_0) = (Cp_a - Cp_{10})(T_a - T_0)$

TABLE 15

DISTRIBUTION OF THE ENTROPY $(S_1 - S_0)$ IN 20-KILOMETER STRATA AT DIFFERENT HEIGHTS

	Z Kilometers	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	
	25000	−1561.0	−54651.2							
	20000	−1138.2	− 6361.3					(M. K. S.)		
	15000	− 851.7	− 2660.5	∞						
	10000	− 651.6	− 1288.1	−7182.0						
	5000	− 525.4	− 692.0	−1259.0	∞	∞				
	1000	− 437.9	− 487.5	− 480.8	−875.9	−15502.6				
	800	− 433.2	− 479.6	− 459.6	−790.4	− 3979.3	∞			
	600	− 428.5	− 471.7	− 439.3	−725.4	− 1957.1	−8695.2	∞		
	400	− 423.9	− 464.1	− 419.9	−675.5	− 1116.7	−1703.4	−6056.9	∞	
	200	− 419.3	− 456.5	− 401.4	−625.8	− 717.5	−1015.4	−1683.5	−12178.6	
	100	− 417.0	− 452.8	− 392.4	−596.0	− 610.0	− 604.8	− 721.4	− 1113.9	
	80	− 416.5	− 452.0	− 390.7	−590.2	− 591.0	− 574.2	− 626.6	− 862.5	
	60	− 416.1	− 451.3	− 388.9	−584.5	− 572.5	− 545.2	− 560.7	− 694.4	
	40	− 415.6	− 450.5	− 387.2	−578.8	− 554.7	− 517.6	− 510.0	− 572.5	
	20	− 415.2	− 449.8	− 385.4	−573.2	− 537.4	− 491.4	− 459.9	− 473.7	
Photos.	0	− 414.7	− 449.1	− 383.7	−567.6	− 520.6	− 466.6	− 403.2	− 376.7	−447.8
	−20	− 414.2	− 448.1	− 380.7	−560.4	− 496.7	− 418.7	− 337.7	− 252.7	
	−40	− 413.6	− 447.2	− 377.8	−553.4	− 473.8	− 375.8	− 262.0	− 98.0	
	−60	− 413.0	− 446.3	− 375.0	−546.4	− 452.0	− 337.2	− 174.8	− 4.5	
	−80	− 412.4	− 445.4	− 372.1	−539.5	− 431.2	− 302.6	− 78.0	—	
	−100	− 411.9	− 444.5	− 369.2	−532.7	− 411.4	− 271.6	− 13.6	—	
	−200	− 409.0	− 439.9	− 355.4	−499.9	− 239.7	− 3.2	—	—	
	−400	− 403.4	− 431.0	− 329.1	−420.7	—	—	—	—	
	−600	− 397.8	− 422.2	− 304.8	−327.9	—	—	—	—	
	−800	− 392.4	− 413.6	− 282.3	−219.5	—	—	—	—	
	−1000	− 387.0	− 405.2	− 261.5	−100.3	—	—	—	—	
	−2000	− 361.0	− 344.4	− 103.3	—	—	—	—	—	
	−4000	− 300.0	− 230.3	—	—	—	—	—	—	
	−6000	− 230.4	− 97.8	—	—	—	—	—	—	
	−8000	− 151.0	− 9.5	—	—	—	—	—	—	
	−10000	− 85.7	—	—	—	—	—	—	—	
	−12000	− 8.4	—	—	—	—	—	—	—	
	−14000	—	—	—	—	—	—	—	—	

(37), $(S_1 - S_0) = (Cpa - Cp_{10})\ (Ta - T_0)/T_{10}$.

there is an increase in the value of the kinetic energy of the free heat per degree on the same level, counting from hydrogen to mercury, while below the photosphere there is decrease from hydrogen to mercury along the same level. The former increases up to the common maximum 6500000, and the latter decreases to the common vanishing value 0. The Table 16 shows that $(W_1 - W_0) = 0$ on the vanishing planes, while Table 17 indicates that the inner energy $(U_1 - U_0)$ reaches the same maximum as $(Q_1 - Q_0)$. Hence,

$$(Q_1 - Q_0) = (S_1 - S_0)\, T_{10} = (W_1 - W_0) + (U_1 - U_0)$$
$$(S_1 - S_0)\, T_{10} = (U_1 - U_0) = 6500000 \text{ per } 20 \text{ km.}$$

$$(100) \quad \frac{S_1 - S_0}{U_1 - U_0} = \frac{(S_1 - S_0)}{6500000} = \frac{1}{T_{10}} = \frac{\infty}{6500000} = \frac{1}{0},$$

since $(S_1 - S_0) = \infty$ and $T_{10} = 0$ on the vanishing plane.

Planck's formulas, $\frac{\partial L}{\partial K} = \frac{\partial s}{\partial u} = \frac{1}{T}$, (Thermodynamik, 135),

upon which the functions for T the temperature, and $\overline{K}$ the polarized specific intensity, are founded,

$$(101) \quad T = \nu F_1\left(\frac{c^2 \overline{K}}{\nu^3}\right), \ \overline{K} = \frac{\nu^3}{c^2} F_2\left(\frac{T}{\nu}\right), \ E_\lambda = \frac{c^2}{\lambda^5} F\left(\frac{\lambda T}{c}\right),$$

that is (136), (137), (106), (107), and others, are really adiabatic, indeterminate forms on the vanishing planes of gases, and are not applicable within the gaseous atmospheres, omitting the $(W_1 - W_0)$ term.

It should be noted that the mean value on the photosphere, — 3443372, is about the mean between the common minimum 0 in the adiabatic levels, and the common maximum — 6656690 on the vanishing planes.

Table 15 for the computed values of the entropy $(S_1 - S_0)$ in 20-kilometer strata shows that the distribution is similar to that for $(Q_1 - Q_0)$, namely, *a common value* — 447.8 *for all the elements on the photosphere*, an increase along the several levels from hydrogen to mercury above the photosphere to the maximun ∞, and a decrease on the levels below the photosphere to the value 0. Hence, the mean temperature of the photosphere

from these data becomes $T = -3443372/-447.8 = 7689.5°$, while the date of Table 6 made the temperature 7693.1°. The entropy is always negative, and it increases in the vertical columns from 0 on the adiabatic level to ∞ on the vanishing plane. These data are useful in testing the theories of radiation which have been founded upon various hypotheses and assumptions.

Table 16 gives the distribution of the work of expansion $(W_1 - W_0)$ in 20-kilometer strata, and it is of an exactly inverse model to that of $(Q_1 - Q_0)$ and $(U_1 - U_0)$. *The mean value on the photosphere is* 2009412, and on the levels above the photosphere there is decrease from hydrogen toward mercury, from a maximum to vanishing values, while below the photosphere there is increase toward a maximum. The mean value of this maximum of work is 4122458, and it is the common value of all the elements in the adiabatic strata. It diminishes to 0 on the vanishing planes of the several gases. It should be noted that the mean value on the photosphere is about the mean of the common maximum 4122458, which is, also, the same as that for hydrogen, and the common minimum 0 on the vanishing planes.

The distribution of the inner energy is like that of $(Q_1 - Q_0)$. *It has the mean value* — 5467133 *on the photosphere;* on the levels above the photosphere it increases from a minimum value, namely, that of monatomic hydrogen, $H_1 = 1.00$, at — 4122458 to a maximum value which is — 6656690 for all the elements; below the photosphere it decreases to the same minimum of hydrogen — 4122458. It should be noted that the mean value on the photosphere is about the mean of the maximum —6656690 and the minimum — 4122458. Mean values are applied to individual gases.

The Equation of Conservation of Energy has three cases:

The First Law of Thermodynamics,

$$(Q_1 - Q_0) = (W_1 - W_0) + (U_1 - U_0).$$

(1) On the vanishing planes,

$$-6656690 = 0000000 - 6656690.$$

(2) On the photosphere, (approximate),

$$-3443372 = 2009412 - 5467133.$$

TABLE 16

DISTRIBUTION OF THE WORK OF EXPANSION $(W_1 - W_0)$ IN 20-KILOMETER STRATA AT DIFFERENT HEIGHTS

	Z Kilometers	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	
	25000	508190	689							
	20000	722080	176796					(M. K. S.)		
	15000	928450	779560							
	10000	1116920	1564544	32094						
	5000	1389940	1959080	767000	—	—				
	1000	1766760	2319120	1633456	1039800	13.8				
	800	1788900	2362940	1716280	1135740	200.0	—			
	600	1811250	2407440	1803250	1262850	468440	52881			
	400	1833900	2452800	1894650	1448970	848320	486776	—	—	
	200	1856800	2499060	1990700	1682840	1185960	1013360	703670	13466	0000000
	100	1868350	2522530	2040500	1813750	1411200	1278880	1223400	741750	
	80	1870650	2527300	2050560	1841100	1480670	1383750	1272440	896180	
	60	1872960	2532060	2060700	1868870	1553500	1497570	1370600	1062140	
	40	1875300	2536770	2070950	1897100	1629890	1619850	1537900	1253400	
	20	1877630	2541500	2081240	1925740	1701040	1752640	1764100	1504000	
Photos.	0	1879960	2546280	2091552	1954830	1794160	1896240	2027410	1884860	2009412
	−20	1882300	2550900	2102000	1984200	1886260	2054600	2330820	2419720	
	−40	1884650	2555500	2112500	2014050	1983000	2226200	2680470	3107140	
	−60	1887000	2560200	2123000	2044400	2084670	2412170	3082360	3699400	
	−80	1889360	2564820	2133650	2075150	2191600	2613650	3525580	3743240	4122458
	−100	1891700	2569500	2144400	2106400	2303960	2832000	3819180	"	
	−200	1903600	259310	2198350	2269650	3098200	4027288	3885360	"	
	−400	1927500	2640770	2310700	2637560	4189400	4045000	"	"	
	−600	1951800	2689380	2428730	3067270	"	"	"	"	
	−800	1976300	2738900	2552800	3544690	"	"	"	"	
	−1000	2001200	2789280	2683256	4085450	"	"	"	"	
	−2000	2130200	3117080	3442720	4545600	"	"	"	"	
	−4000	2414000	3743160	3781040	"	"	"	"	"	
	−6000	2735380	4461300	"	"	"	"	"	"	
	−8000	3101430	4939120	"	"	"	"	"	"	
	−10000	3229700	4995060	"	"	"	"	"	"	
	−12000	3689500	"	"	"	"	"	"	"	
	−14000	3794960	"	"	"	"	"	"	"	

(35). $(W_1 - W_0) = (R_{10} - Cp_{10})(Ta - T_0) = P_{10}(U_1 - U_0)$.

TABLE 17

DISTRIBUTION OF THE INNER ENERGY $(U_1 - U_0)$ IN 20-KILOMETER STRATA AT DIFFERENT HEIGHTS

Z Kilometers	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	
25000....	−5985250	−6558920							
20000....	−5843860	−6570080					(M. K. S.)		
15000....	−5705700	−6413480	—						
10000....	−5581890	−6188720	−6280440						
5000....	−5395000	−6128160	−5790440	—	—				
1000....	−5144700	−6061920	−5315000	−6929000	−6990080				
800....	−5130250	−6047400	−5259800	−6866000	−6865600	—			
600....	−5115600	−6033120	−5205500	−6781300	−6633600	−6889600			
400....	−5101000	−6018900	−5151670	−6657170	−6389880	−6418160	—	—	
200....	−5086500	−6004600	−5098440	−6501290	−6208400	−6065120	−6006430	−6220760	−6656690
100....	−5079250	−5997400	−5072000	−6409900	−6055440	−6088960	−5659870	−5718500	
80....	−5077900	−5995860	−5066750	−6391710	−5999710	−5963250	−5627290	−5614360	
60....	−5076330	−5994400	−5061500	−6373710	−5956620	−5839710	−5561880	−5500390	
40....	−5074900	−5993120	−5056250	−6355670	−5913860	−5718750	−5450130	−5364370	
20....	−5073440	−5991700	−5051000	−6337710	−5871370	−5600250	−5299560	−5179240	
Photos. 0....	−5072000	−5989960	−5045760	−6319710	−5802920	−5484240	−5124000	−4898670	−5467133
−20....	−5070250	−5988140	−5038220	−6299140	−5753880	−5352100	−4921670	−4517320	
−40....	−5068620	−5986140	−5031100	−6278600	−5679750	−5223000	−4688560	−4044030	
−60....	−5066900	−5984300	−5023800	−6258000	−5606630	−5097000	−4420800	−3757790	
−80....	−5065220	−5982300	−5016500	−6237600	−5534370	−4973900	−4125270	−3743240	−4122458
−100....	−5063500	−5980430	−5009220	−6217300	−5463120	−4853840	−3929450	"	
−200....	−5055000	−5970250	−4972900	−6109430	−4933680	−4056888	−3885360	"	
−400....	−5038150	−5950430	−4901100	−5864630	−4189400	−4045000	"	"	
−600....	−5021330	−5930620	−4830220	−5578120	"	"	"	"	
−800....	−5004620	−5910900	−4760440	−5212670	"	"	"	"	
−1000....	−4987900	−5891200	−4691680	−4852100	"	"	"	"	
−2000....	−4905110	−5758000	−4232960	−4545600	"	"	"	"	
−4000....	−4715890	−5503760	−3781040	"	"	"	"	"	
−6000....	−4501700	−5212000	"	"	"	"	"	"	
−8000....	−4257640	−5017760	"	"	"	"	"	"	
−10000....	−4058000	−4995060	"	"	"	"	"	"	
−12000....	−3821480	"	"	"	"	"	"	"	
−14000....	−3794960								

(36), $(U_1 - U_0) = (Cpa - R_{10})(Ta - T_0) = (Q_1 - Q_0) - (W_1 - W_0)$.

(3) On the adiabatic levels,

$$0000000 = 4122458 - 4122458.$$

The Second Law of Thermodynamics, $T_{10}(S_1 - S_0) = (Q_1 - Q_0)$, on the photosphere is $7689.5 \times -447.8 = -3443372$.

The results of Tables 14, 16, 17 have been plotted on Fig. 16 in order to exhibit the general mutual relations of the free heat $(Q_1 - Q_0)$, work of expansion $(W_1 - W_0)$, and the inner energy $(U_1 - U_0)$, of the different elements in their distribution with the distance in a vertical direction from the level of the photosphere. Allowance is made for imperfections in the results of the computations by the method of trials, the essential facts being that the lines pass through the points indicated in the mean values under Table 15 at the respective heights of the top of the adiabatic layers, the plane of the photosphere, and the vanishing planes. The first law of thermodynamics is conserved on every level, and the heights as ordinates over any value, taking in the three terms, are distributed by the law of the equilateral hyperbola, $x\,y = c$, already given. The corresponding constants can easily be computed by collecting the values from the tables, but they will be omitted in this place. The necessary conclusion is that the thermodynamic system (*Q. W. U.*), as well as the dynamic system (*P. ρ. R. T.*), are distributed according to the exponential law of decay, counting from the adiabatic region upward. These data can, therefore, be utilized in constructing the general laws of solar physics in their relations to molecular and atomic conditions. We shall proceed to develop their consequences in relation to the "solar constant of radiation."

It will be convenient to compute the radiation potential, $K_{10} = \dfrac{U_1 - U_0}{v_1 - v_0} = c\,T^{\alpha}$, exponent (α), and the $(\log c)$, in the law of radiation, $K_{10} = c\,T^{\alpha}$, in order to study their connection with the Stefan Law $J_0 = \sigma\,T^4$.

The specific volume $v = 1/\rho$ is computed from the values collected in Table 12, and thence the values,

$$K_{10} = \frac{U_1 - U_0}{v_1 - v_0} = c\,T^{\alpha},$$

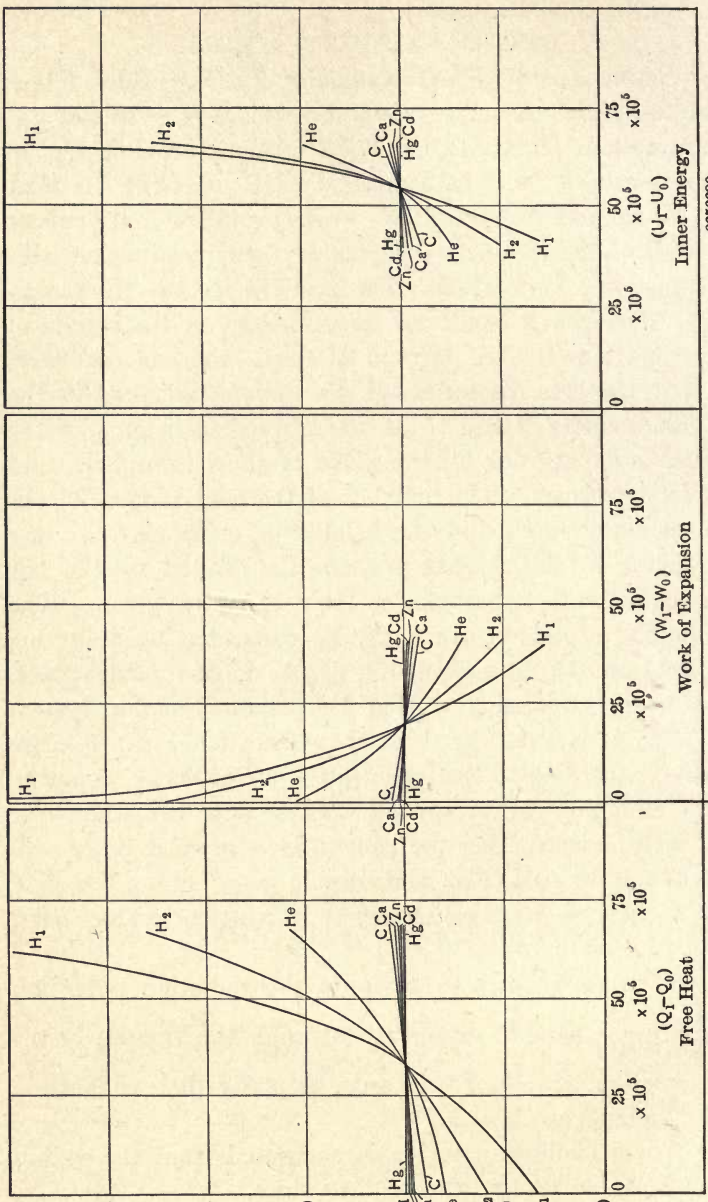


FIG. 16. The relative Distribution of $(Q_1 - Q_0)$, $(W_1 - W_0)$, $(U_1 - U_0)$ with the Height for Eight Chemical Elements on the Sun.

TABLE 18

Distribution of the Radiation Potential K_{10}

$$K_{10} = \frac{(U_1 - U_0)}{(v_1 - v_0)}$$

Z Kilometers	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	
25000.....	−43687	-1.161×10^{-10}							
20000.....	−135500	−2.4					(M. K. S.)		
15000.....	−335390	−1184							
10000.....	−734067	−37869	−19						
5000.....	−1252000	−373220	−110956	—	—				
1000.....	−1746720	−1256260	−1386030	−760300	−0.5				
800.....	−1772480	−1299640	−1480130	−1038830	−1856	—			
600.....	−1798630	−1344500	−1580600	−1355970	−41330	−30.9			
400.....	−1825270	−1390900	−1687900	−1670200	−267712	−37350	—	—	
200.....	−1852100	−1438930	−1802450	−2074710	−919400	−510444	−54177	−0.272	
100.....	−1865700	−1463570	−1862600	−2111750	−1439700	−1210260	−632230	−116460	
80.....	−1868440	−1468520	−1874900	−2149300	−1548360	−1344900	−908050	−260165	
60.....	−1871210	−1473530	−1887260	−2187550	−1665310	−1494600	−1205280	−496544	
40.....	−1873950	−1478530	−1899700	−2226470	−1791040	−1660900	−1492620	−846720	
20.....	−1876650	−1483600	−1912220	−2266100	−1926320	−1845700	−1775830	−1316240	
Photos. 0.....	−1879480	−1488550	−1924770	−2475120	−2071800	−2051050	−2149850	−1880900	−1990190
−20.....	−1882520	−1495400	−1935260	−2526900	−2198350	−2257700	−2495500	−2497820	
−40.....	−1885820	−1502230	−1945720	−2579700	−2332780	−2484940	−2964860	−3288770	
−60.....	−1889040	−1509100	−1956220	−2633600	−2475350	−2735120	−3410080	−4500300	
−80.....	−1892260	−1516000	−1966800	−2688630	−2626700	−30105000	−3715670	−7521400	
−100.....	−1895430	−1522900	−1977400	−2744800	−2787270	−3313620	−4801330	−11612600	
−200.....	−1911600	−1558040	−2041050	−3043930	−3726100	−5336140	−16911700		
−400.....	−1944360	−1630730	−2164400	−3586420	−889980	−22060000			
−600.....	−1977700	−1706800	−2292900	−4289200	−20812400				
−800.....	−2011600	−1786500	−2434000	−4997780	−39003300				
−1000.....	−2046050	−1869740	−2581120	−5862860	−64278600				
−2000.....	−2227350	−2397200	−3323380	−22083000	−324620000				
−4000.....	−2532830	−3558200	−7636600						
−6000.....	−2965130	−5259000	−17216400						
−8000.....	−3288310	−8386500							
−10000.....	−3424620	−14147000							
−12000.....	−4223800	−22559500							
−14000.....	−5590750	−35278300							

The arrows indicate rapidly increasing values of K_{10}.

of Table 18 have been found. K_{10} follows somewhat closely the values of P in Table 9, only noting that while P is the value of the pressure at the height given, K_{10} is the mean value in the stratum whose lower surface is the height of P. Since, originally,

$$(U_1 - U_0) = (Q_1 - Q_0) - (W_1 - W_0) = (Q_1 - Q_0) - P_{10}(v_1 - v_0),$$

and

$$K_{10} = \frac{U_1 - U_0}{v_1 - v_0} = \frac{Q_1 - Q_0}{v_1 - v_0} - P_{10},$$

it follows that the mean values for the stratum K_{10} and P_{10} are separated by the term $\frac{Q_1 - Q_0}{v_1 - v_0}$. It only remains to note that the value is always $K_{10} = 0$ on the vanishing plane, $K_{10} = -1990190$ on the photosphere, and that K_{10} increases to indefinitely large values in the adiabatic region along with P_{10}.

Method of Computing the Coefficients and Exponents in the Formula of Radiation

Taking helium, $m = 4.00$, for an example of the computed values of (A, log C), Table 19, we note that these immediately divide themselves into three parts, corresponding with the non-adiabatic, the isothermal, and the adiabatic regions; in the former, A approximates 4.00 and log $C - 10.00000$ more or less; in the isothermal, A is very large and negative, while log C is very large and positive, due to the large value of $n = -(T_a - T_0)/-(T_1 - T_0)$ in these layers, where the temperature changes slowly and $(T_1 - T_0)$ is small; in the adiabatic region A is about 2.40 and log C is about -3.20000. In order to reduce these values of (A, log C) to (a, log c) the graphs (A, log C) are made on Fig. 17, and it is seen that they separate into two very different distributions. In the adiabatic layers (A, log C) concentrates about a narrow area, contained in the small square, while in the non-adiabatic layers (A, log C) is extended along a straight line, from ($A = 2.00$, log $C = 0.00000$) at the slope -0.35000 for Δ log C and -0.100 for ΔA. The isothermal data can not be plotted on a small diagram. I have taken the slope (A, log C) in the adiabatic region as if

TABLE 19

EXAMPLE OF THE METHOD OF CHANGING (A, LOG C) INTO (α LOG c) AND LOG 60 σ_A, HELIUM

Z	A	Log C	α	Log c	Log 60 σ_A	
		(M. K. S.)		(M. K. S.)	(C. G. S.)	
11500..	—	—	—	—	—	
11000..	5.6131	−16.11147	3.6310	−11.20850	−16.36494	
10500..	5.7773	−16.12239	3.7750	−11.71250	−16.86894	
10000..	5.6911	−16.53935	3.8474	−11.96590	−15.12234	
9500..	5.5861	−16.94719	3.8927	−10.12445	−15.28089	
9000..	5.4619	−15.38863	3.9240	−10.23400	−15.39044	
8500..	5.5085	−15.28425	3.9482	−10.31870	−15.47514	
8000..	5.5506	−15.17602	3.9682	−10.38870	−15.54514	
7500..	5.4179	−15.63871	3.9845	−10.44575	−15.60219	
7000..	5.1278	−14.63633	3.9970	−10.48950	−15.64594	
6500..	4.8676	−13.54427	4.0065	−10.52275	−15.67919	
6000..	4.6697	−12.24446	4.0125	−10.54375	−15.70019	
5500..	4.2709	−11.66218	4.0178	−10.56230	−15.71874	
5000..	4.0625	−10.42998	4.0205	−10.57175	−15.72819	
4500..	3.6789	−8.81261	4.0198	−10.56930	−15.72574	
4000..	3.4939	−8.50161	4.0185	−10.56475	−15.72119	
3500..	3.4420	−8.70003	4.0160	−10.55600	−15.71244	
3000..	3.3830	−8.92801	4.0134	−10.54690	−15.70334	
2500..	3.4597	−8.64222	4.0113	−10.53955	−15.69599	
2000..	3.0021	−6.40393	4.0090	−10.53150	−15.68794	
1500..	4.9972	−13.10266	4.0100	−10.53500	−15.69144	
1000..	**10.415**	**−35.68904**	4.0144	−10.55040	−15.70684	
500..	−130.879	613.47648	4.0220	−10.57700	−15.73344	
0..	−102.267	402.82988	4.0324	−10.61340	−15.76984	
−500..	−123.089	484.03055	4.0402	−10.64070	−15.79714	
−1000..	−92.394	363.42851	4.0502	−10.67570	−15.83414	
−1500..	−101.807	400.96355	4.0574	−10.70090	−15.85734	
−2000..	− 41.316	166.03588	4.0658	−10.73030	−15.88674	
	Sudden transformation of the values on entering the adiabatic strata					
−2500..	**63.4100**	**−240.23076**	2.4177	−3.16071	−8.31715	
−3000..	2.6809	−4.17847	2.4252	−3.17796	−8.33440	
−3500..	2.3819	−3.35719	2.4265	−3.18094	−8.33738	
−4000..	2.3857	−3.34790	2.4270	−3.18210	−8.33854	
−4500..	2.4080	−3.26224	2.4276	−3.18348	−8.33992	
−5000..	2.3938	−3.32427	2.4280	−3.18440	−8.34084	
−5500..	2.4178	−3.22897	2.4285	−3.18555	−8.34199	
−6000..	2.4151	−3.24294	2.4288	−3.18624	−8.34268	
−6500..	2.4252	−3.20316	2.4292	−3.18716	−8.34360	
−7000..	2.4291	−3.18916	2.4295	−3.18885	−8.34529	
↓	↓	↓	↓	↓	↓	
Can	be extended	downward	in	definitely		

Non-adiabatic region | Isothermal layers | Adiabatic region

it extended from the same point (1.00, 0.00000), though there may be a question whether it should not be parallel to the non-adiabatic line, the shift being parallel rather than triangular. The practical effect is the same because (a, log c) extends for so short a distance in the square that no important change could come from a slightly different slope. The practical range of (α, log c) in the non-adiabatic region is in the small rectangle. Table 20A contains the mean values adopted in computing

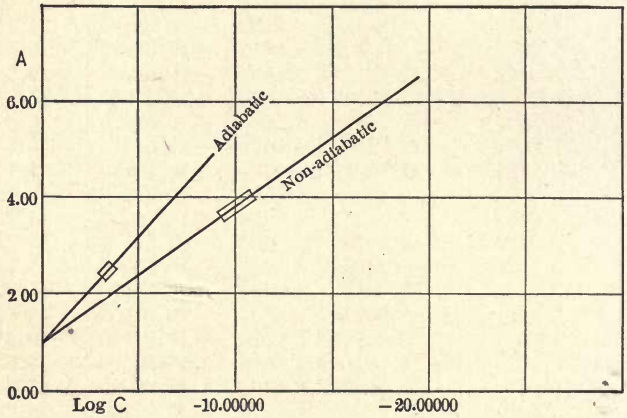


FIG. 17. Helium. The (A_1 log C) Lines in the Adiabatic and the Non-adiabatic Regions.

the pair values (α, log c) for helium. Each of the elements H_1, H_2, *He*, *C*, *Ca*, *Zn*, *Cd*, *Hg*, has been computed in the same way, and these auxiliary tables differ from one another only by minor variations, dependent upon the uncertainties due to the method of trials. Thus, collecting together the initial values for $\alpha = 4.00$, we have the following results for log c. See Table 19.

$$\log A = \frac{\log (\log K_1 - \log K_0)}{\log (\log T_1 - \log T_0)}, \quad \text{Formula (40).}$$

$$\log C = \log K_{10} - A \log T_{10}, \text{ Formula (39).}$$

Plot (A, log C) and construct the auxiliary table for (α, log c).

Take trial values of α (α_1, α_2 . . .) and compute log c = log $K_{10} - \alpha \log T_{10}$; then take the corresponding pair values (α, log c) from the table. If α_1 (trial) = α (computed) the pair (α, log c) is correct. If $\alpha_1 \gtrless \alpha$, take $\alpha_2 = \frac{1}{2}(\alpha_1 + \alpha)$ and proceed to a second trial. When $\alpha_u = \alpha$, this is the adopted (α, log c).

These mean values of Table 20B might be used to construct a common table for all the chemical elements, to serve in a recomputation based upon the series of mean values of the different hyperbolic constants that have already been obtained. At present, the results given have all been computed for each element, quite independent of one another.

Using the auxiliary Table 20A, the pair values (α, log c)

TABLE 20A

THE WORKING VALUES (α, LOG c)

Adiabatic (α Log c)			Non-adiabatic (α Log c)		
	α	log c		α	log c
	4.00	−6.80000		4.08	−10.78000
	—	—		4.07	−10.74500
	3.50	−4.50000		4.06	−10.71000
	—	—		4.05	−10.67500
	2.50	−3.35000		4.04	−10.64000
	2.49	−3.32700		4.03	−10.60500
	2.48	−3.30400		4.02	−10.57000
	2.47	−3.28100		4.01	−10.53500
	2.46	−3.25800		4.00	−10.50000
	2.45	−3.23500		3.99	−10.46500
	2.44	−3.21200		3.98	−10.43000
	2.43	−3.18900		3.97	−10.39500
	2.42	−3.16600		3.96	−10.36000
	2.41	−3.14300		3.95	−10.32500
	2.40	−3.12000		3.94	−10.29000
	2.39	−3.09700		3.93	−10.25500
	2.38	−3.07400		3.92	−10.22000
	2.37	−3.05100		3.91	−10.18500
	2.36	−3.02800		3.90	−10.15000
	2.35	−3.00500		—	—
	2.34	−4.98200		3.80	−11.80000
	2.33	−4.95900		—	—
	2.32	−4.93700		3.70	−11.45000
	2.31	−4.91300		—	—
	2.30	−4.89000		3.60	−11.10000
Slope	−0.100	−0.23000	Slope	−0.100	−0.35000

It is practically limited to the small square.

It is practically limited to the small rectangle.

TABLE 21

DISTRIBUTION OF THE EXPONENT (α) IN $K_{10} = cT^{\alpha}$

The abrupt transformation between the adiabatic and the non-adiabatic strata

	Z Kilometers	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	Non-adiabatic Means	Adiabatic Means
	25000	3.9525	2.3050							3.9525	—
	20000	3.9613	3.5884							3.7749	—
	15000	3.9633	3.8214	—						3.8924	—
	10000	3.9628	3.9124	3.8474						3.9075	—
	5000	3.9696	3.9298	4.0205	—	—				3.9733	—
	1000	3.9862	3.9489	4.0144	4.0095	3.7716				3.9898	—
	800	3.9870	3.9510	4.0179	4.0104	3.9811	—			3.9895	—
	600	3.9878	3.9532	4.0215	4.0148	4.0176	3.9141			3.9848	—
	400	3.9887	3.9553	4.0251	4.0235	4.0206	4.0005	—	—	4.0023	—
	200	3.9895	3.9574	4.0287	4.0370	4.0114	4.0080	4.0200	3.7000	4.0074	—
	100	3.9900	3.9585	4.0305	4.0421	4.0183	4.0075	4.0060	3.9386	3.9989	—
	80	3.9901	3.9587	4.0309	4.0431	4.0220	4.0131	3.9987	3.9367	3.9992	—
	60	3.9902	3.9589	4.0313	4.0441	4.0257	4.0187	3.9982	3.9369	4.0005	—
	40	3.9903	3.9591	4.0317	4.0451	4.0294	4.0243	4.0047	3.9395	4.0030	—
	20	3.9903	3.9593	4.0320	4.0462	4.0331	4.0299	4.0143	3.9469	4.0065	—
Photos.	0	3.9904	3.9595	4.0324	4.0472	4.0367	4.0357	4.0257	3.9636	4.0114	—
	−20	3.9904	3.9597	4.0328	4.0484	4.0402	4.0415	4.0348	3.9808	4.0161	—
	−40	3.9904	3.9599	4.0332	4.0496	4.0437	4.0493	4.0451	3.9989	4.0210	—
	−60	3.9904	3.9601	4.0336	4.0508	4.0472	4.0531	4.0534	2.3997	4.0269	2.3997
	−80	3.9904	3.9603	4.0340	4.0521	4.0507	4.0589	2.3963	2.4022	4.0244	2.3993
	−100	3.9904	3.9605	4.0343	4.0535	4.0544	4.0647	2.4053	2.4038	4.0263	2.4046
	−200	3.9905	3.9614	4.0361	4.0597	4.0732	2.4213	2.4104		4.0242	2.4159
	−400	3.9905	3.9638	4.0396	4.0690	2.4285	2.4242			4.0156	2.4264
	−600	3.9905	3.9660	4.0432	4.0810	2.4305	2.4262			4.0202	2.4284
	−800	3.9906	3.9682	4.0467	2.4155	2.4324				4.0018	2.4240
	−1000	3.9906	3.9704	4.0502	2.4230	2.4337				4.0037	2.4284
	−2000	4.0007	3.9849	4.0658	2.4352	2.4358				4.0171	2.4355
	−4000	4.0084	4.0094	2.4270						4.0089	2.4270
	−6000	4.0177	3.3323	2.4288						4.0177	2.4288
	−8000	4.0241	3.3458	2.4302						4.0241	2.4302
	−10000	2.4236	3.3483							—	2.4236
	−12000	2.4255	3.3492							—	2.4255
	−14000	2.4275	3.3522							—	2.4275

The arrow signifies increasing values of α.

TABLE 22

DISTRIBUTION OF THE COEFFICIENT (LOG c) IN $K_{10} = c\,T^{\alpha}$ (M. K. S.)

The abrupt transformation between the adiabatic and the non-adiabatic strata

Z Kilometers	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	Non-adiabatic Means	Adiabatic Means
25000	−10.62882	−15.39040							—	—
20000	−10.66068	−11.59995					(M. K. S.)		—	—
15000	−10.66788	−10.36419							—	—
10000	−10.66608	−10.66169	−11.96590						—	—
5000	−10.69038	−10.71974	−10.57175	—	—				−10.70506	—
1000	−10.75050	−10.78239	−10.55040	−10.53325	−11.70060				−10.76645	—
800	−10.75348	−10.78934	−10.56300	−10.53640	−10.43385	—			−10.61521	—
600	−10.75647	−10.79630	−10.57560	−10.55180	−10.56160	−10.19935			−10.64835	—
400	−10.75946	−10.80325	−10.58820	−10.58225	−10.57210	−10.50175	—		−10.63450	
200	−10.76245	−10.81021	−10.60080	−10.62950	−10.53990	−10.50280	−10.67067	−10.10100	−10.64606	—
100	−10.76395	−10.81369	−10.60710	−10.64735	−10.56450	−10.52625	−10.62191	−10.79534	−10.66751	—
80	−10.76425	−10.81439	−10.60836	−10.65092	−10.57729	−10.54599	−10.59541	−10.78900	−10.66821	—
60	−10.76455	−10.81509	−10.60962	−10.65449	−10.59008	−10.56573	−10.59365	−10.78967	−10.67361	—
40	−10.76485	−10.81579	−10.61088	−10.65806	−10.60287	−10.58547	−10.61660	−10.79833	−10.68161	—
20	−10.76515	−10.81648	−10.61214	−10.66165	−10.61566	−10.60521	−10.65052	−10.82300	−10.69373	—
Photos. 0	−10.76544	−10.81716	−10.61340	−10.66520	−10.62845	−10.62495	−10.69081	−10.87867	−10.71026	—
−20	−10.76580	−10.81787	−10.61465	−10.66955	−10.64084	−10.64525	−10.72296	−10.93600	−10.72662	—
−40	−10.76617	−10.81859	−10.61589	−10.67393	−10.65323	−10.66555	−10.75935	−10.99634	−10.74364	—
−60	−10.76654	−10.81930	−10.61714	−10.67831	−10.66562	−10.68585	−10.78867	− 3.26597	−10.71735	−3.26597
−80	−10.76691	−10.82001	−10.61838	−10.68269	−10.67801	−10.70615	− 3.25804	− 3.27181	−10.71203	−3.26493
−100	−10.76728	−10.82072	−10.61963	−10.68707	−10.69040	−10.72645	− 3.27904	− 3.27554	−10.71859	−3.27729
−200	−10.76914	−10.82430	−10.62586	−10.70895	−10.76070	− 3.26899	− 3.29093		−10.73779	−3.27996
−400	−10.77285	−10.83146	−10.63832	−10.74150	− 3.28555	− 3.27565			−10.74603	−3.28060
−600	−10.77656	−10.83862	−10.65078	−10.78350	− 3.29003				−10.76238	−3.29003
−800	−10.78027	−10.84578	−10.66324	− 3.32643	− 3.29441				−10.76310	−3.31042
−1000	−10.78398	−10.85294	−10.67570	− 3.34405	− 3.29763				−10.77087	−3.32084
−2000	−10.80252	−10.90047	−10.73030	− 3.37272	− 3.30223				−10.81110	−3.33748
−4000	−10.83024	−10.98083	− 3.18210						−10.90554	−3.18210
−6000	−10.86374	− 7.77434	− 3.18624						−10.86374	−3.18624
−8000	−10.88676	− 7.81934	− 3.19146						−10.88676	−3.19146
−10000	− 3.17428	− 7.82767	↓	↓	↓	↓	↓	↓	—	−3.17428
−12000	− 3.17865	− 7.83067							—	−3.17865
−14000	− 3.18325	− 7.84068							—	−3.18325

The arrow signifies increasing values of (log c).

TABLE 20B

THE INITIAL VALUES OF α AND LOG c FOR THE DIFFERENT CHEMICAL ELEMENTS

Element	Exponent α	Adiabatic Log c	Non-adiabatic Log c
Hydrogen H_1	4.00	−6.80000	−10.80000
Hydrogen H_2	4.00	−6.80000	−10.95000
Helium He	4.00	−6.80000	−10.50000
Carbon C	4.00	−7.0500	−10.50000
Calcium Ca	4.00	−6.90000	−10.50000
Zinc Zn	4.00	−6.90000	−10.50000
Cadmium Cd	4.00	−7.00000	−10.60000
Mercury Hg	4.00	**−7.00000**	**−10.99900**
Means	4.00	−6.90556	−10.66900

have been computed by trials with the results in the fourth and fifth columns of Table 19. It is seen that the very large values of (A, log C) in the isothermal region have given place to the smooth values of (α, log c) throughout the non-adiabatic region, including the isothermal layer. In this region the value of the exponent increases slowly from 3.6310 at $z = 11500$ kilometers to 4.0658 at $-$ 2000 kilometers, while log c changes from $-$ 11.20850 to $-$ 10.73030 at these respective heights, counted from the level of the photosphere. *In passing into the adiabatic levels a sudden and important transformation occurs*, such that the exponent α changes from 4.0658 into 2.4177, and the coefficient log c changes from $-$ 10.73030 into $-$ 3.16071. That is to say, in the adiabatic region we have (α, log c) = (2.4177, $-$ 3.16071), and in the non-adiabatic layers at the bottom of the isothermal layers we have (α, log c) = (4.0658, $-$ 10.73030). *The radiation terms therefore pass through a saltum at the bottom of the isothermal strata.*

Table 21 contains the values of the exponent α on the selected levels, and it shows the same remarkable transition from values a little greater than 4.00 to about 2.4200. There is one conspicuous exception to this rule, namely, hydrogen as a diatomic molecule, $H_2 = 2.00$, where the transformation of the exponent is from 4.0094 to 3.3323 instead of to 2.4200. This pronounced difference of the behavior of the diatomic element

H_2 from the monatomic H_1 is very significant. *It has been necessary to assume that* $H_1 = 1.00$ *is the condition which is to be used in the adiabatic levels, while the coronal observations demonstrate that hydrogen reaches its normal limit at the top of the inner corona, and must therefore have become* $H_2 = 2.00$ *above the level of the transformation. This is interpreted as the evidence of the dissociation of hydrogen in the adiabatic strata.*

Table 22 shows that there is the same transformation of log c in all the elements, from about -10.78000 to about -3.20000. The exception again is diatomic hydrogen, $H_2 = 2.00$, where the transition is from -10.98083 to -7.77434, instead of to -3.20000, *which is again to be associated with the process of dissociation at the levels* -4000 *to* -6000 *kilometers below the level of the photosphere.* Tables 21, 22, indicate plainly that the level of the transformation is always near the bottom of the isothermal layer, at the depth determined by the hyperbolic law, and that it is connected with the entire set of physical processes at these levels, which are contained in the preceding tables. The mean values of (α, log c) in both regions are given in the last columns of the Tables 21, 22.

The "Solar-Constant" of Radiation

On Table 22 the transition between the values of log c at the bottom of the isothermal layer appears to stand on a diagonal straight line, but this is merely due to the selection of the vertical height that has been employed. As a matter of fact, this boundary forms an equilateral hyperbola according to the law $z\,m = c$. By interpolating the probable depths below the plane of the photosphere where this transition takes place, it is found that

$$z_R \,.\, m = -9000.$$

By Table 7, the bottom of the isothermal layer is z_A, so that

$$z_A \,.\, m = -12000.$$

Hence, for each different element we obtain the depth of z_R

for radiation, and of z_A the depth of the transition to adiabatic conditions.

TABLE 23

POSITION OF THE RADIATION LAYER z_R IN THE ISOTHERMAL LAYER

Element m	z_A	z_R	z_I	$z_T=0$
H_1 1.00.....	−12000	−9000	+3600	+45400
H_2 2.00.....	−6000	−4500	+1800	+22700
He 4	−3000	−2250	+900	+11350
C 12	−1000	−750	+300	+3780
Ca 40	−300	−225	+90	+1135
Zn 65	−185	−140	+55	+700
Cd 112	−107	−80	+32	+405
Hg 198	−60	−45	+18	+230

z_A = the bottom of the isothermal layer. Table 5.
z_R = the depth of the radiation layer. Tables 21, 22.
z_I = the top of the isothermal layer. Table 7.
$z_{T=0}$ = the height of the vanishing plane. Table 8.

The position of the radiation stratum in the midst of the isothermal layer appears to be at three-fourths of the distance of the bottom of the isothermal layer z_A from the photosphere. Compare figures 2–9. On Fig. 18 the hyperbolic curves for the constants at the top of Table 23 show the relative positions of z_A, z_R, z_T, $z_{T=0}$, corresponding with the atomic weights of the chemical elements. *The asymptotic condition is apparently reached for the heaviest chemical elements, Radium* 226, *Thorium* 232, *Uranium* 238.

Mean Values of the Coefficient and the Exponent of Radiation in the Stefan Law

The computations show that at whatever depth the radiation transformation may occur for the different elements, the preceding and the following values have only small changes as compared with the abrupt transition itself. They may, therefore, be collected together without regard to the values of z for which the computations were actually made, that is, for the arbitrarily selected intervals $(z_1 - z_0)$. These were chosen in order to make fifty or sixty points in each computation, and

they varied approximately in the following intervals: H_2 2000, H_1 500, *He* 200, *C* 200, *Ca* 50, *Zn* 25, *Cd* 20, *Hg* 10 kilometers, respectively, for the $(z_1 - z_0)$ strata.

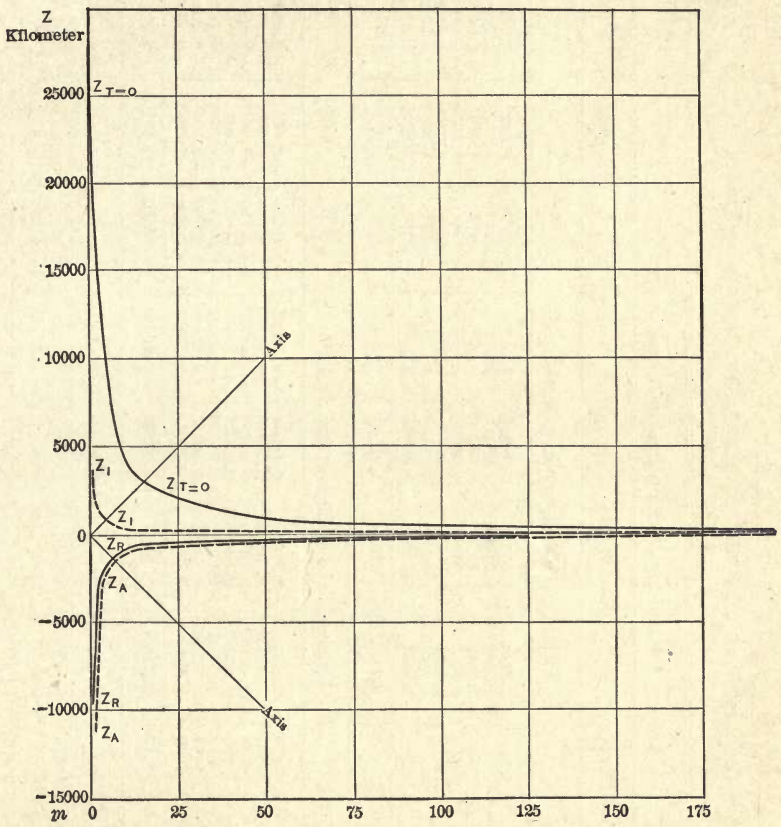


FIG. 18. Four Typical Hyperbolic Curves.

The hyperbolic curve for $m\,z_{T=0}$ = +45400, Vanishing Plane.
The hyperbolic curve for $m\,z_I$ = + 3600, Top of Isothermal Layer.
The hyperbolic curve for $m\,z_R$ = − 9000, Radiation Stratum.
The hyperbolic curve for $m\,z_A$ = −12000, Bottom of Isothermal Layer.

Table 24 contains the values of $(\alpha, \log c)$ for the last ten values in the isothermal layer, 1, 2, . . . 10, and the first ten values in the adiabatic strata, 10, 9, . . . 1. *The transformation of these values takes place in the radiation layer which lies*

TABLE 24

VALUES OF α AND LOG c IN THE ISOTHERMAL AND ADIABATIC STRATA

(C. G. S.)

Values of α and log c within the isothermal layers

Number	1	2	3	4	5	6	7	8	9	10
I Values of the exponent α (isothermal)										
H_1 1	4.0241	4.0177	4.0084	4.0007	3.9904	3.9821	3.9729	3.9662	3.9621	3.9628
He 4	4.0658	4.0574	4.0502	4.0402	4.0324	4.0220	4.0144	4.0100	4.0090	4.0113
C 12	4.0810	4.0690	4.0597	4.0472	4.0370	4.0235	4.0148	4.0104	4.0095	4.0095
Ca 40	4.0732	4.0597	4.0544	4.0440	4.0367	4.0246	4.0183	4.0139	4.0114	4.0112
Zn 65	4.0721	4.0647	4.0593	4.0518	4.0445	4.0357	4.0290	4.0209	4.0129	4.0075
Cd 112	4.0534	4.0451	4.0348	4.0257	4.0143	4.0047	3.9982	3.9987	4.0062	4.0160
Hg 198	3.9989	3.9880	3.9808	3.9699	3.9636	3.9536	3.9469	3.9428	3.9395	3.9373
Means	4.0526	4.0431	4.0354	4.0256	4.0170	4.0066	3.9992	3.9990	3.9929	3.9937
II Values of the coefficient log c (isothermal)										
H_1 1	−9.88676	−9.86372	−9.83024	−9.80252	−9.76544	−9.73556	−9.70244	−9.67832	−9.66355	−9.66608
He 4	−9.73030	−9.70090	−9.67570	−9.64070	−9.61340	−9.57700	−9.55040	−9.53500	−9.53150	−9.53955
C 12	−9.78350	−9.74150	−9.70895	−9.66520	−9.62950	−9.58225	−9.55180	−9.53640	−9.53325	−9.53325
Ca 40	−9.76070	−9.70895	−9.69040	−9.65400	−9.62845	−9.58610	−9.56450	−9.55315	−9.53990	−9.53920
Zn 65	−9.75235	−9.72645	−9.67920	−9.68130	−9.65575	−9.62495	−9.60150	−9.57315	−9.54515	−9.52625
Cd 112	−9.78867	−9.75935	−9.72296	−9.69081	−9.65052	−9.61660	−9.59365	−9.59541	−9.62191	−9.65643
Hg 198	−9.99634	−9.96000	−9.93600	−9.89967	−9.87867	−9.84534	−9.82300	−9.80933	−9.79833	−9.79100
Means	−9.81409	−9.78012	−9.74906	−9.71917	−9.68882	−9.65254	−9.62675	−9.61154	−9.60766	−9.60739

Values of α and log c in the adiabatic strata

Number	10	9	8	7	6	5	4	3	2	1

III Values of the exponent α (adiabatic)

H_1 1	2.4265	2.4284	2.4280	2.4277	2.4275	2.4275	2.4269	2.4255	2.4214	2.4236
He 4	2.4295	2.4292	2.4288	2.4285	2.4280	2.4276	2.4270	2.4265	2.4252	2.4177
C 12	2.4400	2.4458	2.4325	2.4352	2.4302	2.4330	2.4315	2.4300	2.4230	2.4155
Ca 40	2.4347	2.4340	2.4335	2.4328	2.4319	2.4310	2.4299	2.4287	2.4283	2.4183
Zn 65	2.4241	2.4240	2.4238	2.4234	2.4232	2.4227	2.4223	2.4213	2.4175	2.4123
Cd 112	2.4100	2.4100	2.4112	2.4104	2.4098	2.4091	2.4085	2.4076	2.4053	2.3963
Hg 198	2.4040	2.4040	2.4047	2.4030	2.4038	2.4022	2.4022	2.4021	2.3997	2.3935
Means	2.4241	2.4251	2.4232	2.4230	2.4221	2.4219	2.4212	2.4202	2.4172	2.4110

IV Values of the coefficient log c (adiabatic)

H_1 1	−2.18095	−2.18532	−2.18440	−2.18371	−2.18325	−2.18325	−2.18187	−2.17865	−2.16922	−2.17428
He 4	−2.18885	−2.18716	−2.18624	−2.18555	−2.18440	−2.18348	−2.18210	−2.18094	−2.17796	−2.16071
C 12	−2.38400	−2.39763	−2.36638	−2.37272	−2.36097	−2.36755	−2.36403	−2.36050	−2.34405	−2.32643
Ca 40	−2.29981	−2.29820	−2.29705	−2.29544	−2.29337	−2.29130	−2.28875	−2.28601	−2.28509	−2.26209
Zn 65	−2.27543	−2.27520	−2.27474	−2.27382	−2.27336	−2.27221	−2.27129	−2.26899	−2.26025	−2.24829
Cd 112	−2.29000	−2.29000	−2.29280	−2.29093	−2.28954	−2.28790	−2.28650	−2.28440	−2.27904	−2.25804
Hg 198	−2.27600	−2.27600	−2.27763	−2.27367	−2.27554	−2.27181	−2.27181	−2.27157	−2.26597	−2.25151
Means	−2.27072	−2.27280	−2.26846	−2.26798	−2.27578	−2.26536	−2.26376	−2.26157	−2.25451	−2.24019

V The same data for diatomic hydrogen, $H_2 = 2.00$

Isothermal Layers

α	4.0094	3.9974	3.9849	3.9763	3.9704	3.9650	3.9595	3.9545	3.9489	3.9440
log c	−9.98083	−9.94147	−9.90047	−9.87226	−9.85294	−9.83520	−9.81716	−9.80076	−9.78239	−9.76632

Adiabatic Strata

α	3.3522	3.3497	3.3492	3.3487	3.3483	3.3477	3.3458	3.3411	3.3323	3.3213
log c	−6.84068	−6.83233	−6.83067	−6.82900	−6.82767	−6.82566	−6.81934	−6.80366	−6.77434	−6.73767

The minus sign affects only the characteristic of the logarithm.

among them at various depths. Section I contains the mean values of the exponent α in the isothermal layer, and it ranges from 4.0526 to 3.9937, approximately 4.00 on the average; Section II contains the coefficient log c, which ranges from $-$ 9.81409 down to $-$ 9.60739 (C. G. S.); Section III contains the values of the exponent α in the adiabatic layer, and it ranges from 2.4241 down to 2.4110; Section IV contains the values of the coefficient log c, and it ranges from $-$ 2.27072 down to $-$ 2.24019 (C. G. S.); *Section V contains the corresponding values for the diatomic hydrogens* $H_2 = 2.00$, *and it shows that the system* (α, *log c*) *is the same in the isothermal layer, but that it is very different in the adiabatic strata, since the exponent* α *ranges from* 3.3522 *down to* 3.3213 *and the coefficient* log c from $-$ 6.84068 down to $-$ 6.73767 (C. G. S.). The minus sign affects only the characteristic of the logarithm.

An examination of Table 24 indicates that, although there are minor differences among the elements, there prevails a general harmony in all respects. The same range of the mean values is found for each element. Such differences as exist depend upon the inaccuracies with which the adiabatic *Ra* and Cp_a were computed by the method of trials. The most important data are the adiabatic values just before (2.1) entering the radiation layer, and those values which occur just within (1.2) the isothermal layer. In the values of log c of the adiabatic strata, the hydrogen H_1 and helium *He* are apparently a little too small in comparison with the other elements. It is believed that an adjusted computation, following the hyperbolic constants throughout, will produce a mean value under (1) about like that now under (2). The mean $-$ 2.25450 has, therefore, been adopted for the following discussions regarding the value of the solar constant of radiation. The general problem is to interpret the following data:

Isothermal Layer, $\alpha = 4.00$, log $c = -$ 9.81400 (C. G. S.).
Adiabatic Strata, $\alpha = 2.4172$, log $c = -$ 2.25450 "

The sudden transition from the adiabatic to the isothermal conditions seems physically like the release of a powerful spring, the discharge of a cannon, or a radioactive process, in which the

compressed state of the adiabatic strata is per saltum transformed into the weak state of the isothermal layer, the coefficient c being one three-millionth less in value. It is assumed that this represents the process of transformation in the configuration of the electrons, atoms, molecules, the change of state being accompanied by a powerful radiation into space. This is the source of the solar constant of radiation, whatever its history of depletion may become. We have therefore to evaluate the mean value of the coefficient log c, of which the limits have been determined, supposing this mean value to be the σ of the Stefan Law, $J_0 = \sigma T^4$.

In making the evaluation of the mean log c_0 it is not proper to take the mean value of the extremes, $\frac{1}{2}$ (log c_1 + log c_2), because this assumes that the process of decay or discharge is arithmetical. *On the other hand, all analogies suggest that the process of depletion must be logarithmic.* We must assume that this process begins as a tangent to a logarithmic spiral on the level log $\sigma_1 = -2.25450$, and ends as a normal to this curve at log $\sigma_2 = -9.81400$. We have to determine the angle of a logarithmic spiral whose origin is on the axis of ordinates, which passes -2.25450 as a tangent, and becomes perpendicular to -9.81400. Such a logarithmic curve, $r = e^{a\theta}$, has the angle $\alpha = 35° 10'$, so that $a = \cot \alpha = 1.4198$. It will be convenient at this point to collect together the analytic conditions of a logarithmic spiral.

Logarithmic Spiral

(102) General Equation. $r = e^{a\theta}$. r = radius, θ = angle, a = constant.

(103) Differentials. $\dfrac{dr}{d\theta} = a\, e^{a\theta} = a\, r$. $\dfrac{ds}{d\theta} = e^{a\theta}(1 + a^2)^{\frac{1}{2}} = r\,(1 + a^2)^{\frac{1}{2}}$.

$$\frac{dr}{ds} = \frac{a}{(1 + a^2)^{\frac{1}{2}}}.$$

(104) Element of the curve $ds = (dr^2 + r^2\, d\theta)^{\frac{1}{2}} =$

$$e^{a\theta}\,(1 + a^2)^{\frac{1}{2}}\, d\theta = \frac{(1 + a^2)^{\frac{1}{2}}}{a}\, dr.$$

(105) Trigonometric functions.

$$\sin \alpha = \frac{r\,d\theta}{ds} = \frac{1}{(1+a^2)^{\frac{1}{2}}}.$$

$$\cos \alpha = \frac{dr}{ds} = \frac{a}{(1+a^2)^{\frac{1}{2}}}.$$

$$\tan \alpha = \frac{r\,d\theta}{dr} = \frac{1}{a}. \quad \cot \alpha = \frac{dr}{r\,d\theta} = a.$$

(106) Tangent and Normal. $T = r\dfrac{ds}{dr} = \dfrac{r}{a}(1+a^2)^{\frac{1}{2}}.$

$$N = \frac{ds}{d\theta} = r\,(1+a^2)^{\frac{1}{2}}.$$

(107) Subtangent, Subnormal.

$$S.T. = r^2\frac{d\theta}{dr} = \frac{r}{a}. \qquad S.N. = \frac{dr}{d\theta} = a\,r.$$

(108) Perpendiculars on T, N.

$$p_t = r^2\frac{d\theta}{ds} = \frac{r}{(1+a^2)^{\frac{1}{2}}}. \quad p_r = r\frac{dr}{ds} = \frac{a\,r}{(1+a^2)^{\frac{1}{2}}}.$$

(109) Line integral. $(s_1 - s_0) = \int \left(r^2 + \dfrac{dr^2}{d\theta^2}\right)^{\frac{1}{2}} d\theta =$

$$\int \left(1 + r^2\frac{d\theta^2}{dr^2}\right)^{\frac{1}{2}} dr.$$

Line integral. $(s_1 - s_0) = \left(1 + \dfrac{r^2}{a^2\,r^2}\right)(r_1 - r_0) =$

$$\frac{(1+a^2)^{\frac{1}{2}}}{a}(r_1 - r_0).$$

(110) Area or surface integral.

$$A = \tfrac{1}{2}\int r^2\,d\theta.$$

(111) Volume of revolution.

$$V = \pi\int y^2\,dx = \pi\int x^2\,dy.$$

(112) Center of gravity. $g_0 = \tfrac{1}{2}\dfrac{\int_a^b y^2\,dx}{\int_a^b y\,dx}.$

Compare the International Cloud Report, page 515.

The Logarithmic Spiral for $\alpha = 35° 10'$

The angle 35° 10′ is the constant angle at which the logarithmic curve crosses the radius from the pole to any point on the curve. See Fig. 19. The computations proceed by the following formulas:

$$\alpha = 35° 10', \cot \alpha = a = 1.4193.$$

Compute the value for $\theta = 45° = \frac{\pi}{4}$, $\log r = M a \frac{\pi}{4} b$.

Take the factor b as a series of multiples of 45°, $b = 0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.1 \ldots 3.5$.

TABLE 25

POINTS ON THE LOGARITHMIC SPIRAL OF 35° 10′

r	θ	r	θ	r	θ
	Deg.		Deg.		Deg.
0.2000	0.0	1.0626	67.5	3.2594	112.5
0.2499	9.0	1.1901	72.0	3.6282	117.0
0.3024	18.0	1.3303	76.5	4.0562	121.5
0.3904	27.0	1.4873	81.0	4.5346	126.0
0.4878	36.0	1.6628	85.5	5.0690	130.5
0.6097	45.0	1.8588	90.0	5.6666	135.0
0.6816	49.5	2.0780	94.5	6.3348	139.5
0.7620	54.0	2.5230	99.0	7.0822	144.0
0.6518	58.5	2.5970	103.5	7.9170	148.5
0.9523	63.0	2.9032	108.0	8.8510	153.0

The point of tangency to − 2.25450 must occur at the angle $\theta = 90° - 35°.167 = 54°.833$, and the normal to − 9.81400 must occur at 54°.833 + 90° = 144°.833.

The pair values are, $\begin{cases} r_1 = 0.77790 \text{ and} \\ \theta_1 = 54°.833 \end{cases} \begin{cases} r_2 = 7.23000, \\ \theta_2 = 144°.833. \end{cases}$

On the scale of the drawing the values of r_1, r_2 are such that the unit is 1.00000 on the logarithm of the ordinates. Take the line integral from s_1 (r_1, θ_1) to s_2 (r_2, θ_2), and we have, $(s_2 - s_1) = 7.8926$. Hence, the middle point is at $(s_0 - s_1) = 3.9463$. The pair values for the middle point are $r_0 = 4.0040$, $\theta_0 = 120°.98$. This is on the ordinate for $\log \sigma_0 = - 5.74000$ by the drawing.

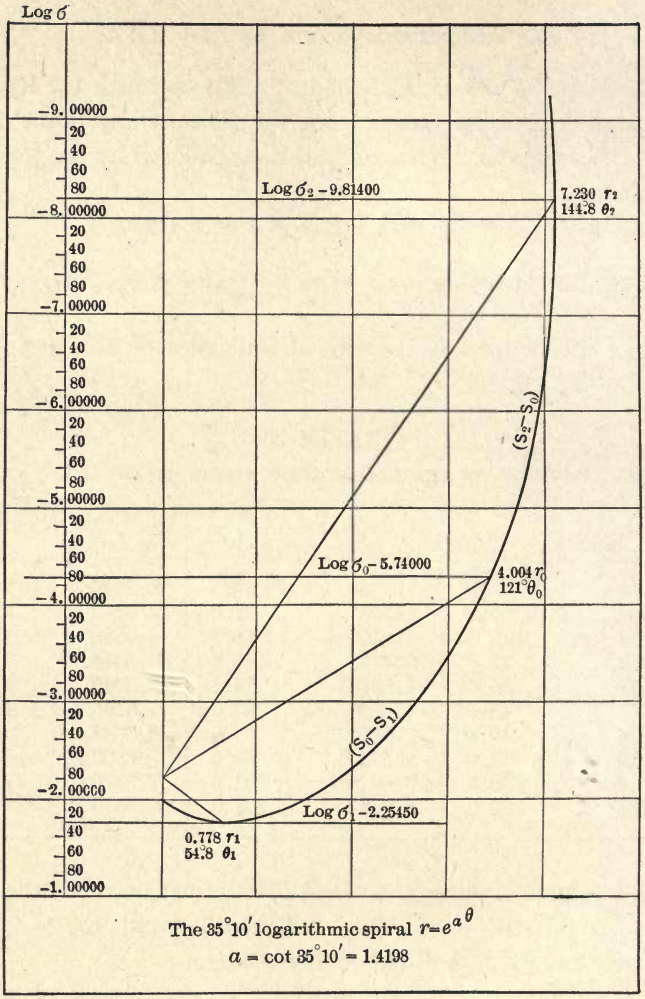


FIG. 19. The Mean Coefficient of Radiation σ_0 in the Stefan Law, $J_0 = \sigma_0 T^4$

Resolving the component $(r_1 \,.\, \theta_1)$, $0.7779 \cos 54°.10^1$,	0.45540
" " " $(r_0 \,.\, \theta_0)$, $4.0040 \cos 59°.0$,	2.06220
Sum of the two component ordinates (σ_1 to σ_0),	2.51760
Value of the log σ_1 at the initial point,	− 2.25450
" " " log σ_0 " " middle point,	− 5.73690
" " " log σ_2 " " final point,	− 9.81400

The result is that the coefficient of radiation corresponding to the middle point of the logarithmic curve is $-$ 5.73690 *by the analysis, and* $-$ 5.74000 *by scaling from the drawing.*

It only remains to compare this result with the values of the Kurlbaum coefficient in the Stefan Law as determined by the laboratory experiments. The following values are collected in the (M. K. S.) and (C. G. S.) systems of units. The dimensions of σ in this place are the same as those of equation (38) and its equivalents, (M. K. S.) $\times$ 10 = (C. G. S.), for the computed σ. This is to be distinguished from 60 σ/A, the equivalent in gram calories per square centimeter per minute. (Table 19.) The coefficient $\sigma = \frac{a\,c}{4}$, as may be seen in the formula for spatial density of the energy of radiation by Formula (76) and others following. As it is not proper to press too strongly the accuracy of the details of such a computation, I have provisionally adopted log σ = $-$ 5.74000 (C. G. S.), as in the following collected results.

TABLE 26

VALUES OF THE KURLBAUM COEFFICIENT IN $J_0 = \sigma T^4 = \frac{c\,u}{4}\left[\frac{ML^2}{T^2}\cdot\frac{1}{L^3} = \frac{M}{LT^2}\right]$.

Authority	M. K. S.		C. G. S.			
	Log σ	$\sigma = \frac{ac}{4}$	Log σ	Log α	Log 60 σA	$\frac{\text{gr. cal.}}{\text{cm.}^2\ \text{min.}}$
Planck	$-$6.72393	5.2956 $\times 10^{-6}$	$-$5.72393	$-$15.84887	$-$11.88037	7.5922 $\times 10^{-11}$
Schwärze	$-$6.73480	5.4300 "	$-$5.73480	$-$15.85974	$-$11.89124	7.7847 "
Berlin Phys. Inst.	$-$6.73719	5.4600 "	$-$5.73719	$-$15.86213	$-$11.89363	7.8276 "
Westphal	$-$6.74370	5.5424 "	$-$5.74370	$-$15.86864	$-$11.90014	7.9458 "
Shakspear	$-$6.75358	5.6700 "	$-$5.75358	$-$15.87852	$-$11.91002	8.1287 "
Bigelow	$-$6.74000	5.4954 "	$-$5.74000	$-$15.86494	$-$11.89644	7.8784 "

The value of 60 σ_A (C. G. S.) adopted in Bulletins No. 3, No. 4, O. M. A., and in the Meteorological Treatise, page 280, is 7.90×10^{-11}.

From this summary it is at least safe to infer that the coefficient of radiation in $J_0 = \sigma T^4$ *is the same in the solar atmosphere, in the special layer of radiation described, that it is in the terrestrial laboratories.*

We have already given the temperatures in the isothermal layer, Table 6, the mean value in that collection being 7686°.7.

It is found in the computations that in the isothermal layer of each gas there is a gradual small decrease in the temperature of each element from the top to the bottom of these strata, just as there is in the earth's isothermal layer, 40000 to 12000 meters above the sea level. We therefore take from the computations the values of the temperature of each element at the depth below the plane of the photosphere assigned to the radiation stratum, where the abrupt transformations of Table 24 take place.

TABLE 27

THE TEMPERATURE IN THE RADIATION LEVELS

Element	Depth in Kilometers	Temperature of Radiation
H_1 1.00	−9000	7656.6
H_2 2.00	−4500	7645.9
He 4	−2250	7658.3
C 12	−750	7655.0
Ca 40	−225	7658.3
Zn 65	−140	7657.0
Cd 112	−80	7651.0
Hg 198	−45	7653.6
	Mean	7654.5

The mean temperature of radiation is 7654°.5 absolute.

If the radius of the sun is $R = 694800$ kilometers, log 5.84186, and the distance to the earth is $D = 149340900$ kilometers, log 8.17418, then $\log \left(\frac{R}{D}\right)^2 = -5.33536$ is the reduction factor to the mean distance of the earth for the solar constant of radiation. In the (C. G. S.) system the factor $\frac{60}{4.1851 \times 10^7} = 0.0000014336$, $\log - 6.15644$. We compute the values of the radiation by the Stefan Law,

$$J_0 = \left(\frac{R}{D}\right)^2 60\, \sigma_A\, T^4, \text{ (C. G. S.)}, \tag{113}$$

at the sun and at the distance of the earth.

TABLE 28

EVALUATION OF THE FORMULA,

$$J_0 = \left(\frac{R}{D}\right)^2 60\,\sigma_A\,T^4 \text{ (C. G. S.)},$$

to determine the "solar constant" of radiation.

Formula	Sun	Earth
	°	°
T	7654.5	7654.5
Log T	3.88392	3.88392
Log T^4	15.53568	15.53568
Log σ	−5.74000	−5.74000
Log $60/4.1851\times10^7$	−6.15644	−6.15644
Log $\left(\frac{R}{D}\right)^2$	0.00000	−5.33536
	5.43212	0.76748
J $\frac{\text{gr. cal.}}{\text{cm.}^2\text{ min.}}$	270470.	5.8544 Solar Constant

At the sun the emission of radiation is 270470 *gram calories per square centimeter per minute, and at the distance of the earth, without any depletion, this is equivalent to* 5.8544 *gr. cal./cm.² min.*

The usual value assigned to the solar constant of radiation by the observers of the Smithsonian Institution is about 1.940 gr. cal./cm.² min., as determined by the pyrheliometer when the reductions are made by the Langley-Abbot method of extrapolation for a special assumption in the Bouguer formula. By the thermodynamics of the earth's atmosphere, Bul. No. 4, O. M. A., it was found *that* 3.980 *gr. cal./cm.² min. is the effective amount received and utilized in the atmosphere in order to maintain the existing temperatures, pressures, densities, gas efficiencies that actually exist.* We must, therefore, infer that the amount lost by scattering or internal reflection and by absorption between the radiation layer and the vanishing plane in the sun's atmosphere amounts to about

$$5.85 - 3.98 = 1.87 \text{ gr. cal./cm.}^2 \text{ min.},$$

which is about one-third of the original amount. Hence, we may summarize briefly from Bul. No. 4:

Depleted by scattering or absorption at the sun.....1.87
" " " " earth....2.48
Received on the sea level by the pyrheliometer.....1.50
Total solar radiation.....5.85

TABLE 29

THE AVERAGE PHYSICAL CONDITIONS IN THE SOLAR STRATA WHERE THE RADIATION ORIGINATES

Element	Depth in Kilometers	T Temperature	P Pressure	A Atmospheres	ρ Density	R Gas Coefficient	In 20—Kilometers		
							Q_1-Q_0 Free Heat	W_1-W_0 Work	U_1-U_0 Inner Energy
H_1 1.00.	−9000	7656.6	2426700	23.950	.0014609	216950	−978630	3164920	−4156600
H_2 2.00.	−4500	7645.9	2429700	23.980	.0028738	110580	−1498200	3912000	−5433120
He 4....	−2250	7658.3	2432050	24.003	.0058514	54273	−598870	3475200	−4095440
C 12....	−750	7655.0	2587250	25.535	.018218	18553	−1856200	3418780	−5428500
Ca 40....	−225	7658.3	2421600	23.900	.058363	5418	−1173310	3407620	−4711480
Zn 65....	−140	7657.0	2567800	25.343	.098010	3422	−1142800	3336560	−4513140
Cd 112....	−80	7651.0	2839900	28.028	.17902	2073	−599590	3525580	−4125270
Hg 198....	−45	7653.6	2493280	24.047	.29056	1096	−686080	3286780	−4042200
		7654.5	2524785	24.848	$.0014889 \times m = \rho$	$\frac{220772}{m} = R$	−1066710	3440930	−4563218

It is desirable to know the average dynamic and thermodynamic conditions which prevail in the solar strata where the thermodynamic radiation has its source. That is to say, those are the physical conditions which are suitable to set in operation the readjustment of the electrons in the atoms and molecules. They are as follows:

Temperature..... 7654°.5 A.
Pressure in kilograms per square meter 2524785
Pressure in terrestrial atmospheres.... 24.848
Density in kilograms per cubic meter. $0.0014889 \times m$
Gas coefficient in velocity square per degree..... $220772/m$
The free heat in work per cubic meter − 1066710 (20 − kilom.)
The work of external expansion..... 3440930 "
The inner energy..... − 4563218 "

Dividing the last three by 20000, we shall have the work or kinetic energy per cubic meter, Joules per m^3.

$(Q_1 - Q_0)$ the free heat.....	$-$ 53.3355	$\frac{\text{Joules}}{m^3}$
$(W_1 - W_0)$ the work of expansion....	172.0465	"
$(U_1 - U_0)$ the inner energy.....	$-$ 228.1609	"

Multiply by 10 to obtain ergs per cubic centimeter.

These values must be incorporated into the theory of the physics of solar radioactive radiation.

$$\textit{The Evaluation of } J_a = c_1 T_1^{\alpha_1} - c_0 T_0^{\alpha_0} \textit{ in } \frac{gr.\ cal.}{cm.^2\ min.}$$

With the computed values of log c and α, as in Tables 19 to 22, the value of the absorbed energy in the several strata has been obtained. For convenience of comparison the log c has been transformed to log $60\,\sigma_A$ C. G. S. in order to have the result in gr. cal./cm.^{2} min. As it is not practical to transcribe the entire computation, the values of J_a have been summed in strata as indicated in Table 30 under z in kilometers for each element. The sum ΣJ_a for each stratum is placed against the z_1 for the top of the stratum, the bottom being the next value z_0 below it. The approximate position of the top of the isothermal layer z_I, the photosphere 000, the radiating stratum z_R, and the top of the adiabatic layer z_A are indicated on the Table 30. The depths of the strata differ from element to element, so that the classification is complex. It is found, however, by taking the sum ΣJ_a from the vanishing plane down to the adiabatic strata that the *amount of energy in these strata averages* 94.46 *gr. cal./cm.*^{2} *min. for each chemical element.* This is the amount of energy required within the gases to maintain the existing solar T. P. ρ. R. and the prevailing thermodynamic conditions. *In the earth's atmosphere this amounts to about* 2.47 *gr. cal./cm.*^{2} *min. down to the sea level in middle latitudes.*

From the sums ΣJ_a, and the heights $z_1 - z_0$, the value of $\Delta J = \Sigma J_a / (z_1 - z_0)$ gives the gradient of the heat contents

per kilometer. When plotted, these gradients form a fan-shaped diagram from small values on the top line, close to the abscissas of the atomic weights, up to a high diagonal within the adiabatic strata (lowest line on the table). These gradients are essential in a theory of the solar radiation. It should be noted that while (α, log c) differ abruptly in passing through the radiation stratum, Table 24, *it yet appears that the gradient of the heat contents does not change per saltum, so that radiation does not seem to be a common thermodynamic process, as treated by Planck.*

TABLE 30

EVALUATION OF THE ABSORBED ENERGY $\Sigma J_a = \Sigma (c_1 T_1^{\alpha_1} - c_0 T_0^{\alpha_0})$ AS A TOTAL IN SELECTED STRATA, AND AS A GRADIENT OR AVERAGE AMOUNT PER KILOMETER

HYDROGEN 1.00			HYDROGEN 2.00			HELIUM 4.00			CARBON 12.00		
Z	ΣJ_a	ΔJ_a	Z	ΣJ_a	ΔJ_a	Z	ΣJ_a	ΔJ_a	Z	ΣJ_a	ΔJ_a
48000...	0.026	.0000	25000	0.002	.0000	11000	0.143	.0001	4000	0.233	.0001
34000...	0.622	.0005	17000	0.143	.0018	7000	2.813	.0025	2400	4.005	.0040
24000...	3.723	.0012	12000	1.901	.0026	4000	16.358	.0043	1400	19.717	.0150
14000...	14.648	.0018	7000	12.845	.0034	1000	4.029	.0058	400	6.013	.0200
	Z_I			Z_I			Z_I			Z_I	
4000...	3.982	.0020	2000	4.147	.0036	500	4.349	.0068	200	6.352	.0240
2000...	3.546	.0020	1000	3.556	.0038	.....	.....	.0075	.....	.....	.0300
000...	5.100	.0021	000	4.321	.0046	000	3.775	.0077	000	7.342	.0380
−2000...	4.320	.0022	−1000	7.335	.0052	−500	4.682	.0085	−200	7.207	.0420
−4000...	6.132	.0023	−2000	8.151	.0064	−1000	4.600	.0100	−400	11.211	.0500
−6000...	4.701	.0032	−3000	8.344	.0080	−1500	6.196	.0126	−600	12.064	.0600
−8000...	8.178	.0040	−4000	11.281	.0100	−2000	9.131	.0180	−800	6.997	.0800
	Z_R			Z_R			Z_R			Z_R	
−10000...	5.140	.0052	−5000	12.903	.0129	−2500	7.648	.0240	−1000	11.744	.1000
.....	.....	.0068	−6000	12.740	.0180	−3000	28.214	.0320	−1200	27.606	.1850
	Z_A			Z_A			Z_A			Z_A	
−12000...	17.458	.0087	−7000	50.280	.0251	−3500	40.075	.0401	−1400	89.720	.4000
−14000...	22.436	.0112	−9000	92.020	.0420	−4500	64.480	.0620	−1800	156.070	.6804
−16000...	26.794	.0133	−11000	131.740	.0658	−5500	81.550	.0816	−2200	226.310	1.1316
−18000...	.....	.....	−13000	.....	.....	−6500	.....	.....	−2600	.....	.....
To Z_A	60.118			87.669			91.938			120.491	

TABLE 30—*Continued*

CALCIUM 40			ZINC 65			CADMIUM 112			MERCURY 198		
Z	ΣJ_a	ΔJ_a	Z	ΣJ_a	ΔJ_a	Z	ΣJ_a	ΔJ_a	Z	ΣJ_a	ΔJ_a
1000.....	0.117	.0004	675	0.078	.0008	360	0.156	.0013	200	0.111	.0020
700.....	2.993	.0250	475	2.707	.0450	240	2.819	.0700	140	3.087	.2000
400.....	16.480	.0500	275	17.484	.0710	140	18.211	.1500	80	30.505	.3500
100.....	4.835	.0750	75	0.792	.1000	40	4.297	.1950	20	4.382	.4250
	Z_I			Z_I			Z_I			Z_I	
50.....	5.394	.0800	50	5.342	.1150	20	5.381	.2250	10	4.979	.4500
.....	.....	.0850	25	3.025	.1300	.....	.....	.2500	....	.....	.4700
0.....	3.758	.0900	0	6.177	.1400	0	5.037	.2700	0	2.939	.4960
−50.....	6.274	.1100	−25	2.695	.1600	−20	6.668	.2920	−10	5.872	.5250
−100.....	4.589	.1320	−50	2.255	.1850	−40	6.388	.3200	−20	4.524	.6000
−150.....	11.431	.1700	−75	6.670	.2300	−60	3.654	.3900	−30	8.038	.7200
−200.....	9.773	.2200	−100	6.052	.3100	−80	8.523	.5150	−40	2.048	.9000
	Z_R			Z_R			Z_R			Z_R	
−250.....	11.028	.3750	−125	6.026	.4300	−100	18.563	.7400	−50	7.308	1.1500
−350.....	54.478	.5048	−150	9.100	.7000	−120	27.313	1.0600	−60	15.109	1.5109
	Z_A			Z_A			Z_A			Z_A	
−450.....	172.570	.8250	−200	38.241	1.0500	−140	69.990	1.4500	−70	47.733	2.3867
−650.....	260.340	1.3020	−250	49.430	1.5500	−180	80.830	2.0507	−90	61.340	3.0670
−850.....	373.390	1.8670	−300	62.480	2.1000	−220	104.430	2.7108	−110	78.110	3.9055
−1050.....	.....	.....	−350	.....	.....	−260	.....	.....	−130	.....	.....
To Z_A	131.150			68.403			107.010			88.901	

Z_I = the top of the isothermal layer.
Z_R = the radiation stratum.
Z_A = the top of the adiabatic layer.
ΣJ_a = the sum of the absorbed energy in the stratum down to the next value of Z_0 $(Z_1–Z_0)$
ΔJ_a is the average amount in a stratum 1000 meters deep.

The sums at the bottom of ΣJ_a are the total amounts from the vanishing plane down to the top of the adiabatic layer Z_A. The average amount is 94.46 $\frac{\text{gr. cal.}}{\text{cm.}^2 \text{ min.}}$ reduced to the mean distance of the earth.

NOTES. (1) R. Emden, in his Über Strahlungsgleichgewicht und atmosphärische Strahlung," 1913, page 63, arrives at this theorem: "An isothermal atmosphere which is sufficiently thick emits black radiation." The solar atmospheres conform to this criterion.

(2) In the Appendix to this Treatise a more specific analysis of the physical conditions at the layer of radiation confirms the *saltum* process which is described in this chapter.

(3) It seems that radiation begins suddenly at the bottom of the isothermal layer, and is continued throughout these strata as black or fully effective radiation.

CHAPTER IV

The Coefficients in the Stefan and the Wien-Planck Formulas for Black Body Radiation

The Conversion from the (M. K. S.) System to the (C. G. S.) System

Up to this point the computations have all been made in the (M. K. S.) system of units, because for the data of meteorology it is much more convenient in many respects. The gram and the centimeter are too small for practical purposes in the atmosphere, and their use would entail a set of inconvenient and unmanageable numbers. On the other hand, in the magnified correlative system, the ton = 100^3 grams = 1000 kilograms is a unit of mass too large for the practical man, while the unit of length, 1 meter = 100 centimeters, is very suitable. The unit of time, one second, is the same in both systems. The best compromise for general purposes is the meter-kilogram-second (M. K. S.) system, which has been employed in the computations.

In taking up the problems of radiation the convenient system is evidently the (C. G. S.) system, because the entire literature of the physics of radiation, including the fundamental constants and coefficients, is written on that basis. Accordingly, the remainder of our discussion will be executed in the (C. G. S.) system of units. In making the transition from (M. K. S.) to (C. G. S.), the equation of condition for problems in radiation is that heretofore employed, and it is derived from the

First Law of Thermodynamics, $dQ = dW + dU$.

Since $dW = P\,dv$, this transposes and integrates into,

$$\frac{(Q_1 - Q_0)}{(v_1 - v_0)} - P_{10} = \frac{U_1 - U_0}{v_1 - v_0} = K_{10} = c\,T_{10}{}^{\alpha}.$$

This equation has the dimensions of kinetic energy per unit volume in every term, and it represents the work done in developing the kinetic energy of the moving molecules within the given volume. Thus, $(Q_1 - Q_0)$ is the free heat which is due to the velocity of the molecules in the volume $(v_1 - v_0)$. Since

$$(Q_1 - Q_0) = (Cp_a - Cp_{10})\,(T_a - T_0),$$

it must have the dimensions of the specific heat, $\frac{L^2}{T^2 \text{ deg.}}$, the dimension of $(v_1 - v_0) = \frac{L^3}{M}$; therefore, $\frac{Q_1 - Q_0}{v_1 - v_0}$ has the dimension $\frac{L^3}{T^2 \text{ deg.}} \cdot \frac{M}{L^3} = \frac{M}{L\,T^2 \text{ deg.}}$ (M. K. S.).

For 1 Joule $= \frac{\text{kilog. meter}^2}{\text{second}^2}$, $\frac{M\,L^2}{T^2}$, this is equivalent to

$$\frac{\text{kilogram}}{\text{meter second}^2 \text{ degree}} = \frac{\text{Joule}}{\text{meter}^3 \text{ deg.}} = \frac{\text{Joule}}{\text{volume deg.}}.$$

Similarly, in the (C. G. S.) system, for 1 erg $= \frac{\text{gr. cm.}^2}{\text{sec.}^2}$, $\frac{Q_1 - Q_0}{v_1 - v_0}$ becomes $\frac{\text{gram}}{\text{cm. sec.}^2} = \frac{\text{erg}}{\text{cm.}^3} = \frac{\text{dyne}}{\text{cm.}^2}$.

Since $\frac{M}{L\,T^2} = 10$, we have, (M. K. S.) $\times$ 10 = (C. G. S.) and the conversion factor is 10 from (M. K. S.) to (C. G. S.).

P_{10} is the pressure as work per unit volume, and not force per unit area, though the dimensions are the same,

Pressure = work per unit volume $= M\,L^2\,T^{-2} \cdot L^{-3} = M\,L^{-1}\,T^{-2}$.

Pressure = force per unit area $= M\,L\,T^{-2} \cdot L^{-2} = M\,L^{-1}\,T^{-2}$.

Its conversion factor from (M. K. S.) to (C. G. S.) is 10. $(U_1 - U_0)$ is the inner energy per unit volume, due again to the kinetic energy of the molecular motions. Since $(U_1 - U_0) = (Cp_a - R_{10})\,(T_a - T_0)$, it follows that the same factor of conversion applies as for $(Q_1 - Q_0)$.

K_{10} is the energy of the electromagnetic radiation per unit volume, and it is so defined by all authorities. Since T_{10} and the exponent α are without dimensions, it follows that the coefficient c is the energy per (degree)$^\alpha$ in the unit volume.

It may be noted in passing that Mr. Frank W. Very, of the Westwood Astrophysical Observatory, insists upon interpreting this equation of condition as if it were,

$$\frac{\text{Joules}}{\text{area}} \text{ instead of } \frac{\text{Joules}}{\text{volume}} \text{ (M. K. S.).}$$

$$\text{Very, } \frac{\text{Joules}}{\text{area}} = \frac{\text{Joules}}{m^2} = \frac{\text{kilog. } m^2}{\text{sec.}^2} \cdot \frac{1}{m^2}, \left[\frac{M}{T^2}\right], \text{ Factor} = 1000.$$

$$\text{Bigelow, } \frac{\text{Joules}}{\text{volume}} = \frac{\text{Joules}}{m^3} = \frac{\text{kilog. } m^2}{\text{sec.}^2} \cdot \frac{1}{m^3}, \left[\frac{M}{L\,T^2}\right], \text{ Factor} = 10.$$

He has published an erroneous criticism regarding my computations in asserting that the conversion factor is 1000 instead of 10. This is evidently contrary to the common law of pressure in meteorology, which converts the pressure $P = 101323.5 \frac{\text{kilograms}}{\text{meter}^2}$ into $1013235. \frac{\text{grams,}}{\text{cm.}^2}$, and not into $101323500 \frac{\text{grams}}{\text{cm.}^2}$. That would destroy the entire meteorological system in practice.

It may be proper in this connection to make a few quotations as to the meaning of $u = a\,T^4$ when referring to the Stefan formula; $K_{10} = \sigma\,T^4$ evidently has identical dimensions.

Lorentz, "The Theory of Electrons," page 74. "σ is a constant. *The total energy per unit volume*, or the total emissivity of a black body, must be proportional to the fourth power of the temperature."

Arthur L. Day and C. E. Van Ostrand, "Astrophysical Journal," January, 1904, page 4. "$d\,Q$ = differential of radiant heat entering the cylinder, $d\,W$ = the external work, $d\,U$ = the increase of internal energy. $J_0 = U/u$ = *the density of the energy or intensity per unit volume.*"

Richardson, "The Electron Theory of Matter," p. 334. "It follows that the *energy per unit volume, in vacuo*, of radiation in equilibrium in an enclosure at the absolute temperature T is equal to a universal constant multiplied by the fourth power of the absolute temperature."

Planck, "Die Theorie der Wärmestrahlung,"

"P. 62, u = *specific intensity* of the black radiation.

" P. 23, u = the *volume density* of the total radiation.

" P. 63, u, the *volume density* and the *specific intensity* of black radiation, are proportional to T^4.

" P. 63, $a = 7.061 \times 10^{-15}$ $\frac{\text{ergs}}{\text{cm.}^3 \text{ deg.}^4}$, $\frac{\text{ergs}}{\text{volume deg.}^4}$."

This is the energy in a volume in the pure ether assumed to be enclosed in a vessel of perfectly reflecting mirror walls, or in which the *vessel is maintained at a certain temperature* T.

This confined radiant energy when put in motion with the velocity of light c becomes the radiating energy in one direction, so that a column having the base of 1 cm.^{2} and the length c is the *amount which, as a radiating flux, can enter an enclosure per second.* If this is a sphere the area of the surface is $\pi r^2 \times 4$, and that of the area on the diameter is πr^2, so that $\sigma = \frac{c}{4} a$ is the *amount of the flux which crosses a unit area per second in one direction.* Hence, $J_0 = \sigma T^4 = \frac{c\,a}{4} T^4$ is the surface flux of black radiation. $\sigma = \frac{a}{4} \times$ velocity. This is the simple interpretation of the Poynting Law which connects the surface flux with the volume density (66).

Planck defines the Kirchhoff Theorem in the following terms, page 133:

"A vacuum surrounded by mirror walls in which arbitrary emitting and absorbing bodies are distributed in an arbitrary arrangement, *in the course of time come to the stationary state of black radiation*, which is fully determined through one parameter, *the temperature*, and therefore does not depend upon the number, the kind, and the arrangement of the ponderable bodies.

The general problem of radiation consists of three distinct parts:

(1) The electromagnetic black radiation confined for convenience in a mirror enclosure.

(2) The ponderable bodies or gases in the same enclosure having any distribution.

(3) The stationary state of equilibrium at the temperature T, between the electromagnetic radiant energy and the kinetic energy of molecular motion, brought about by the mechanism of the molecules, atoms and electrons, which are involved in the conservation of energy for the system, during the transformation from the radiant to the kinetic motions of ponderables. Without knowing what the structural mechanism really is, there is the transformation of a definite amount from the radiation energy per volume into the temperature corresponding with the given kinetic energy.

Lorentz expresses the general problem as follows in terms of Planck's resonators, on page 79. "If a body is enclosed within perfectly reflecting walls, there will be a state of equilibrium, on the one hand between the resonators and the radiation in the ether, and on the other hand between the resonators and the ordinary heat motion of the molecules and atoms constituting the ponderable matter. The first of these equilibria can be examined by means of the electromagnetic equations, and, in order to understand the second, one could try to trace the interchange of energy between the resonators and the ordinary particles."

These extracts sufficiently indicate the nature of the problem that has arisen in prosecuting this research. We cannot practically compute from the electromagnetic radiation to the temperature of the ponderable matter in the assumed ideal enclosure, but we must proceed from the temperature of the free gases in an atmosphere back to the radiation energy which originated it. We cannot isolate a given volume of the æther having a certain amount of radiant agitation in space, somewhere between the sun and the earth, place within it a ponderable mass of gas, and note its temperature. Furthermore, it is not necessary to have resort to this ideal enclosing vessel, which is useful only for analytical purposes, because the single fact which is essential is that the amount of *radiant energy in the unit volume should remain constant for the temperature* T.

This constancy of radiant energy per volume can be secured by opening the ends of the vessel along the path of the solar radiation, and so allow the volume energy of radiation to flow through it with the velocity of light. In this way all unit spaces surrounding the sun are filled with a given amount of energy fixed proportionally to its distance from the sun. The earth's atmosphere is immersed in such a flowing radiation, whose intensity per unit volume within the atmosphere is equivalent to $\sigma = \frac{a\,c}{4}$ in a given volume per degree^3. Since our computations are strictly thermodynamic, being based upon the ponderable gas temperatures of the successive strata, we obtain at once the σ of the Stefan, or quasi-Stefan law, and not the primitive energy of the æther at rest in a vacuum outside the atmosphere. The computations showed immediately that in solving the expression,

$$\frac{U_1 - U_0}{v_1 - v_0} = K_{10} = c\,T^{\alpha}\ ,$$

the c is to be identified with the coefficient σ, and this is the method which we have pursued. Having K_{10} and T it was necessary to solve the equation for two unknown quantities (c and α), as has been done by the method of pairs ($\log c, \alpha$). This value of c is so closely identical with the Stefan σ, in $J_0 = \sigma\,T^4$, that the problem of the relations between c and σ become of primary interest. The outcome of this method is such as clearly to justify the procedure, and to bring forward many important facts which will be of value in solving this great physical problem of the mechanism of the transformation of the radiant energy of the æther into ponderable kinetic energy at the temperature T, or the reverse. It has been shown that the value of $\log \sigma$ is discontinuous near the bottom of the isothermal layers of the gases of the sun, but that the mean logarithmic value agrees closely with the Kurlbaum value of this coefficient. Further examples of this relation will be given in this chapter.

(1) The model for the analysis of radiant electromagnetic energy consists in enclosing it within a certain volume by means

of perfectly reflecting mirror walls, or in an absorbing enclosure at the fixed temperature T. Such an amount is indestructible.

(2) The same volume density of radiant energy can be maintained at the same point in the pure æther of space by the continuous flux of the constant radiation from the sun outwards,

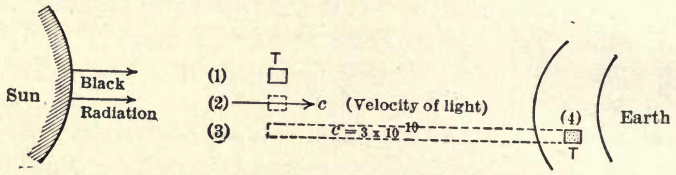


FIG 20. The Several Stages in the Theory of the Transformation of Radiant Energy into Molecular Kinetic Energy at the Stationary Temperature T.

after enclosing walls have been removed. The amount of flux through this volume (2) per second is 3×10^{10} times the amount in the first æther volume enclosed in (1).

(3) The amount arriving at any equivalent volume (3) in the earth's gaseous atmosphere per second is equal to the pure æther electromagnetic energy which is contained in a volume 3×10^{10} cm. long $\times$ 1 cm.^{2} $= 3 \times 10^{10}$ cm.^{3}

(4) If this amount of energy is instantaneously enclosed in 1 cm.^{3} of the gaseous atmosphere of the earth, in the course of time this electromagnetic energy of the æther will be converted into the kinetic energy of the molecular motion of the gas at a fixed equivalent temperature T. The electromagnetic energy in the æther possesses no temperature, and it is of an entirely different physical order from the temperature of the gas, but the two kinds of energy are exactly equivalent. The dimensions of the two amounts of energy are always equivalent to the kinetic energy of the gas per volume,

$$\left[\frac{M\,L^2}{T^2} \cdot \frac{1}{L^3}\right], \frac{\text{kinetic energy}}{\text{volume}},$$

and this is not to be confused with the rate at which the æther energy flows through the boundary of the gaseous volume (4),

$$\left[\frac{M\,L^2}{T^2} \cdot \frac{1}{L^3} \cdot \frac{L}{T}\right], \frac{\text{kinetic energy}}{\text{area, second}} \text{ (C. G. S.)}.$$

The reason for this is that the stationary *temperature T has been acquired only after the lapse of time, which is necessary for a certain amount of electromagnetic energy in this volume to go through the physical transformations.* The flux of energy per second through a surface of the volume, as in case (2) of the pure æther, merely serves to maintain the fixed amount of radiant energy which can ultimately generate the gaseous temperature T. It is easy to perceive that the flux, velocity per second, in one direction through the æther, from the sun to the earth, is broken up into the innumerable, irregular, directional velocities pertaining to the heat motions of the molecules which maintain T, so that we cannot utilize the flux-equations, but only the volume-equations, in this connection. Hence, the values of,

$$\frac{Q_1 - Q_0}{v_1 - v_0} - P_{10} = \frac{U_1 - U_0}{v_1 - v_0} = \sigma T^{\alpha}$$

are all for volume energy in the gases, and $a = \frac{4\sigma}{c}$ is a volume-energy at one point of a homogeneous column which is $c = 3 \times 10^{10}$ cm. long. It is, also, the pure æther energy within the gaseous volume without any concentration, that is, it becomes equivalent to the true electromagnetic radiant energy of the pure æther.

The Coefficients in the Wien-Planck Formula of Spectrum Radiation

$$(91)\quad J_0 = \frac{8\pi}{c} E_\lambda = \frac{4\pi}{\lambda^2} \mathbf{K}_\nu = \frac{c}{\lambda^2} u = \frac{8\pi h \nu^3}{\lambda^2 c^3} \cdot \frac{1}{e^{\frac{h\nu}{kT}} - 1} = \frac{8\pi c h}{\lambda^5} \cdot \frac{1}{e^{\frac{ch}{k\lambda T}} - 1}.$$

This formula is constructed on the basis of simple black body radiation, which is adiabatically confined in the perfectly reflecting mirror walls of the enclosing vessel. This would correspond to the pure radiation outside the vanishing plane of an atmosphere, but in the passage of such radiation through a non-adiabatic atmosphere the distribution becomes different, and it has been the purpose of this research to discover some

of the facts which are concerned with the generation, propagation, and dissipation of the solar radiant energy in the atmospheres of the sun and of the earth. The Wien-Planck formula of distribution is closely related to the thermodynamic conditions by the law of conservation,

$$(114)\quad T_{10}(S_1 - S_0) = (Q_1 - Q_0) = (W_1 - W_0) + (U_1 - U_0) = P_{10}(v_1 - v_0) + (U_1 - U_0).$$

When the term for work, $(W_1 - W_0) = P_{10}(v_1 - v_0) = 0$, so that there is no volume expansion to take into consideration, this reduces in the æther to,

$$(115)\quad \frac{S_1 - S_0}{U_1 - U_0} = \frac{1}{T_{10}}.$$

On entering an atmosphere, the term $(W_1 - W_0) = P_{10}(v_1 - v_0)$ is valid, changing the adiabatic conditions outside the gaseous envelope into the non-adiabatic conditions within the atmosphere, these prevailing down to the deep adiabatic layers, where $(W_1 - W_0) = (U_1 - U_0)$, and true adiabatic conditions are restored. In the non-adiabatic layers,

$$(116)\quad \frac{U_1 - U_0}{v_1 - v_0} = \frac{Q_1 - Q_0}{v_1 - v_0} - P_{10} = \frac{T_{10}(S_1 - S_0)}{v_1 - v_0} - P_{10}.$$

Since the gas coefficient R becomes a variable in the non-adiabatic strata, it follows that all the quantities depending upon R are, also, variable coefficients, as, for example,

(117) General gas coefficient, $K = m R.$

(118) Boltzmann's entropy coefficient, $k = \frac{K}{N} = \frac{m R}{N}.$

(119) Planck's Wirkungsquantum, $h = \frac{k^{4/3}}{c}\left(\frac{48\pi\alpha}{a}\right)^{1/3}.$

(120) Wien-Planck coefficient, $c_1 = 8\pi c h.$

" " " $c_2 = \frac{c h}{k}.$

(121) Stefan's coefficient, $\sigma = \frac{a\,c}{4}$, where

$$a = \frac{6\,\alpha\,c_1}{c_2^4},$$

σ = variable, corresponding with σ in

$$K_{10} = \frac{U_1 - U_0}{v_1 - v_0} = \sigma\,T^4.$$

(122) Spatial density, $u = a\,T^4$.

(123) Radiation intensity, $\mathbf{K} = \frac{c}{4\pi} \cdot a\,T^4$,

c = velocity of light.

(124) Entropy (specific), $s = \frac{4}{3}\,a\,T^3$.

(125) Radiation of Entropy, $L = \frac{c}{4\pi} \cdot a\,T^3$.

The entire thermodynamics of radiation, therefore, must be based upon a series of non-adiabatic variable coefficients, instead of upon a set of adiabatic constants, as has been assumed in previous discussions. Furthermore, we have already indicated that radiation originates in a *per saltum* process of readjustment of the electronic circulation; in a sudden transformation, and not in a thermodynamic change of the kinetic energy of the circulation of the atoms and the molecules. It seems that the radiation originates in a radioactive process, and that this radiant energy is depleted by a series of thermodynamic processes involving absorption, with temperature and other dependent changes, as well as scattering by reflection without temperature variations. We shall proceed to collect the results of the terms that are concerned with these processes of thermodynamic distribution in the Stefan and the Wien-Planck laws, now computing the variable coefficients which replace the adiabatic constants.

The Formulas of Computation

The computed values of $\log\,\sigma$ in $K_{10} = \sigma\,T^{\alpha}$ (M. K. S.) have been transformed into $\frac{\text{gram calories}}{\text{centimeter}^2\ \text{minute}}$ by the conver-

sion factor $\frac{10 \times 60}{4.1851 \times 10^7} = 0.000014336$, log $[-5.15644]$. We shall now proceed to reach the identical results by an entirely different method of computation, namely, by using the formulas of Table 1 in the following order. It will facilitate the interpretation of the data from step to step, by thinking of the energy in terms of work per unit volume, which is the kinetic energy of the molecular motion per unit volume, whose dimensions should be written,

$$\frac{M\,L^2}{T^2} \cdot \frac{1}{L^3} = \frac{\text{mass} \times \text{velocity square}}{\text{volume}} = \frac{M}{L\,T^2}.$$

The computations proceed in order:

(126) $H = \frac{3}{2} P, \left[\frac{M\,L^2}{T^2} \cdot \frac{1}{L^3}\right]$, the molecular kinetic energy.

(127) $U = Cv\rho T, \left[\frac{L^2}{T^2 \text{deg.}} \cdot \frac{M}{L^3} = \frac{M}{L\,T^2 \text{deg.}}\right]$, the total inner energy.

Generally, $U = H + J$, the molecular kinetic energy plus the inter-atomic and inter-molecular energies J. In the monatomic molecules $U = H$ and $J = 0$. Our computations have been limited to monatomic gases on account of the difficulty of determining J in the sun. We have, however, carried out this work for hydrogen, $H_2 = 2.00$ diatomic, because this gas vanishes at 25000 kilometers above the photosphere, so that $H_1 = 1.00$ monatomic is not applicable. For $H_2 = 2.00$ the value of $\frac{H}{U} = 0.6094$. Hence, $J = U - H = 0.3906\,U$. This was derived by assuming the ratio of the specific heats $\kappa = Cp/Cv = 1.4063$ for $H_2 = 2.00$, the same as for atmospheric air. This assumption seems to have been justified for hydrogen, $H_2 = 2.00$, but the difficulty of determining $\kappa = Cp/Cv$ for more complex solar gases is such that this part of the research is still under consideration.

As these thermodynamic quantities are based upon the velocity of the molecular motions, the next pair of values is necessarily,

(128) $\bar{q}^2 = 3\,R\,T = 3\,P\,v$, the mean square velocity, which is three times the volume kinetic energy for the unit m.

(129) $\gamma^2 = \frac{8}{3\,\pi}\bar{q}^2$ = the arithmetical mean velocity square, used in computing the Maxwell free path length l_{max}, and the number of collisions per second ν. Up to this stage the computations have employed the specific kinetic energy formula,

$$P\,v = R\,T, \text{ or } P = \rho\,R\,T,$$

where $m = 1$, and the density ρ is for the specific volume $v = \frac{1}{\rho}$.

We must now use the *molecular volume*, $V = m\,v = \frac{m}{\rho}$, because we require the actual amount of energy in the operation. The factor m readily eliminates from the group of formulas heretofore employed, so that it was not convenient to use it. We, therefore, take the general formulas,

(130) $P\,v\,m = m\,R\,T, \quad P\,V = K\,T, \quad P = \frac{K\,T}{V} = \rho\,R\,T.$

The value K thus computed is required in determining the Boltzmann factor $k = \frac{K}{N}$.

(131) $K = m\,R$, the general thermal efficiency, $\left[\frac{L^2}{T^2\ \text{deg.}}\right]$

(132) $n = \frac{H}{E_0}$, $\left[\frac{1}{L^3}\right]$, the number of H-atoms per unit volume.

(133) $E_0 = \frac{3\,P}{2\,n} = \frac{1}{2}\,m\,u^2$, $\left[\frac{L^2}{T^2}\right]$, the mean kinetic energy of one H-atom. It is a constant, whose value is 5.5718×10^{-14} $[-14.74599]$ in the earth's atmosphere, and 4.3769×10^{-11} $[-11.64117]$ in the sun's atmosphere.

$$\frac{\text{transformation of } E_0 \text{ (earth)} \times \gamma^2 = E_0 \text{ (sun)}}{\text{M. K. S.} \qquad\qquad \text{C. G. S.}}$$

Earth, 5.5718×10^{-21} $[-21.74599]$, 5.5718×10^{-14} $[-14.74599]$ $\left[\frac{M\,L^2}{T^2}\right]$.

Sun, 4.3769×10^{-18} [$-$ 18.64117], 4.3769×10^{-11} [$-$ 11.64117] $\left[\frac{M L^2}{T^2}\right]$.

The transformation is E_0 (earth) $\times \gamma^2 = E_0$ (sun).

$$\gamma^2 = (28.028)^2 = 785.56, [2.89518].$$

(134) $u^2 = \frac{3 P v}{N} = \frac{3 R T}{N} = \frac{2 E_0}{m}, \left[\frac{M L^2}{T^2}\right]$, is the mean square velocity of the single molecule, and hence the mean kinetic energy of each molecule in the unit mass, the gram. This is not a constant for all gases because m is in the denominator. Having computed u^2 for any gas under terrestrial conditions, u^2 (earth) $\gamma^2 = u^2$ (sun); also, u^2 (M. K. S.) $\times 10^7 = u^2$ (C. G. S.).

(135) $N = \frac{n m}{\rho} = V n = \frac{K}{k} = \frac{1}{m_H}, \left[\frac{1}{M}\right]$, the number of molecules per unit mass, the gram in (C. G. S.). This is supposed to be a universal constant, the reciprocal of the mass of the hydrogen atom, $\frac{1}{m_H} = N$, but the computations in non-adiabatic atmospheres show that there is some variability. Since R is variable, $K = m R$ is variable, and this variability proceeds throughout the entire system, changing constants into variable coefficients. Next we compute,

(136) $k = \frac{K}{N} = \frac{2 E_0}{3 T} = R m \times m_H \left[\frac{M L^2}{T^2 \text{ deg.}}\right], \frac{\text{erg}}{\text{deg.}}$, the Boltzmann entropy coefficient.

(137) $h = \frac{k^4}{c}\left(\frac{48 \pi \alpha}{a}\right)^{1/3}, \left[\frac{M L^2}{T^2} \cdot T\right]$, erg sec., the Planck Wirkungsquantum. $\alpha = 1.0823$ as developed by Planck, "Die Theorie der Wärmestrahlung," page 165,

$$\alpha = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \ldots = 1.0823.$$

(138) $a = \frac{4 \sigma}{c}$ as determined from $K_{10} = \sigma T^{\alpha}$.

Since k is a variable coefficient in non-adiabatic atmospheres, it follows that h is, also, a variable coefficient.

(139) $c_1 = 8\pi c h, \left[\frac{M L^2}{T^2} . L\right]$, erg × cm.

(140) $c_2 = \frac{c h}{k}$, [L . deg.], cm. deg.

} Wien-Planck coefficients.

(141) $a = \frac{6 \alpha c_1}{c_2^4}, \left[\frac{M L^2}{T^2} \cdot \frac{1}{L^3} \cdot \frac{1}{\text{deg}^4}\right], \frac{\text{erg,}}{\text{cm.}^3 \text{ deg.}^4}$, Stefan coefficients.

(142) $\sigma = \frac{c}{4} a, \left[\frac{M L^2}{T^2} \cdot \frac{1}{L^3} \cdot \frac{L}{T} \cdot \frac{1}{\text{deg.}^4} = \frac{M}{T^3 \text{ deg}^4}.\right], \frac{\text{gram}}{\text{sec.}^3 \text{ deg.}^4}$

$= \frac{\text{erg}}{\text{cm.}^2 \text{ sec. deg.}^4}$

(143) $A = 4.1851 \times 10^7, \left[\frac{ML^2}{T^2}\right]$, ergs, mechanical equivalent of heat in small calories per sec.^{2}

$\frac{60 \sigma}{A} = \frac{\text{gram calories}}{\text{cm.}^2 \text{ min.}}$, the measure of the kinetic energy of molecular motion in calories of heat. Since all the terms of the equations of condition represent kinetic energy of molecular motion, $\left[\frac{M L^2}{T^2}\right]$, in one form or another, and since A represents a definite amount of this kinetic energy, or work done per unit volume, it follows that they can all be reduced to the corresponding number of calories. The computations prove that $\frac{60 \sigma}{A} \times 10$ derived directly from the (log c, α) set of pairs (M. K. S.) through the factor 0.000014336, is exactly the same in value as when derived through the general formula,

(144) $60 \sigma_A \left(\frac{\text{gr. cal.}}{\text{cm.}^2 \text{ min.}}\right) = \frac{6 \alpha c_1}{c_2^4} \cdot \frac{c}{4} \cdot \frac{60}{4.1851 \times 10^7}$ (C. G. M.)

Indeed, all the resulting values of σ (M. K. S.) have been checked by this independent computation in the (C. G. S.) to verify the clerical work of the computations.

The following tables contain the data of the several elements, as they are collected together for intercomparisons, and for the derivation of the general laws which underlie the solar thermo-

dynamics. This fragmentary collection, apart from the continuity of the computations, necessarily deprives the data of much of its original force. The number of points found in the tables is about one-third of the number for which the computations were made. They are quite full near the photosphere, but in the high and the low levels the data have been very much diminished in the amount to be included in the tables.

The Thermodynamics of Radiation in the Solar Atmospheres

The table for $H = \frac{3}{2}P$ has been omitted because it can be immediately computed from Table 9; similarly, the table for $U = C\,v\,\rho\,T = H$ is omitted because our compilations are for monatomic gases, except hydrogen, $H_2 = 2.00$, as has been stated; also, the table for $\gamma = \left(\frac{8}{3\pi}\right)^{\frac{1}{2}} q = 0.92132\ q$ [9.96441] is omitted. The computations for the Maxwell mean free path length are not computed on account of the difficulty of determining the coefficient η in $l_{max} = \frac{\eta}{0.30967\ \rho\ \gamma}$, even if the coefficient 0.30967 is still available. Consequently, the computation for the number of collisions per second, $\nu = \frac{\gamma}{l_{max}}$, must be deferred.

Table 31 contains the square root of the mean square-velocity $\bar{q} = \sqrt{q^2}$. An examination of the results along the common *plane of the photosphere* for the different elements gives the general value,

$q = 54289 / \sqrt{m}$. Hence, $\frac{1}{2}\ m\ q^2 = 1.4736 \times 10^9$ [9.16838].

The universal mean kinetic energy is equivalent to,

$$(145)\quad \frac{1}{2}m\,q^2 = \frac{3}{2}P\,mv = \frac{3}{2}mRT = \frac{3}{2}P\,V = \frac{3}{2}K\,T = 1.4736 \times 10^9,$$

which is the same as for monatomic hydrogen. This conforms to the Clausius law of the constant mean kinetic energy at the photosphere.

TABLE 31. $\overline{q} = \sqrt{q^2}$ = THE SQUARE ROOT OF THE MEAN SQUARE OF THE ACTUAL MOLECULAR VELOCITIES

$q^2 = 3\ Pv = 3\ RT = Nu^2 = 2\ Hv$, IN METERS PER SECOND, $\left[\frac{L^2}{T^2}\right]$ (M. K. S.)

Height in Kilometers	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	Means
Z 25000	18046	34.011							
20000	24758	3751.9							
15000	34492	11883							
10000	38718	22121	891.58						
5000	45724	32134	11094						
1000	51787	39359	23684	10600	88.84				
800	52108	39707	24246	11539	1139.2				
600	52430	40060	24851	12485	2785.7	273.58			
400	52755	40415	25472	13458	4648.3	2141.2			
200	53081	40775	26109	14504	6381.7	4350.3	1872.4	333.14	
100	53245	40951	26433	15048	7287.0	5479.8	3553.5	1714.8	
80	53278	40989	26499	15160	7471.7	5703.0	3927.6	2121.6	
60	53310	41026	26564	15271	7660.7	5935.4	4239.4	2524.8	
40	53344	41063	26630	15384	7854.5	6177.3	4551.0	2930.7	
20	53376	41100	26696	15497	8053.2	6429.1	4877.7	3348.3	
Photos. 0	53410	41137	26761	15612	8257.0	6691.1	5230.1	3793.5	
−20	53444	41173	26827	15730	8463.7	6965.3	5601.3	4294.6	
−40	53478	41210	26892	15848	8675.8	7250.3	6006.1	4864.3	
−60	53511	41247	26960	15967	8893.2	7547.2	6434.0	5497.3	
−80	53544	41284	27026	16087	9116.2	7856.0	6898.6	6137.0	
−100	53578	41321	27092	16208	9344.6	8177.5	7392.0	6719.4	
−200	53744	41508	27427	16826	10807	10005	9662.7		
−400	54080	41884	28111	18110	13778	13450			
−600	54419	42264	28811	19519	16558				
−800	54760	42647	29528	21029	18930				
−1000	55103	43034	30263	22662	21034				
−2000	56849	45007	34233	30981	29371				
−4000	60445	49241	43343						
−6000	64337	53874	51330						
−8000	68407	58871	58090						
−10000	72847	63806							
−12000	77518	68410							
−14000	82243	72723							
$\sqrt{m}$	1.0	1.4	2.0	3.5	6.3	8.1	10.6	14.1	Means
$q\sqrt{m}$	53410	57592	53522	54642	52019	54198	55439	53488	54289

On the photosphere the value of $q = \frac{54289}{\sqrt{m}}$

Z_R	−9000	−4500	−2250	−750	−225	−140	−80	−45	
q	70627	50399	35372	20651	11178	8808	6899	5023	
$q\sqrt{m}$	70627	70559	70744	72279	70421	71345	73129	70824	71241

At the radiation layers, $q = \frac{71241}{\sqrt{m}}$

TABLE 32. n = THE NUMBER OF MOLECULES PER CUBIC CENTIMETER (C. G. S.)

$$n = \frac{PN}{KT} = \frac{N\rho}{m} = \frac{H}{E_0} = \frac{N\rho R}{K} = \frac{k}{K}\frac{N^2}{V} = \frac{N}{V} = \frac{3}{2}\frac{P}{E_0}.$$

Height in Kilometers	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	Means
Z 25000	9.0746×10^{14}	1.4058							
20000	4.4132×10^{15}	1.2947×10^{10}							
15000	1.5147×10^{16}	3.7875×10^{13}							
10000	4.1280 "	2.7984×10^{15}	8.5530×10^{9}						
5000	9.4826 "	3.7112×10^{16}	3.6911×10^{15}						
1000	1.7670×10^{17}	1.5109×10^{17}	1.1242×10^{17}	3.1669×10^{16}	2.6597×10^{7}				
800	1.8224 "	1.6063 "	1.2718 "	4.8418 "	9.2122×10^{12}				
600	1.8795 "	1.7076 "	1.4389 "	7.1775 "	8.0548×10^{14}	2.4630×10^{10}			
400	1.9384 "	1.8153 "	1.6280 "	1.0446×10^{17}	1.0420×10^{16}	7.2327×10^{14}			
200	1.9992 "	1.9299 "	1.8419 "	1.5190 "	5.0816 "	2.5035×10^{16}	1.4343×10^{15}	1.0794×10^{7}	
100	2.0302 "	1.9899 "	1.9592 "	1.8258 "	9.8642 "	7.9394 "	3.7805×10^{16}	3.8999×10^{15}	
80	2.0365 "	2.0020 "	1.9835 "	1.8942 "	1.1177×10^{17}	9.6944 "	5.8226 "	1.1309×10^{16}	
60	2.0428 "	2.0143 "	2.0081 "	1.9652 "	1.2665 "	1.1837×10^{17}	8.5300 "	2.6987 "	
40	2.0491 "	2.0267 "	2.0330 "	2.0389 "	1.4350 "	1.4454 "	1.2163×10^{17}	5.6875 "	
20	2.0555 "	2.0391 "	2.0583 "	2.1153 "	1.6260 "	1.7649 "	1.7199 "	1.1072×10^{17}	
Photos. 0	2.0618 "	2.0516 "	2.0834 "	2.1946 "	1.8425 "	2.1550 "	2.4946 "	2.0664 "	2.1187×10^{17}
−20	2.0682 "	2.0645 "	2.1095 "	2.2785 "	2.0850 "	2.6336 "	3.4349 "	3.8428 "	
−40	2.0747 "	2.0774 "	2.1357 "	2.3653 "	2.3597 "	3.2187 "	4.8689 "	**7.1640** "	
−60	2.0812 "	2.0904 "	2.1620 "	2.4555 "	2.6706 "	3.9337 "	6.8750 "	1.3207×10^{18}	
−80	2.0878 "	2.1035 "	2.1888 "	2.5492 "	3.0223 "	4.8074 "	**9.7324** "	2.2902 "	
−100	2.0943 "	2.1167 "	2.2159 "	2.6465 "	3.4204 "	**5.8754** "	1.3748×10^{18}	3.6033 "	
−200	2.1273 "	2.1837 "	2.3563 "	3.1914 "	**7.0765** "	1.6110×10^{18}	5.2469 "		
−400	2.1947 "	2.3243 "	2.6645 "	4.6106 "	2.3836×10^{18}	7.0801 "			
−600	2.2642 "	2.4741 "	3.0138 "	**6.7049** "	5.9731 "				
−800	2.3360 "	2.6334 "	3.4075 "	9.7324 "	1.1669×10^{19}				
−1000	2.4100 "	2.8031 "	3.8534 "	1.4144×10^{18}	1.9767 "				
−2000	2.8169 "	3.8228 "	**7.1377** "	6.7532 "	1.0491×10^{20}				
−4000	3.8277 "	**7.1240** "	2.3223×10^{18}						
−6000	5.2296 "	1.3276×10^{18}	5.4100 "						
−8000	**7.1060** "	2.4533 "	1.0155×10^{19}						
−10000	9.7324 "	4.2834 "							
−12000	1.3279×10^{18}	6.9387 "							
−14000	1.7852 "	1.0595×10^{19}							
At sea level for dry air, $n = 2.7278\times10^{19}$									
Z_R	−9000	−4500	−2250	−750	−225	−140	−80	−45	
n_R	8.4692	8.5620	9.1484	7.4618	9.1714	9.9692	9.7324	8.6748	8.8990×10^{17}

The data for the planes of the radiation, z_R, give a similar result, but with a different constant, $q = 71241/\sqrt{m}$. Hence, $\frac{1}{2} m q^2 = 2.5376 \times 10^9$, [9.40443].

Finally, on the vanishing planes $q = 0$ and the mean kinetic energy vanishes. There must, therefore, be a complex gradual diminution of the general mean kinetic energy from level to level, counting from the inner layers to the vanishing planes. The values in the lower levels can be computed as long as the gaseous law $P = \rho R T$, or $P V = K T$, continues to represent the natural conditions, that is, before viscous coefficients become necessary.

The value of q^2 is important in several formulas.

The general kinetic energy, $P V = K T = \frac{1}{3} m q^2$.

The specific kinetic energy, $P v = R T = \frac{1}{3} q^2$.

The hydrostatic pressure, $P = \rho R T = \frac{1}{3} \rho q^2$.

The molecular kinetic energy, $H = \frac{3}{2} P = \frac{1}{2} \rho q^2$.

By Table 9 the mean pressure along the layers where the black radiation originates is $P = 2593212$ (M. K. S.).

Since $H = \frac{3}{2} P = E_0 n$, we have in M.K.S. for $n = 8.8990 \times 10^{23}$

$H = 3889800$, and $E_0 n = 3895000$,

which checks the procedure approximately, especially the results by the method of trials upon which the entire series of data depends. The average number of molecules per cubic centimeter on the plane of the photosphere is 2.1187×10^{17}, about one-fourth the number on the radiation levels. The number diminishes to zero on the vanishing planes, very rapidly in the topmost strata. The number of molecules per cu. cm. on the plane of the photosphere is about one-hundredth the amount in the earth's atmosphere near the sea level, 2.1187×10^{17} against 2.7278×10^{19}.

TABLE 33. $K = mR$ = THE THERMAL COEFFICIENT IN $PV = KT$ (M. K. S.)

$$K = \frac{PV}{T} = knmv = kN = \frac{PN}{T_n} = \frac{PNE_0}{TH} = \frac{k}{m_H}$$

	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	Means
Z 25000	32688	9.6374							
20000	47516	9825.6							
15000	62307	45802							
10000	75712	93338	1432.3						
5000	92316	116872	50324						
1000	116170	134874	98156	69680	350.8				
800	117590	136908	92492	76088	11536				
600	119030	139384	107020	83128	35075	2534			
400	120490	141904	112136	93900	61300	31841			
200	121975	144474	117884	109214	79856	69030	45125	292.9	
100	122723	145776	120872	117631	94404	86749	85011	49128	
80	122871	146040	121476	119396	98128	92846	87263	60014	
60	123020	146304	122084	121176	102004	99366	91914	71306	
40	123170	146564	122696	122988	106652	107023	101080	83982	
20	123322	146828	123312	124824	112172	115999	115505	99994	
Photos. 0	123470	147094	123932	126696	117980	125736	132900	123827	$127704K_p$
−20	123630	147366	124544	128628	123976	136483	152678	158849	
−40	123780	147644	125156	130584	130272	147927	175616	203991	
−60	123940	147916	125776	132576	136960	160453	201825	**255717**	$221672K_R$
−80	124100	148194	126392	134592	144044	174038	232198	261841	$282628K_A$
−100	124250	148470	127016	136632	151492	188773	**264230**	"	
−200	125040	149856	130180	147360	**203148**	**281691**	273549	"	
−400	126630	152672	136752	170940	294360	284187	"	"	
−600	128230	155494	143724	198828	294360	"	"	"	
−800	129860	158310	151116	231180	294360	"	"	"	
−1000	131510	161180	158888	**268872**	"	"	"	"	
−2000	140070	176254	**203852**	320604	"	"	"	"	
−4000	158552	211572	265032	"	"	"	"	"	
−6000	179867	252440	265032	"	"	"	"	"	
−8000	203595	**288806**	265032	"	"	"	"	"	
−10000	231177	295428	"	"	"	"	"	"	
−12000	**260465**	295428	"	"	"	"	"	"	
−14000	266020	295428	"	"	"	"	"	"	
Z_R	−9000	−4500	−2250	−750	−225	−140	−80	−45	
K_R	217386	221789	211500	223092	214550	225940	232198	226923	221672

TABLE 34. N = THE NUMBER OF MOLECULES IN ONE GRAM (C. G. S.)

$$N = Vn = mvn = \frac{K}{k} = \frac{1}{m_H} = \frac{PV}{kT} = \frac{3Pv}{u^2} = \frac{q^2}{u^2} = \frac{2Hv}{u^2} = \frac{3}{2}\frac{K}{\epsilon}$$

	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	Means
Z 25000	3.7189×10^{22}	4.2543×10^{18}							
20000	7.0020 "	3.2161×10^{21}							
15000	1.1467×10^{23}	3.2258×10^{22}							
10000	1.7125 "	1.1180×10^{23}	3.6322×10^{20}						
5000	2.3884 "	2.3591 "	6.5191×10^{22}						
1000	3.0636 "	3.5393 "	2.5565×10^{23}	1.5403×10^{23}	3.6066×10^{19}				
800	3.1017 "	3.6026 "	2.6860 "	1.8253 "	5.9300×10^{21}				
600	3.1402 "	3.6668 "	2.8219 "	2.1366 "	3.5459×10^{22}	5.5359×10^{20}			
400	3.1792 "	3.7321 "	2.9648 "	2.4827 "	9.8733 "	3.3965×10^{22}			
200	3.2187 "	3.7986 "	3.1148 "	2.8838 "	1.8609×10^{23}	1.4019×10^{23}	4.4657×10^{22}	2.5154×10^{19}	
100	3.2386 "	3.8323 "	3.1826 "	3.1041 "	2.4264 "	2.2245 "	1.6533×10^{23}	6.6643×10^{22}	
80	3.2427 "	3.8389 "	3.2086 "	3.1501 "	2.5507 "	2.4094 "	1.9650 "	1.0202×10^{23}	
60	3.2467 "	3.8457 "	3.2244 "	3.1968 "	2.6814 "	2.6098 "	2.2946 "	1.4447 "	
40	3.2508 "	3.8525 "	3.2403 "	3.2442 "	2.8189 "	2.8269 "	2.6384 "	1.9467 "	
20	3.2546 "	3.8594 "	3.2564 "	3.2922 "	2.9635 "	3.0620 "	3.0305 "	2.5411 "	
Photos. 0	3.2586 "	3.8663 "	3.2724 "	3.3410 "	3.1153 "	3.3167 "	3.4844 "	3.2616 "	$3.2929\times10^{23}N_P$
−20	3.2628 "	3.8734 "	3.2885 "	3.3913 "	3.2735 "	3.5938 "	3.9966 "	4.1801 "	
−40	3.2668 "	3.8803 "	3.3047 "	3.4425 "	3.4395 "	3.8940 "	4.5951 "	5.3629 "	$5.8270\times10^{23}N_R$
−60	3.2709 "	3.8873 "	3.3210 "	3.4945 "	3.6140 "	4.2194 "	5.2746 "	**6.8493** "	
−80	3.2750 "	3.8943 "	3.3374 "	3.5473 "	3.7974 "	4.5719 "	6.0620 "	8.5365 "	$6.9210\times10^{23}N_A$
−100	3.2791 "	3.9013 "	3.3539 "	3.6009 "	3.9900 "	**4.9539** "	**6.9602** "	1.0233×10^{24}	
−200	3.2996 "	3.9366 "	3.4374 "	3.8809 "	**5.3365** "	7.4158 "	1.1893×10^{24}		
−400	3.3410 "	4.0083 "	3.6107 "	4.4961 "	8.6742 "	1.3405×10^{24}			
−600	3.3830 "	4.0812 "	3.7927 "	5.2226 "	1.2528×10^{24}				
−800	3.4255 "	4.1555 "	3.9839 "	6.0620 "	1.6374 "				
−1000	3.4685 "	4.2311 "	4.1847 "	**7.0398** "	2.0217 "				
−2000	3.6076 "	4.6279 "	**5.3550** "	1.3157×10^{24}	3.9415 "				
−4000	4.1736 "	5.5394 "	8.5840 "						
−6000	4.7284 "	**6.6310** "	1.2039×10^{24}						
−8000	5.3455 "	7.9178 "	1.5498 "						
−10000	6.0620 "	9.3010 "							
−12000	**6.8643** "	1.0692×10^{24}							
−14000	7.7268 "	1.2083 "							
At Z_A	6.8643	6.6310	6.9695	7.0398	7.0054	7.0465	6.9828	6.8493	6.9210×10^{23}
At Z_R	5.7038	5.8123	5.7586	5.8521	5.7537	5.9387	6.0620	5.7345	5.8270×10^{23}

The values of $K = m R$ in Table 33 are in the (M. K. S.) system, to be converted into (C. G. S.) by the factor 10^4. There are three principal constant values, $K_A = 282628$ at the top of the adiabatic region; $K_R = 221672$ in the strata where the black body radiation originates; and $K_P = 127704$ on the plane of the photosphere, the decrease in the value of K continuing to zero on the vanishing planes. The variability of $K = m R$ necessarily follows the variability of R. It is a constant in the adiabatic region, but falls in value in the isothermal and the non-adiabatic regions till it vanishes. The vertical decrease in K is slow for hydrogen, and it is very rapid for mercury. The family of curves can be plotted, using the mean points, and their relations to the height will be seen to be hyperbolic. Since K is variable it is probable that $K = k N$ are all variables.

The number of molecules in the unit mass, here the gram molecule, is again constant in certain layers, but these are slowly diminishing with the height. At the top of the adiabatic layers $N_A = 6.9210 \times 10^{23}$; at the radiation layer $N_R = 5.8270 \times 10^{23}$; at the photosphere $N_P = 3.2929 \times 10^{23}$.

We now encounter a difficult physical problem.

(146) $N E_0 = \frac{3}{2} PV = \frac{3}{2} K T = \frac{3}{2} m R T = \frac{1}{2} m q^2$ is mutually satisfied for each gas at a given level; the several gases have certain common values as in Table 34, but changing from one level to another, it follows that N, the number of molecules per unit mass, and E_0, the kinetic energy of one molecule, $E_0 = \frac{3}{2}\frac{P}{n} = \frac{1}{2} m u^2 = \frac{3}{2}\frac{m R T}{N} = \frac{3}{2}\frac{K T}{N}$ *cannot both be fixed universal constants at the same time.* It has been commonly assumed in physics that the kinetic energy of each molecule of a gas in collision to produce the pressure P is an indestructible quantity, primitive to the constitution of matter itself. It is the force that maintains constant the structure of material bodies, whatever its nature. It is of the same primitive character as gravitation, or the electromagnetic forces, and on this account E_0 *has been retained a constant* while N becomes a variable. It

TABLE 35. $\frac{1}{2}mq^2$ = MEAN KINETIC ENERGY OF MOLECULAR MOTION.

$$\frac{1}{2}mq^2 = NE_0 = \frac{3}{2}PV = HV = \frac{3}{2}KT = \frac{3}{2}mRT = (U_1 - U_0), \left[\frac{ML^2}{T^2}\right].$$

	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	Means
Z 25000....	1.6280×10^8	1.1567×10^8							
20000....	3.0647 "	1.4074×10^7							
15000....	5.0190 "	1.4119×10^8							
10000....	7.4953 "	4.8935 "	1.5898×10^6						
5000....	1.0453×10^9	1.0326×10^9	2.8534×10^8						
1000....	1.3409 "	1.5491 "	1.1190×10^9	6.7416×10^8	1.5785×10^5				
800....	1.3580 "	1.5778 "	1.1815 "	7.9893 "	2.5955×10^7				
600....	1.3751 "	1.6064 "	1.2442 "	9.3520 "	1.5520×10^8	2.4271×10^6			
400....	1.3921 "	1.6350 "	1.3069 "	1.0866×10^9	4.3215 "	1.4866×10^8			
200....	1.4092 "	1.6636 "	1.3696 "	1.2622 "	8.1452 "	6.1361 "	1.9546×10^8	1.1010×10^5	
100....	1.4178 "	1.6779 "	1.4010 "	1.3623 "	1.0620×10^9	9.7364 "	7.2362 "	2.9169×10^8	
80....	1.4195 "	1.6806 "	1.4072 "	1.3823 "	1.1223 "	1.0693×10^9	8.6008 "	4.4655 "	
60....	1.4212 "	1.6835 "	1.4134 "	1.4023 "	1.1826 "	1.1649 "	1.0020×10^9	6.3236 "	
40....	1.4229 "	1.6864 "	1.4197 "	1.4223 "	1.2429 "	1.2605 "	1.1548 "	8.5206 "	
20....	1.4246 "	1.6893 "	1.4260 "	1.4423 "	1.3032 "	1.3561 "	1.3264 "	1.1122×10^9	
Photos. 0....	1.4263 "	1.6922 "	1.4323 "	1.4623 "	1.3635 "	1.4517 "	1.5251 "	1.4273 "	1.4412×10^9
−20....	1.4281 "	1.6956 "	1.4402 "	1.4865 "	1.4400 "	1.5951 "	1.7493 "	1.8296 "	
−40....	1.4300 "	1.6988 "	1.4482 "	1.5105 "	1.5166 "	1.7384 "	2.0113 "	**2.3473** "	
−60....	1.4318 "	1.7020 "	1.4562 "	1.5345 "	1.5932 "	1.8817 "	2.3087 "	2.9979 "	2.5507×10^9
−80....	1.4336 "	1.7052 "	1.4642 "	1.5585 "	1.6698 "	2.0250 "	**2.6533** "	3.7364 "	3.0593×10^9
−100....	1.4354 "	1.7084 "	1.4722 "	1.5825 "	1.7464 "	**2.1683** "	3.0464 "	4.4790 "	
−200....	1.4445 "	1.7243 "	1.5120 "	1.7026 "	**2.3358** "	3.2459 "	5.2056 "		
−400....	1.4629 "	1.7562 "	1.5919 "	1.9679 "	3.7966 "	3.8673 "			
−600....	1.4813 "	1.7881 "	1.6718 "	**2.2859** "	5.4833 "				
−800....	1.4997 "	1.8200 "	1.7517 "	2.6533 "	7.1665 "				
−1000....	1.5181 "	1.8519 "	1.8316 "	3.0813 "	8.8487 "				
−2000....	1.6159 "	2.0256 "	**2.3438** "	5.7588 "	1.7257 "				
−4000....	1.8268 "	2.4245 "	3.7572 "						
−6000....	2.0696 "	**2.9023** "	5.2695 "						
−8000....	**2.3397** "	3.4656 "	6.7760 "						
−10000....	2.6533 "	4.0710 "							
−12000....	**3.0045** "	4.6797 "							
−14000....	3.3820 "	5.2888 "							
Z_R.....	−9000	−4500	−2250	−750	−225	−140	−80	−45	
$(\frac{1}{2}mq^2)_R$.....	2.4965	2.5439	2.5205	2.5614	2.5184	2.5993	2.6533	2.5100	2.5507×10^9
Z_A.....	−12000	−6000	−3000	−1000	−300	−185	−107	−60	
$(\frac{1}{2}mq^2)_A$.....	3.0045	2.9023	3.0505	3.0813	3.0662	3.0843	3.1975	2.9979	3.0593×10^9

TABLE 36. k = THE BOLTZMANN'S ENTROPY COEFFICIENT

$$k = \frac{K}{N} = \frac{P}{nT} = \frac{\rho R}{n} = \frac{K}{Vn} = \frac{K}{mnu} = \frac{K}{m_H} = \frac{2}{3}\epsilon = \frac{2}{3}\frac{E_0}{T} = \frac{2}{3}\frac{H}{nT} \cdot \left[\frac{ML^2}{T^2} \cdot \frac{1}{\text{deg.}} = \frac{\text{erg}}{\text{deg.}}\right].$$

	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	Means
Z 25000...	8.7894×10^{-15}	3.6466×10^{-13}							
20000...	6.7861 "	3.0551×10^{-14}							
15000...	5.4336 "	1.4199 "							
10000...	4.4212 "	8.3486×10^{-15}	3.9432×10^{-14}						
5000...	3.8653 "	4.9530 "	7.7192×10^{-15}						
1000...	3.7918 "	3.7995 "	3.8395 "	4.5240×10^{-15}	9.7268×10^{-14}				3.8103×10^{-15}
800...	3.7913 "	3.8005 "	3.8290 "	4.1685 "	1.9453 "				3.8069 "
600...	3.7908 "	3.8015 "	3.8185 "	3.8906 "	9.8917×10^{-15}	4.5592×10^{-14}			3.8254 "
400...	3.7903 "	3.8025 "	3.8080 "	3.7822 "	6.2086 "	9.3530×10^{-15}			3.7958 "
200...	3.7898 "	3.8035 "	3.7975 "	3.7871 "	4.2912 "	4.9126 "	1.0063×10^{-14}	5.3053×10^{-14}	3.7960 "
100...	3.7895 "	3.8040 "	3.7923 "	3.7896 "	3.8906 "	3.8907 "	5.1194×10^{-15}	7.3870×10^{-15}	3.8261 "
80...	3.7894 "	3.8041 "	3.7913 "	3.7901 "	3.8698 "	3.8690 "	4.4213 "	5.8946 "	3.8190 "
60...	3.7894 "	3.8042 "	3.7903 "	3.7906 "	3.8491 "	3.8473 "	3.9974 "	4.9457 "	3.8383 "
40...	3.7893 "	3.8043 "	3.7892 "	3.7911 "	3.8284 "	3.8256 "	3.8145 "	4.3229 "	3.8061 "
20...	3.7893 "	3.8044 "	3.7882 "	3.7915 "	3.8077 "	3.8039 "	3.7946 "	3.8535 "	3.8041 "
Photos. 0...	3.7892 "	3.8045 "	3.7871 "	3.7920 "	3.7870 "	3.7822 "	3.7974 "	3.8044 "	3.7930 "
−20...	3.7892 "	3.8046 "	3.7873 "	3.7925 "	3.7890 "	3.7861 "	3.8009 "	3.8079 "	3.7947 "
−40...	3.7893 "	3.8047 "	3.7874 "	3.7930 "	3.7910 "	3.7900 "	3.8051 "	3.8116 "	3.7965 "
−60...	3.7893 "	3.8048 "	3.7876 "	3.7935 "	3.7930 "	3.7939 "	3.8093 "	3.7410 "	3.7890 "
−80...	3.7894 "	3.8049 "	3.7877 "	3.7940 "	3.7959 "	3.7978 "	3.8135 "	3.0736 "	3.7976 "
−100...	3.7894 "	3.8050 "	3.7879 "	3.7945 "	3.7968 "	3.8018 "	3.7797 "	2.5640 "	3.7936 "
−200...	3.7897 "	3.8055 "	3.7888 "	3.7970 "	3.8067 "	3.8233 "	2.2899 "		3.8018 "
−400...	3.7902 "	3.8065 "	3.7908 "	3.8019 "	3.3936 "	2.1200 "			3.7974 "
−600...	3.7907 "	3.8075 "	3.7928 "	3.8069 "	2.3497 "				3.7995 "
−800...	3.7912 "	3.8085 "	3.7948 "	3.8135 "	1.7978 "				3.7995 "
−1000...	3.7917 "	3.8095 "	3.7968 "	3.8193 "	1.4560 "				3.8043 "
−2000...	3.7941 "	3.8085 "	3.8068 "	2.4368 "	7.4684×10^{-16}				3.8031 "
−4000...	3.7989 "	3.8195 "	3.0875 "						3.8092 "
−6000...	3.8039 "	3.8069 "	2.2012 "						3.8054 "
−8000...	3.8087 "	3.6475 "	1.3000 "						3.8087 "
−10000...	3.8135 "	3.1763 "							3.8135 "
−12000...	3.7945 "	2.7631 "							3.7945 "
−14000...	3.4428 "	2.4451 "							

$\log k = [-15.58033]$ 3.8048×10^{-15}

seems more probable that the number of molecules in the gram, under the extreme solar conditions, should be the variable. However, since $N = \frac{1}{m_H}$, and $m_H = \frac{1}{N}$ is the mass of the hydrogen atom, *it follows that this is a computed variable in free atmospheres*. If it is insisted that m_H is to be invariable, it follows that E_0 is to be made a variable. The only escape from this dilemma is that the simple thermodynamic laws laid at the basis of the computations, while they conform to the analytical concepts regarding the kinetic theory of matter, nevertheless are too simple to represent the facts in nature. The electron theory of matter has opened up such extensive possibilities as to the physical structures of atoms, molecules, and their relations through ionization and free electrons that the kinetic theory of thermodynamics may be inadequate. It is our purpose to present the facts that must follow from the variability of R in free atmospheres, though it carries with it a revolutionary modification of many old and current hypotheses, especially regarding the origin of radiation and its emission and absorption in free gaseous atmospheres. The common formulas are based upon strictly ideal models, as black radiation in a mirror enclosure, apart from influencing ponderable matter. It is necessary to define the existing thermodynamic conditions in atmospheric gases before proceeding to the corresponding processes in electromagnetic radiations.

Table 35 contains the mean kinetic energy $\frac{1}{2} m q^2$, and several equivalent forms. It has a constant value for all gases on certain levels, at the top of the adiabatic strata $(\frac{1}{2} m q^2)_A = 3.0593 \times 10^9$; on the radiation layers $(\frac{1}{2} m q^2)_R = 2.5507 \times 10^9$; on the plane of the photosphere $(\frac{1}{2} m q^2)_P = 1.4412 \times 10^9$; on the vanishing planes the kinetic energy is zero. There is a gradual decrease in the value of the kinetic energy from the deep layers beneath the photosphere to the vanishing planes at whatever height they may occur.

Returning to the problem of the coefficients, it is seen from the general equation that we have,

$$(147)\quad E_0 = \frac{\frac{1}{2} m q^2}{N} = \frac{\frac{3}{2} P V}{N} = \frac{H V}{N} = \frac{H}{n} = \frac{\frac{3}{2} K T}{N} = \frac{\frac{3}{2} m R T}{N}.$$

If K and N are universal constants, and $k = \frac{K}{N}$, $E_0 = \frac{3}{2} k T$, it follows that the Boltzmann entropy product $k T$ is, also, a constant. If K is a variable, $K = m R$, as it must be if R is a variable, it follows that k is a variable unless N varies in the same ratio as $K = m R$ varies in the non-adiabatic strata. At the time of making the computations for trial, it was not known what any of the relations are between $N \,.\, n \,.\, H \,.\, E_0 \,.\, P \,.\, V \,.\, K \,.\, T$. They were computed in the order $K = m R_0$, $n = \frac{H}{E_0}$, $N = n \frac{m}{\rho} = \frac{H}{E_0} \frac{m}{\rho}$, and checked by $u^2 = \frac{q^2}{N}$. It is perfectly evident that if there results a value of N inconsistent with $N = \frac{1}{m_H}$, where m_H is the mass of the hydrogen atom, computed for $E_0 =$ constant, $\log E_0 = -18.64117$, it follows that N or E_0 must one or both be variables. The former as a variable implies that the structure of the mass of the gram-molecule depends upon other forces than the mere summation of independent mass molecules $\sum_1^N m_H$, such as are being so widely discussed in the electron theory of matter. The latter as a variable admits that the molecular kinetic energy of individual atoms and molecules is variable, and that E_0 per degree has other conditions affecting it than the temperature, $T = \frac{P V}{K}$. The theory of the free path lengths, collisions, invariability of the primitive kinetic energy, are all involved. The choice is so difficult that it is thought proper to present the problem in as concrete a form as possible. It may be noted that unless N diminishes with the height, as in Table 34, the variability of the Boltzmann entropy coefficient $k = \frac{K}{N}$ will be somewhat greater than it has been found to be in Table 36; and the Planck Wirkungsquantum will be a greater variable than that given in Table 38. Furthermore, the values of c_1 in Table 40, c_2 in Table 42, a in Table 44, will be much wider variables. Compare the last Chapter VIII where N is proved to be variable.

The Boltzmann entropy coefficient, $k = \frac{K}{N}$, is computed directly from K and N, each variables, and it results in a common mean value throughout the isothermal region, including the radiation layer.

$$k = 3.8048 \times 10^{-15} \text{ [log } k = -15.58033\text{] erg/deg.}$$

The common value for terrestrial atmospheric air at the sea level is approximately 1.3606×10^{-16} [-16.13374] as derived in the laboratory. From balloon ascensions, Bulletin No. 4, O. M. A., we collect the following data:

TABLE 37

$k = \frac{K}{N}$ — AS DERIVED FROM BALLOON ASCENSIONS.

1911	Uccle, June 9	Uccle, Sept. 13	Uccle, Nov. 9	Means up to 60000 meters	
$z = 90000$.....	.....	.....	3.7118×10^{-14}		
80000.....	.....	3.7139×10^{-14}	9.2830×10^{-16}		
70000.....	3.7125×10^{-14}	8.2530×10^{-16}	5.3048 "		
60000.....	*14.0890×10^{-16}	4.6427 "	3.7123 "	4.1775×10^{-16}	3.7066×10^{-16}
50000.....	*6.5164 "	3.3766 "	3.0947 "	3.2357 "	
40000.....	2.2789 "	2.2374 "	2.1975 "	2.2379 "	mean k at 55,000 meters.
30000.....	1.6962 "	1.6656 "	1.6481 "	1.6701 "	
20000.....	1.7246 "	1.6904 "	1.6641 "	1.6930 "	
10000.....	1.6443 "	1.5895 "	1.6800 "	1.6379 "	
0000.....	1.2832 "	1.2699 "	1.3419 "	1.2983 "	
The laboratory	value.....	.....	.....	1.3606×10^{-16}	-16.13374

The ratio $\frac{k \text{ (sun)}}{k \text{ (earth)}} = \frac{38.0480 \times 10^{-16}}{1.3606 \times 10^{-16}} = 27.963$, nearly $\gamma = 28.028$.

* Omitted from the means.

Table 38 gives the Planck Wirkungsquantum h, which in the isothermal layer is nearly constant, but it really increases slowly with the height. Its mean value is $h = 1.2113 \times 10^{-23}$ [-23.08325] in these strata, generally above the level of radiation proper. Below these levels we have the following values,

z_R — 9000 — 4500 — 2250 — 750 — 255 — 140 — 80 — 45.

h_A 8.6264×10^{-26} 1.1698×10^{-24} 6.4703×10^{-26}
7.6756×10^{-26} 6.7790×10^{-26} 8.0490×10^{-26}
8.0893×10^{-26} 7.8370×10^{-26} 7.6467×10^{-26}.

Omitting hydrogen, $H_2 = 2.00$, we have the result:

h_2 *above* the radiation layer 1.2113×10^{-23}, -23.08325.

h_1 *below* " " " 7.6467×10^{-26}, -26.88347.

TABLE 38. h = PLANCK'S WIRKUNGSQUANTUM.

$$h = \frac{k^{4/3}}{c}\left(\frac{48\,\pi\,\alpha}{a}\right)^{1/3}. \qquad \alpha = 1.0823 = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \ldots, \qquad \left[\frac{M\,L^2}{T^2}.\quad T = \text{erg. sec.}\right].$$

	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	Means
Z 25000	4.0846×10^{-23}	3.1956×10^{-19}							
20000	2.7590 "	4.6296×10^{-22}							
15000	2.0400 "	9.2700×10^{-23}							
10000	1.5518 "	3.6341 "	4.9126×10^{-22}						
5000	1.2733 "	1.7332 "	3.5074×10^{-23}						
1000	1.1852 "	1.1596 "	1.4051 "	1.7719×10^{-23}	2.0071×10^{-21}				1.2500×10^{-23}
800	1.1823 "	1.1539 "	1.3869 "	1.5849 "	1.3342×10^{-22}				1.2410 "
600	1.1794 "	1.1482 "	1.3688 "	1.4286 "	4.9201×10^{-23}	4.9837×10^{-22}			1.2813 "
400	1.1764 "	1.1425 "	1.3507 "	1.3439 "	2.6228 "	4.7808×10^{-23}			1.2534 "
200	1.1734 "	1.1368 "	1.3326 "	1.2984 "	1.6429 "	2.0244 "	4.6297×10^{-23}	5.6016×10^{-22}	1.2353 "
100	1.1719 "	1.1340 "	1.3236 "	1.2819 "	1.4147 "	1:4569 "	1.9520 "	2.7861×10^{-23}	1.2972 "
80	1.1716 "	1.1335 "	1.3217 "	1.2786 "	1.3917 "	1.4257 "	1.6384 "	2.0721 "	1.2871 "
60	1.1713 "	1.1329 "	1.3199 "	1.2753 "	1.3687 "	1.3945 "	1.4342 "	1.6389 "	1.2995 "
40	1.1710 "	1.1323 "	1.3181 "	1.2720 "	1.3456 "	1.3632 "	1.3239 "	1.3607 "	1.2752 "
20	1.1707 "	1.1317 "	1.3163 "	1.2687 "	1.3225 "	1.3319 "	1.2810 "	1.1811 "	1.2505 "
Photos. 0	1.1704 "	1.1311 "	1.3145 "	1.2654 "	1.2994 "	1.3006 "	1.2432 "	1.0789 "	1.2254 "
−20	1.1701 "	1.1306 "	1.3140 "	1.2615 "	1.2881 "	1.2827 "	1.2144 "	1.0337×10^{-23}	1.2119 "
−40	1.1699 "	1.1300 "	1.3134 "	1.2576 "	1.2769 "	1.2649 "	1.1826 "	9.8820×10^{-24}	1.2279 "
−60	1.1696 "	1.1294 "	1.3129 "	1.2536 "	1.2657 "	1.2471 "	1.1580 "	7.8370×10^{-26}	1.2195 "
−80	1.1693 "	1.1289 "	1.3123 "	1.2496 "	1.2545 "	1.2293 "	8.0893×10^{-26}	6.0036 "	1.2240 "
−100	1.1690 "	1.1283 "	1.3118 "	1.2456 "	1.2433 "	1.2115×10^{-23}	7.8660 "	4.7011 "	1.2183 "
−200	1.1674 "	1.1255 "	1.3030 "	1.2258 "	1.1821×10^{-23}	8.0490×10^{-26}	3.9957 "		1.2008 "
−400	1.1643 "	1.1197 "	1.2916 "	1.1976 "	6.7790×10^{-26}	3.6200 "			1.1933 "
−600	1.1612 "	1.1139 "	1.2802 "	1.1616×10^{-23}	4.1383 "				1.1792 "
−800	1.1581 "	1.1081 "	1.2688 "	7.6756×10^{-26}	2.8862 "				1.1783 "
−1000	1.1550 "	1.1023 "	1.2574 "	7.5878 "	2.1736 "				1.1716 "
−2000	1.1397 "	1.0625 "	1.2100×10^{-23}	4.0769 "	8.8926×10^{-27}				1.1377 "
−4000	1.1176 "	1.0028×10^{-23}	6.4703×10^{-26}						1.0602 "
−6000	1.0912 "	1.1698×10^{-24}	4.1083 "						1.0912 "
−8000	1.0738×10^{-23}	1.0675 "	2.7275 "						1.0738 "
−10000	8.6264×10^{-26}	8.8204×10^{-25}							
−12000	8.5400 "	7.3078 "							
−14000	7.4750 "	6.1609 "							

Mean value.....[−23.08325] $1-2113\times10^{-23}$

We have here a sudden transition in the value of h, similar to that discussed for σ under Table 24, by the method of the logarithmic spiral,

$\log \sigma_2$ for the final point, $-$ 9.81400.
$\log \sigma_0$ " " middle " $-$ 5.73690.
$\log \sigma_1$ " " initial " $-$ 2.25450.

The distance of $\log \sigma_0$ from $\log \sigma_1 = 0.461$ of the total distance from $\log \sigma_1$ to $\log \sigma_2$. Similarly, the logarithmic mean value of h_0 can be obtained from h_2 and h_1 by applying the factor 0.461 to the interval.

$\log h_2$ = the final value, $-$ 23.08325, approximately.
$\log h_0$ = " middle " $-$ 25.13167, "
$\log h_1$ = " initial " $-$ 26.88347, "

Hence, the logarithmic mean value of h_0 is,

$$\log h_0 = -25.13167, \quad h_0 = 1.3542 \times 10^{-25}.$$

		Logarithm
The common laboratory value is,	6.545×10^{-27}	$-$ 27.81590
The solar value h (earth) $\times \gamma^{4/3}$ (= 85.134),	5.5721×10^{-25}	$-$ 25.74602
Computed solar value,	1.3542×10^{-25}	$-$ 25.13167

We collect from Bulletin No. 4, O. M. A., the values of h as computed from three balloon ascensions:

TABLE 39

$$h = \frac{k^{4/3}}{c}\left(\frac{48\,\pi\,\alpha}{a}\right)^{1/3}$$ FROM THREE BALLOON ASCENSIONS

1911	Uccle, June 9	Uccle, Sept. 13	Uccle, Nov. 9	Mean Values up to 60000 Meters
$z = 90000$	.....	.....	3.0212×10^{-20}	
80000	.....	3.1279×10^{-22}	2.6632×10^{-25}	
70000	5.3350×10^{-22}	1.3909×10^{-25}	6.9295×10^{-26}	
60000	2.5722×10^{-25}*	6.0879×10^{-26}	3.0141 "	45.5100×10^{-27}
50000	4.2185×10^{-26}*	1.9790 "	1.7646 "	18.718 "
40000	1.0498 "	1.0213 "	9.9955×10^{-27}	10.2355 "
30000	6.5330×10^{-27}	6.3470×10^{-27}	6.2650 "	6.3817 "
20000	5.5854 "	5.4506 "	5.3626 "	5.4662 "
10000	4.5545 "	4.4074 "	4.6475 "	4.5368 "
0000	3.1791 "	3.1587 "	3.2970 "	3.2116 "
The laboratory value of h	.....	.....	.....	6.545×10^{-27}

The value of h from balloon ascensions is about the same at 30000 meters as the laboratory value. The terrestrial value increases upward, as does the corresponding solar value, but more rapidly. If the change from terrestrial values of h is through the factor $\gamma^{4/3} = (28.028)^{4/3} = 85.138$ [1.93012], to the solar value, then this system for h is in excellent general agreement. But if h is subject to changes with $\gamma = \frac{G\,(\text{sun})}{g_0\,(\text{earth})}$, the ratio of the gravitations, it cannot be regarded as a universal constant. It is apparently a complex variable having significant values in radiation processes as a potential coefficient.

In the formula $h = \frac{k^{4/3}}{c}\left(\frac{48\,\pi\,\alpha}{a}\right)^{1/3}$, c is the velocity of light, $3 \times 10^{10}\,\frac{\text{cm.}}{\text{sec.}}$, and a is the value of the coefficient in $u = a\,T^4$ derived from the computed $K_{10} = \sigma\,T^{\alpha}$ in (M. K. S.), placing (σ, α) in this case to distinguish it from the other notation. The dimension of σ is $\left[\frac{M\,L^2}{T^2} \cdot \frac{1}{L^3} = \frac{M}{L\,T^2}\right]$ the kinetic energy per volume, and the transformation factor from (M. K. S.) to (C. G. S.) is $10 = \frac{1000}{100}$. We now identify this σ (C. G. S.) with σ of the usual Stefan formula, $J_0 = \sigma\,T^4$, since both represent the kinetic energy per unit volume. It expresses the amount of radiant energy which has entered the volume in one second, that is, the amount of electromagnetic energy in a column whose base is one centimeter square and length is c, the velocity of light per second. The unit volume, if spherical, has four times the surface of the area on the diameter. Hence, $a = \frac{4\,\sigma}{c}$, as given in the papers on radiation. It is customary to take a volume energy in the pure ether under radiant agitation a, multiply this by $\frac{c}{4}$ to obtain σ in the Stefan formula. It must be remembered that in this research we are working in the opposite direction, from the effects of radiation in the

TABLE 40. $c_1 = 8\pi c h$—THE WIEN-PLANCK COEFFICIENT $\left[\frac{M L^3}{T^2}\right]$.

	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	Means
Z 25000....	3.0096×10^{-11}	2.4094×10^{-7}							
20000....	2.0802×10^{-11}	3.4906×10^{-10}							
15000....	1.5381×10^{-11}	6.9893×10^{-11}							
10000....	1.1700×10^{-11}	2.7400×10^{-11}	3.7040×10^{-10}						
5000....	9.6004×10^{-12}	1.3067×10^{-11}	2.6445×10^{-11}						
1000....	8.9360×10^{-12}	8.7432×10^{-12}	1.0594×10^{-11}	1.3055×10^{-11}	1.5133×10^{-9}				8.8396×10^{-12}
800....	8.9139×10^{-12}	8.7000×10^{-12}	1.0457×10^{-11}	1.1949×10^{-11}	1.0082×10^{-10}				8.8865×10^{-12}
600....	8.8918×10^{-12}	8.6570×10^{-12}	1.0321×10^{-11}	1.0771×10^{-11}	3.7097×10^{-11}	3.7576×10^{-10}			8.8603×10^{-12}
400....	8.8698×10^{-12}	8.6140×10^{-12}	1.0184×10^{-11}	1.0133×10^{-11}	1.9775×10^{-11}	3.6046×10^{-11}			8.8341×10^{-12}
200....	8.8478×10^{-12}	8.5710×10^{-12}	1.0048×10^{-11}	9.7893×10^{-12}	1.2387×10^{-11}	1.5264×10^{-11}	3.4907×10^{-10}	4.2235×10^{-10}	9.3140×10^{-12}
100....	8.8368×10^{-12}	8.5495×10^{-12}	9.9793×10^{-12}	9.6603×10^{-12}	1.0667×10^{-11}	1.0985×10^{-11}	1.4717×10^{-11}	2.1006×10^{-11}	9.2565×10^{-12}
80....	8.8346×10^{-12}	8.5452×10^{-12}	9.9658×10^{-12}	9.6364×10^{-12}	1.0493×10^{-11}	1.0749×10^{-11}	1.2353×10^{-11}	1.5624×10^{-11}	8.9344×10^{-12}
60....	8.8324×10^{-12}	8.5409×10^{-12}	9.9521×10^{-12}	9.6126×10^{-12}	1.0319×10^{-11}	1.0514×10^{-11}	1.0815×10^{-11}	1.2357×10^{-11}	9.4674×10^{-12}
40....	8.8302×10^{-12}	8.5366×10^{-12}	9.9384×10^{-12}	9.5888×10^{-12}	1.0145×10^{-11}	1.0278×10^{-11}	9.9818×10^{-12}	1.251×10^{-11}	9.5528×10^{-12}
20....	8.8280×10^{-12}	8.5323×10^{-12}	9.9247×10^{-12}	9.5650×10^{-12}	9.9710×10^{-12}	1.0042×10^{-11}	9.6583×10^{-12}	8.9540×10^{-12}	9.4344×10^{-12}
Photos. 0....	8.8258×10^{-12}	8.5280×10^{-12}	9.9110×10^{-12}	9.5412×10^{-12}	9.7970×10^{-12}	9.8064×10^{-12}	9.3750×10^{-12}	8.1345×10^{-12}	9.2399×10^{-12}
−20....	8.8234×10^{-12}	8.5237×10^{-12}	9.9024×10^{-12}	9.5114×10^{-12}	9.7046×10^{-12}	9.6720×10^{-12}	9.1560×10^{-12}	7.7940×10^{-12}	9.4344×10^{-12}
−40....	8.8210×10^{-12}	8.5193×10^{-12}	9.8938×10^{-12}	9.4815×10^{-12}	9.6220×10^{-12}	9.5376×10^{-12}	8.9166×10^{-12}	7.4508×10^{-12}	9.2560×10^{-12}
−60....	8.8187×10^{-12}	8.5150×10^{-12}	9.8852×10^{-12}	9.4516×10^{-12}	9.5394×10^{-12}	9.4032×10^{-12}	8.7312×10^{-12}	5.9095×10^{-12}	9.1920×10^{-12}
−80....	8.8163×10^{-12}	8.5106×10^{-12}	9.8766×10^{-12}	9.4217×10^{-12}	9.4568×10^{-12}	9.2688×10^{-12}	6.0991×10^{-14}	4.5266×10^{-12}	9.2251×10^{-12}
−100....	8.8140×10^{-12}	8.5063×10^{-12}	9.8680×10^{-12}	9.3918×10^{-12}	9.3742×10^{-12}	9.1344×10^{-12}	5.9308×10^{-12}	3.5445×10^{-12}	9.1815×10^{-12}
−200....	8.8022×10^{-12}	8.4845×10^{-12}	9.8249×10^{-12}	9.2423×10^{-12}	8.9127×10^{-12}	6.0687×10^{-14}	3.0127×10^{-12}		9.0533×10^{-12}
−400....	8.7788×10^{-12}	8.4412×10^{-12}	9.7388×10^{-12}	9.0296×10^{-12}	5.1112×10^{-14}	2.7500×10^{-14}			8.9971×10^{-12}
−600....	8.7554×10^{-12}	8.3979×10^{-12}	9.6527×10^{-12}	8.7586×10^{-12}	3.1202×10^{-14}				8.8911×10^{-12}
−800....	8.7320×10^{-12}	8.3546×10^{-12}	9.5666×10^{-12}	5.7873×10^{-14}	2.1762×10^{-14}				8.8811×10^{-12}
−1000....	8.7086×10^{-12}	8.3113×10^{-12}	9.4805×10^{-12}	5.7211×10^{-14}	1.6388×10^{-14}				8.8335×10^{-12}
−2000....	8.5932×10^{-12}	8.0112×10^{-12}	9.1232×10^{-12}	3.0739×10^{-14}					
−4000....	8.4264×10^{-12}	7.5607×10^{-12}	4.8784×10^{-14}						9.1220×10^{-12}
−6000....	8.2270×10^{-12}	8.8202×10^{-13}	3.0976×10^{-14}						
−8000....	8.0964×10^{-12}	8.0486×10^{-14}	1.2000×10^{-14}						
−10000....	6.5041×10^{-14}	6.6504×10^{-14}							
−12000....	6.4390×10^{-14}	5.5100×10^{-14}							
−14000....	5.6360×10^{-14}	4.6451×10^{-14}							
Z_R.....	−9000	−4500	−2250	−750	−225	−140	−80	−45	
Above Z_R.....	8.0964	7.5607	9.1232	8.7586	8.9127	9.1344	8.7312	7.4508	8.4710×10^{-12}
Below Z_R.....	6.5041	.....	4.8784	5.7873	5.1112	6.0687	6.0991	5.9095	5.7655×10^{-14}

kinetic of ponderable gases back to pure ether radiations. We therefore identify our computed σ, derived from atmospheric thermodynamic data, with (σ) immediately, and then pass to (a) of the Stefan formula by the factor $\frac{4}{c}$, since both σ and a must have the same dimensions $\left[\frac{M L^2}{T^2} \cdot \frac{1}{L^3}\right]$ kinetic energy per unit volume. In this way we obtain the coefficients $c_1 = 8\pi c h$, $c_2 = \frac{c h}{k}$, in the Wien-Planck formula for the black body spectrum. Since c_1 and c_2 are both variables the problem becomes very complex.

We have for diatomic hydrogen:

Below the radiation level..... 8.8202×10^{-13}

Mean value in the isothermal layers..... 9.1220×10^{-12}

TABLE 41

COMPUTED VALUES OF $c_1 = 8\pi c h$ FROM THREE BALLOON ASCENSIONS

1911	Uccle, June 9	Uccle, Sept. 13	Uccle, Nov. 9	Mean Values up to 60000 Meters
$z = 90000$.....	.....	.....	2.2780×10^{-8}	
80000.....	.....	2.3584×10^{-10}	2.0080×10^{-13}	
70000.....	4.0225×10^{-10}*	1.0487×10^{-13}	5.2247×10^{-14}	
60000.....	1.9394×10^{-13}*	3.0822×10^{-14}	2.2726 "	26.7740×10^{-15}
50000.....	3.1807×10^{-14}*	1.4922 "	1.3304 "	14.1130 "
40000.....	7.9153×10^{-15}	7.7003×10^{-15}	7.5060×10^{-15}	7.7072 "
30000.....	4.9258 "	4.7956 "	4.7237 "	4.8130 "
20000.....	4.2112 "	4.1097 "	4.0435 "	4.1215 "
10000.....	3.4182 "	3.3232 "	3.5042 "	3.4152 "
0000.....	2.3976 "	2.3816 "	2.4859 "	2.4217 "

*Omitted in the means on account of the unequal heights.

c_1 increases with the height in both of the atmospheres, especially in the upper rarefied gases.

The value of c_1 computed from terrestrial balloon ascensions, at the top of the isothermal layer, 40000 meters, agrees at that point with the laboratory value. Reducing this to the sun by the gravitation factor $\gamma^{\frac{4}{3}} = 85.138$ it agrees with the logarithmic mean value, quite closely.

TABLE 42. $c_2 = \frac{c\,h}{k}$. — THE WIEN-PLANCK COEFFICIENT [L deg.].

	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	Means
Z 25000	136.24	26289.							
20000	121.96	454.60							
15000	112.63	195.86							
10000	105.30	130.59	373.75						
5000	98.826	104.95	136.31						
1000	93.768	91.560	109.790	117.500	619.060				92.664
800	93.549	91.084	108.658	114.060	206.210				97.764
600	93.330	90.610	107.526	110.150	149.220	327.930			97.155
400	93.112	90.136	106.394	106.600	127.630	153.340			96.614
200	92.894	89.662	105.262	102.850	114.860	123.630	138.02	316.750	97.667
100	92.785	89.425	104.696	101.480	109.090	112.350	114.38	113.150	97.097
80	92.763	89.376	104.582	101.206	107.858	110.546	111.17	105.460	99.157
60	92.742	89.329	104.469	100.932	106.626	108.702	107.64	99.415	98.820
40	92.720	89.282	104.356	100.658	105.394	106.858	104.12	94.426	98.211
20	92.698	89.235	104.243	100.384	104.162	105.014	101.270	89.856	98.358
Photos. 0	92.676	89.188	104.130	100.110	102.930	103.170	98.210	85.076	96.936
−20	92.649	89.139	104.036	99.749	101.992	101.654	95.847	81.438	95.813
−40	92.622	89.092	103.940	99.388	101.053	100.140	93.240	77.777	94.657
−60	92.596	89.045	103.844	99.027	100.114	98.626	91.200	0.62846	96.370
−80	92.570	88.998	103.748	98.666	99.175	97.112	0.63639	0.58597	96.711
−100	92.544	88.951	103.652	98.305	98.236	95.598	0.62433	0.55006	96.214
−200	92.412	88.714	103.174	96.850	93.152	0.63157	0.52348		94.860
−400	92.156	88.238	102.218	94.500	0.59930	0.51500			94.278
−600	91.900	87.762	101.262	91.542	0.52836				93.117
−800	91.644	87.286	100.306	0.60381	0.48163				93.079
−1000	91.388	86.810	99.350	0.59600	0.44784				92.516
−2000	90.116	83.695	95.356	0.50191					96.117
−4000	88.256	78.762	0.62869						
−6000	86.054	9.2188	0.55986						
−8000	84.580	8.7798	0.49000						
−10000	0.6786	8.3308							
−12000	0.6752	7.9343							
−14000	0.6513	7.5592							
Z_R	−9000	−4500	−2250	−750	−225	−140	−80	−45	
Above Z_R	84.580	78.762	95.356	91.542	93.152	95.598	91.200	77.777	88.496
Below Z_R	0.6786	.....	0.6287	0.6038	0.5993	0.6316	0.6364	0.6285	0.6296
Diatomic hydrogen below the radiation level									9.2188
Mean value in the isothermal layers									96.117

SUMMARY OF THE RESULTS FOR $c_1 = 8\pi ch$.

	Number	Logarithm
General mean in the isothermal layer.....	9.1220×10^{-12}	−12.96009
$(c_1)_2$ Mean just above the radiation layer.....	8.4710×10^{-12}	−12.92793
$(c_1)_0$ Logarithmic mean, for factor 0.461	8.2393×10^{-13}	−13.91589*
$(c_1)_1$ Mean just below the radiation layer.....	5.7655×10^{-14}	−14.76084
Balloon ascensions at 40000 meters.....	7.7072×10^{-15}	−15.88690
" " reduced to the sun....	6.5617×10^{-13}	−13.81702*
Laboratory value reduced to the sun.....	4.2013×10^{-13}	−13.62338*
Laboratory value at the earth.....	4.9350×10^{-15}	−15.69329
$\gamma^{4/3}$ = factor for c_1 from earth to sun	85.138	1.93012

Compare the * values.

TABLE 43

COMPUTED VALUES OF $c_2 = \frac{ch}{k}$ FROM THREE BALLOON ASCENSIONS

1911	Uccle, June 9	Uccle, Sept. 13	Uccle, Nov. 9	Mean Values up to 60000 Meters
$z = 90000$.....	.....	.....	974.50	
80000.....	.....	252.67	8.6066	
70000.....	862.00*	5.0559	3.9188	
60000.....	5.4771*	2.6415	2.4351	2.5383
50000.....	1.9421*	1.7583	1.7106	1.7345
40000.....	1.3820	1.3694	1.3591	1.3702
30000.....	1.1555	1.1432	1.1401	1.1496
20000.....	0.9716	0.9674	0.9668	0.9686
10000.....	0.8310	0.8318	0.8299	0.8309
0000.....	0.7440	0.7462	0.7371	0.7434

c_2 increases with the height in both of the atmospheres, especially in the upper rarefied gases.

SUMMARY OF THE RESULTS FOR $c_2 = \frac{ch}{k}$

	Number	Logarithm
General mean in the isothermal layer.....	96.117	1.98280
$(c_2)_2$ Mean just above the radiation layer.....	88.496	1.94692
$(c_2)_0$ Logarithmic mean, for factor 0.461.....	40.787	1.61052*
$(c_2)_1$ Mean just below the radiation layer.....	0.6296	−1 79906
Balloon ascensions at 40000 meters.....	1.3702	0.13678
" " reduced to the sun.....	4.1621	0.61931*
Laboratory value reduced to the sun.....	4.3834	0.64181*
Laboratory value at the earth.....	1.44303	0.15928
$\gamma^{1/3}$ = factor for c_2, from earth to sun	3.0376	0.48253

Compare the * values.

TABLE 44. $a\frac{6\alpha c_1}{c_2^4}$.—THE VOLUME INTENSITY COEFFICIENT IN THE ÆTHER $\left[\frac{M}{L\,T^2\,\text{deg.}^4}\right]$.

	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	Means
Z 25000	5.6724×10^{-19}	3.2759×10^{-24}							
20000	6.1040 "	5.3074×10^{-20}							
15000	6.2061 "	3.0842×10^{-19}							
10000	6.1804 "	6.1186 "	1.2327×10^{-19}						
5000	6.5363 "	6.9936 "	4.9738 "						
1000	7.5065 "	8.0785 "	4.7351 "	4.5517×10^{-19}	6.6914×10^{-20}				7.8925×10^{-19}
800	7.5590 "	8.2132 "	4.8829 "	4.5849 "	3.6207×10^{-19}				6.8850 "
600	7.6115 "	8.3479 "	5.0308 "	4.7507 "	4.8587 "	2.1100×10^{-19}			6.9901 "
400	7.6640 "	8.4826 "	5.1787 "	5.0957 "	4.9778 "	4.2333 "			7.1084 "
200	7.7165 "	8.6173 "	5.3266 "	5.6814 "	4.6221 "	4.2435 "	6.2461×10^{-19}	2.7245×10^{-19}	6.8177 "
100	7.7428 "	8.6847 "	5.4006 "	5.9247 "	4.8916 "	4.4790 "	5.5828 "	8.3232 "	6.9382 "
80	7.7483 "	8.6986 "	5.4153 "	5.9732 "	5.0467 "	4.7076 "	5.2523 "	8.2020 "	6.5763 "
60	7.7534 "	8.7115 "	5.4301 "	6.0219 "	5.2019 "	4.9362 "	5.2312 "	8.2152 "	6.6238 "
40	7.7586 "	8.7250 "	5.4449 "	6.0706 "	5.3571 "	5.1648 "	5.5150 "	8.3802 "	6.4202 "
20	7.7638 "	8.7385 "	5.4597 "	6.1193 "	5.5123 "	5.3934 "	5.9627 "	8.8706 "	6.7275 "
Photos. 0	7.7690 "	8.7520 "	5.4745 "	6.1680 "	5.6675 "	5.6220 "	6.5424 "	10.0830 "	7.0098 "
−20	7.7757 "	8.7672 "	5.4914 "	6.2332 "	5.8414 "	5.9180 "	7.0452 "	11.5060 "	6.7245 "
−40	7.7825 "	8.7822 "	5.5083 "	6.2986 "	6.0152 "	6.2140 "	7.6612 "	13.2220×10^{-19}	6.8946 "
−60	7.7893 "	8.7972 "	5.5252 "	6.3640 "	6.1890 "	6.5100 "	8.1962×10^{-19}	2.4597×10^{-12}	7.0530 "
−80	7.7961 "	8.8122 "	5.5421 "	6.4294 "	6.3628 "	6.8060 "	2.4153×10^{-12}	2.4932 "	6.9581 "
−100	7.8029 "	8.8272 "	5.5590 "	6.4948 "	6.5366 "	7.1020×10^{-19}	2.5349 "	2.5145 "	6.7204 "
−200	7.8368 "	8.9024 "	5.6434 "	6.8215 "	7.6858×10^{-19}	2.4769×10^{-12}	2.6033 "		7.2700 "
−400	7.9046 "	9.0527 "	5.8123 "	7.3528 "	2.5733×10^{-12}	2.5150 "			7.5245 "
−600	7.9724 "	9.2030 "	5.9812 "	8.0993×10^{-19}	2.5999 "				7.8140 "
−800	8.0402 "	9.3533 "	6.1501 "	2.8273×10^{-12}	2.6264 "				7.8379 "
−1000	8.1080 "	9.5036 "	6.3190 "	2.9443 "	2.6459 "				7.9769 "
−2000	8.4616 "	1.0602×10^{-18}	7.1655×10^{-19}	3.1454 "					6.8397 "
−4000	9.0194 "	12.7580×10^{-19}	2.0280×10^{-12}						
−6000	9.7423 "	7.9305×10^{-16}	2.0474 "						
−8000	10.2730×10^{-19}	8.7963 "	2.0670 "						
−10000	1.9916×10^{-12}	8.9658 "							
−12000	2.0119 "	9.0284 "							
−14000	2.0332 "	9.0286 "							

	Z_R	−9000	−4500	−2250	−750	−225	−140	−80	−45	
Above	Z_R	10.2730	12.7580	7.1655	8.0993	7.6858	7.1020	8.1962	13.2220	9.3127×10^{-19}
Below	Z_R	1.9916	.....	2.0280	2.8273	2.5733	2.4769	2.4153	2.4597	2.3960×10^{-12}

Diatomic hydrogen below the radiation level ..... 7.9305×10^{-16}

Mean value in the isothermal layers ..... 6.8397×10^{-19}

c_1 at 40000 meters reduced to the sun 6.1657×10^{-13} $- 13.81702$. The value of $c_1 = 8\pi c h$ increases upward from the low levels to the vanishing plane. In the solar atmosphere it passes through a sudden change in the radiation layer, and its mean value, derived from the logarithmic law of depletion, agrees closely with the value of $c_1 = 8\pi c h$, as derived from computations of the terrestrial thermodynamics, at the height of the top of the earth's isothermal layer.

The solar value $(c_1)_0$ 8.2393×10^{-15} is the same as in the earth's atmosphere at the height 42000 meters above the sea level.

The value of c_2, computed from terrestrial balloon ascensions, agrees at the top of the isothermal layer, with the laboratory value 1.3702 and 1.4430, respectively. Reducing the balloon value to the sun by the factor $\gamma^{\frac{1}{3}} = 3.0376$, it becomes 4.1621, while the logarithmic mean value by the factor 0.461 is 40.787 at the sun, about ten times as great. The same abrupt change occurs in c_2 as in c_1 on passing the radiation layer, 0.6296 below and 88.496 above z_R for all the gases computed. Attention is called to diatomic hydrogen, which agrees with all the gases above this layer, but is very different below it, while monatomic hydrogen agrees above and below with the other gases. It is supposed that this indicates dissociation of the hydrogen diatomic molecule below the plane of radioactivity.

TABLE 45

COMPUTED VALUES OF $a = \frac{6\alpha c_1}{c_2^4}$ FROM THREE BALLOON ASCENSIONS

1911	Uccle, June 9	Uccle, Sept. 13	Uccle, Nov. 9	Mean Values up to 60000 Meters	
$z = 90000$....	.....	.....	1.6403×10^{-19}		
80000....	.....	2.2546×10^{-17}	2.3765×10^{-16}		
70000....	4.7310×10^{-21}*	1.0422×10^{-15}	1.4387×10^{-15}		
60000....	1.3995×10^{-15}*	4.1111×10^{-15}	4.1970×10^{-15}	4.1541×10^{-15}	7.134×10^{-15}
50000....	1.4519×10^{-14}*	1.0137×10^{-14}	1.0091×10^{-14}	10.1140 "	The laboratory value is found at 55000 meters above the sea level.
40000....	1.4091 "	1.4221 "	1.4287 "	14.203 "	
30000....	1.7945 "	1.8195 "	1.8153 "	18.098 "	
20000....	3.0686 "	3.0476 "	3.0058 "	30.407 "	
10000....	4.6766 "	4.5073 "	4.7964 "	47.268 "	
0000....	5.0810 "	4.9880 "	5.4694 "	51.795 "	

* Omitted in the means.

a Decreases with the height in the atmospheres of the earth and the sun, especially in the rarefied gases of the uppermost strata.

TABLE 46. $\sigma = \frac{ac}{4}$.—THE KINETIC ENERGY PER UNIT VOLUME DERIVED FROM THE RADIATION PER SECOND, $\left[\frac{M}{T^3} \cdot \frac{1}{\text{deg}^4}\right]$

	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	Means
Z 25000	4.2503×10^{-9}	2.4569×10^{-14}							
20000	4.5779 "	3.9805×10^{-10}							
15000	4.6546 "	2.3670×10^{-9}							
10000	4.6353 "	4.5889 "	9.2450×10^{-10}						
5000	4.9022 "	5.2451 "	3.7303×10^{-9}						
1000	5.6299 "	6.0589 "	3.5513 "	3.4137×10^{-9}	5.0186×10^{-10}				5.8444×10^{-9}
800	5.6692 "	6.1600 "	3.6622 "	3.4387 "	2.7156×10^{-9}				5.1638 "
600	5.7086 "	6.2610 "	3.7731 "	3.5610 "	3.6440 "	1.5825×10^{-9}			5.2476 "
400	5.7480 "	6.3620 "	3.8840 "	3.8217 "	3.7333 "	3.1750 "			5.3313 "
200	5.7874 "	6.4630 "	3.9949 "	4.2610 "	3.4666 "	3.1826 "	4.6846×10^{-9}	2.0434×10^{-9}	5.1266 "
100	5.8071 "	6.5135 "	4.0504 "	4.4435 "	3.6687 "	3.3592 "	4.1871 "	6.2424 "	5.1629 "
80	5.8110 "	6.5236 "	4.0614 "	4.4799 "	3.7850 "	3.5305 "	3.9392 "	6.1514 "	4.9322 "
60	5.8151 "	6.5337 "	4.0725 "	4.5164 "	3.9014 "	3.7020 "	3.9234 "	6.1614 "	4.9678 "
40	5.8190 "	6.5438 "	4.0836 "	4.5529 "	4.0178 "	3.8735 "	4.1363 "	6.2850 "	4.8151 "
20	5.8229 "	6.5539 "	4.0947 "	4.5894 "	4.1342 "	4.0450 "	4.4720 "	6.6529 "	5.0486 "
Photos. 0	5.8268 "	6.5640 "	4.1058 "	4.6259 "	4.7506 "	4.2165 "	4.9068 "	7.5622 "	5.3198 "
−20	5.8319 "	6.5752 "	4.1184 "	4.6750 "	4.3808 "	4.4385 "	5.2839 "	8.6298 "	5.0434 "
−40	5.8370 "	6.5865 "	4.1310 "	4.7240 "	4.5112 "	4.6605 "	5.7495 "	9.9165×10^{-9}	5.1714 "
−60	5.8421 "	6.5978 "	4.1437 "	4.7730 "	4.6416 "	4.8825 "	6.1471×10^{-9}	1.8448×10^{-2}	5.2897 "
−80	5.8472 "	6.6091 "	4.1564 "	4.8220 "	4.7720 "	5.1045 "	1.8115×10^{-2}	1.8699 "	5.2185 "
−100	5.8523 "	6.6204 "	4.1691 "	4.8710 "	4.9024 "	5.3265×10^{-9}	1.9012 "	1.8859 "	5.2903 "
−200	5.8777 "	6.6768 "	4.2324 "	5.1161 "	5.7644×10^{-9}	1.8577×10^{-2}	1.9540 "		5.5335 "
−400	5.9285 "	6.7895 "	4.3591 "	5.5145 "	1.9300×10^{-2}	1.8866 "			5.6479 "
−600	5.9793 "	6.9022 "	4.4858 "	6.0745×10^{-9}	1.9501 "				5.6884 "
−800	6.0301 "	7.0149 "	4.6125 "	2.1204×10^{-2}	1.9697 "				5.5325 "
−1000	6.0809 "	7.1276 "	4.7392 "	2.2082 "	1.9844 "				5.9826 "
−2000	6.3461 "	7.9517 "	5.3741×10^{-9}	2.3590 "					5.3028 "
−4000	6.7646 "	9.5684×10^{-9}	1.5210×10^{-2}						
−6000	7.3067 "	5.9479×10^{-6}	1.5355 "						
−8000	7.7050×10^{-9}	6.5972 "	1.5500 "						
−10000	1.4937×10^{-2}	6.7243 "							
−12000	1.5089 "	6.7713 "							
−14000	1.5249 "	6.9292 "							
Z_R	−9000	−4500	−2250	−750	−225	−140	−80	−45	
Above Z_R	7.7050	9.5684	5.3741	6.0745	5.7644	5.3265	6.1471	9.9165	6.9846×10^{-9}
Below Z_R	1.4937	.....	1.5210	2.1204	1.9300	1.8577	1.8115	1.8448	1.7970×10^{-2}
Diatomic hydrogen below the radiation level									5.9479×10^{-6}
Mean values in the isothermal layers									5.3028×10^{-9}

SUMMARY OF THE RESULTS FOR $a = \frac{6\alpha c_1}{c_2^4}$

	Number	Logarithm
General mean in the isothermal layer	6.8397×10^{-19}	-19.83504
a_2 Mean just above the radiation layer	9.3127×10^{-19}	-19.96907
a_0 Logarithmic mean, for the factor 0.461	7.5738×10^{-15}	-15.87932*
a_1 Mean just below the radiation layer	2.3960×10^{-12}	-12.37949
Balloon ascensions at 55000 meters	7.390×10^{-15}	-15.86864*
" " reduced to the sun	7.390×10^{-15}	-15.86864
Laboratory value reduced to the sun	7.3900×10^{-15}	-15.86864
Laboratory value at the earth	7.3900×10^{-15}	-15.86864*

Compare the * values.

The γ factor is 1.00, since $\frac{6\alpha c_1}{c_2^4}$ gives $\gamma^{4/3} / \gamma^{4/3} = 1.00$, so that for the coefficient a the reduction from the values at the earth to those corresponding with them at the sun disappears. The value of a changes with the height in each atmosphere.

TABLE 47

COMPUTED VALUES OF $\sigma = \frac{ac}{4}$ FROM THREE BALLOON ASCENSIONS

1911	Uccle, June 9	Uccle, Sept. 13	Uccle, Nov. 9	Mean Values up to 60000 Meters	
$z = 90000$....	.....	.....	1.2302×10^{-9}		
80000....	.....	2.8183×10^{-9}	1.7824×10^{-5}		
70000....	3.5482×10^{-11}*	7.8163×10^{-5}	1.0790×10^{-5}		
60000....	1.0496×10^{-5}*	3.0834×10^{-5}	3.1478 "	3.1156×10^{-5}	5.3557×10^{-5}
50000....	1.0889×10^{-4}*	7.6030 "	7.5883 "	7.5957 "	
40000....	1.0568 "	1.0666×10^{-4}	1.0716×10^{-4}	1.0650×10^{-4}	
30000....	1.3458 "	1.3646 "	1.3613 "	1.3572 "	
20000....	2.3015 "	2.2856 "	2.2543 "	2.2805 "	
10000....	3.5075 "	3.3805 "	3.5973 "	3.4934 "	
0000....	3.8101 "	3.7410 "	4.1020 "	3.5510 "	

The laboratory value is found at 55000 meters above the sea level.

* Omitted from the means.

σ Decreases upwards in the atmospheres of the sun and the earth.

The laboratory value 7.390×10^{-15} is encountered again at about 55000 meters above the sea level; it is, also, found in the radiation layers near the bottom of the isothermal layer of each solar gas. It seems as if the solar radiation once originated advances through the solar envelope without much absorption, being depleted chiefly by scattering, and that it arrives unchanged at the outer layers of the earth's gaseous envelope.

TABLE 48

$\frac{60\,\sigma}{A}$, $\frac{\text{gram calories}}{(\text{centimeter})^2,\ \text{minute}}$

	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	Means
Z 25000....	6.0991×10^{-15}	3.5223×10^{-20}							
20000....	6.5631 "	5.7066×10^{-16}							
15000....	6.6730 "	3.3162×10^{-15}							
10000....	6.6454 "	6.5789 "	1.3254×10^{-15}						
5000....	7.0280 "	7.5197 "	5.3480 "						
1000....	8.0712 "	8.6862 "	5.0913 "	4.8941×10^{-15}	7.1948×10^{-16}				8.3787×10^{-15}
800....	8.0280 "	8.8311 "	5.2503 "	4.8176 "	3.8932×10^{-15}				7.3698 "
600....	8.1849 "	8.9760 "	5.4093 "	5.1081 "	5.2242 "	2.2688×10^{-15}			7.5234 "
400....	8.2418 "	9.1208 "	5.5683 "	5.4790 "	5.3521 "	4.5518 "			7.6436 "
200....	8.2987 "	9.2656 "	5.7273 "	6.1087 "	4.9698 "	4.5629 "	6.7160×10^{-15}	2.9295×10^{-15}	7.3501 "
100....	8.3272 "	9.3380 "	5.8068 "	6.3704 "	5.2596 "	4.8159 "	6.0027 "	8.9494 "	7.4606 "
80....	8.3328 "	9.3524 "	5.8227 "	6.4228 "	5.4263 "	5.0617 "	5.6474 "	8.8190 "	7.0710 "
60....	8.3385 "	9.3669 "	5.8386 "	6.4751 "	5.5932 "	5.3075 "	5.6247 "	8.8332 "	7.1225 "
40....	8.3442 "	9.3814 "	5.8545 "	6.5274 "	5.7601 "	5.5533 "	5.9300 "	9.0106 "	6.9035 "
20....	8.3499 "	9.3959 "	5.8704 "	6.5797 "	5.9270 "	5.7991 "	6.4112 "	9.5378 "	7.2339 "
Photos. 0....	8.3556 "	9.4104 "	5.8863 "	6.6320 "	6.0939 "	6.0449 "	7.0347 "	10.8420 "	7.5375 "
−20....	8.3628 "	9.4265 "	5.9043 "	6.7022 "	6.2803 "	6.3630 "	7.5752 "	12.3720 "	7.2306 "
−40....	8.3700 "	9.4427 "	5.9225 "	6.7725 "	6.4668 "	6.6813 "	8.2376 "	14.2170×10^{-15}	7.4177 "
−60....	8.3773 "	9.4589 "	5.9407 "	6.8428 "	6.6533 "	6.9996 "	8.8128×10^{-15}	2.6448×10^{-8}	7.5836 "
−80....	8.3846 "	9.4751 "	5.9589 "	6.9131 "	6.8398 "	7.3179 "	2.5970×10^{-8}	2.6807 "	7.4816 "
−100....	8.3919 "	9.4913 "	5.9771 "	6.9834 "	7.0263 "	7.6362×10^{-16}	2.7256 "	2.7037 "	7.5844 "
−200....	8.4282 "	9.5722 "	6.0679 "	7.3347 "	8.2640×10^{-15}	2.6633×10^{-8}	2.8013 "		7.9334 "
−400....	8.5006 "	9.1339 "	6.2495 "	7.9058 "	2.7699×10^{-8}	2.7040 "			7.9475 "
−600....	8.5730 "	9.8956 "	6.4311 "	8.7086×10^{-15}	2.7954 "				8.4021 "
−800....	8.6454 "	1.0057×10^{-14}	6.6127 "	3.0400×10^{-8}	2.8239 "				8.4384 "
−1000....	8.7178 "	1.0219 "	6.7943 "	3.1658 "	2.8449 "				8.5770 "
−2000....	9.0980 "	1.1400 "	7.7046×10^{-16}	3.3820 "					7.6329 "
−4000....	9.6756 "	1.3718×10^{-14}	2.1805×10^{-8}						
−6000....	1.0475×10^{-14}	8.5270×10^{-12}	2.2014 "						
−8000....	1.1046×10^{-14}	9.4580 "	2.2200 "						
−10000....	2.1414×10^{-8}	9.6403 "							
−12000....	2.1632 "	9.7075 "							
−14000....	2.1862 "	9.9340 "							
Z_R.....	−9000	−4500	−2250	−750	−225	−140	−80	−45	
Above Z_R.....	11.0460	13.7180	7.7046	8.7086	8.2640	7.6362	8.8128	14.2170	10.0134×10^{-15}
Below.....	2.1414		2.1805	3.0400	2.7699	2.6633	2.5970	2.6448	2.2546×10^{-8}
Diatomic hydrogen below the radiation level.....									8.5270×10^{-12}
Mean value in the isothermal layer.....									7.6329×10^{-15}

The excessive changes in a above 55000 meters involves problems of difficulty in the kinetic theory of gases. From the level 55000 meter, where $a = 7.390 \times 10^{-15}$, to the sea level, the value of a steadily increases to 51.795×10^{-15}, where it is about seven times as large as at 55000 meters. This is, however, compensated by a change of the exponent from $\alpha = 4.00$ to $\alpha = 3.82$ on the sea level.

SUMMARY OF THE RESULTS FOR $\sigma = \frac{a}{4} c \left[\frac{M}{T^3 \text{ deg.}^4}\right]$

	Number	Logarithm
General mean in the isothermal layer.....	5.3028×10^{-9}	-9.72450
σ_2 Mean just above the radiation layer.....	6.9846×10^{-9}	-9.84414
σ_0 Logarithmic mean, for the factor 0.461.....	5.6425×10^{-5}	-5.75147*
σ_1 Mean just below the radiation layer.....	1.7970×10^{-2}	-2.25455
Balloon ascensions at 55000 meters.....	5.3557×10^{-5}	-5.72882*
" " reduced to the sun.....	5.3557×10^{-5}	-5.72882
Laboratory value reduced to the sun.....	5.5424×10^{-5}	-5.74370*
Laboratory value at the earth.....	5.5424×10^{-5}	-5.74370

Compare the * values.

The γ factor = 1.00 for σ and for a, since these have the same dimensions. These values of σ should be compared with those in Table 26, from which it is seen that there is close agreement in the results. The ordinary laboratory value 5.5424×10^{-5} is found again near the level 55000 meters in the earth's atmosphere, and, also, in the radiation layer of the solar isothermal strata of the several gases. The exceptional behavior of diatomic hydrogen $H_2 = 2.00$ below the radiation layer should be noted.

It should be remembered that we have taken a and σ to represent a certain amount of kinetic energy per volume, according to the dimensions $\left[\frac{M L^2}{T^2} \cdot \frac{1}{L^3} = \frac{M}{L T^2}\right]$. The first form is more instructive and it should always be employed in the discussion of the radiation problems. Since work = force times length, $\frac{M L}{T^2} \cdot L = \frac{M L^2}{T^2}$, the amount of work done is equivalent

to the kinetic energy per volume. The practical units of work are,

$$\text{(M. K. S.) Joule} = \frac{\text{kilogram meter}^2}{\text{second}^2},$$

$$\text{(C. G. S.) Erg} = \frac{\text{gram centimeter}^2}{\text{second}^2},$$

The dimensions of a and σ both give $\frac{M L^2}{T^2} \cdot \frac{1}{L^3}$, the work per volume, $\frac{\text{Joule}}{\text{vol.}}, \frac{\text{erg.}}{\text{vol.}}$. The interpretation $\frac{\text{Joule}}{\text{area}}$ and $\frac{\text{erg}}{\text{area}}$ is erroneous from every point of view.

TABLE 49

COMPUTED VALUES FROM THREE BALLOON ASCENSIONS, $\frac{60\,\sigma}{A}$, IN GRAM CALORIES PER SQUARE CENTIMETER PER MINUTE

1911	Uccle, June 9	Uccle, Sept. 13	Uccle, Nov. 9	Means	
z 90000....	.....	.....	1.7637×10^{-15}	—	
80000....	.....	2.3250×10^{-13}	2.5553×10^{-12}	—	
70000....	5.0869×10^{-17}	1.1206×10^{-11}	1.5469×10^{-11}	—	
60000....	1.5047×10^{-11}	4.4204 "	4.5127 "	3.4793×10^{-11}	7.9667×10^{-11}
50000....	1.5611×10^{-10}	1.0900×10^{-10}	1.0850×10^{-10}	1.2454×10^{-10}	The laboratory value is found at 55000 meters above sea level.
40000....	1.5151 "	1.5291 "	1.5720 "	1.5387 "	
30000....	1.9294 "	1.9564 "	1.9518 "	1.9459 "	
20000....	2.2995 "	3.2768 "	3.2319 "	3.2694 "	
10000....	5.0285 "	4.8453 "	5.1573 "	5.0104 "	
0000....	5.4633 "	5.3630 "	5.8809 "	5.3691 "	

The corresponding flux per unit area is given instead of the volume kinetic energy.

SUMMARY OF THE RESULTS FOR $\frac{60\,\sigma}{A}$, $\frac{\text{gram calories}}{\text{cm.}^2\ \text{minute}}$.

		Number	Logarithm
	General mean in the isothermal layer.	7.6329×10^{-15}	−15.88269
$(60\ \sigma_A)_2$	Mean just above the radiation layer..	10.0134×10^{-15}	−14.00058
$(60\ \sigma_A)_0$	Logarithmic mean, for the factor 0.461	7.6015×10^{-11}	−11.88090*
$(60\ \sigma_A)_1$	Mean just below the radiation layer..	2.2546×10^{-8}	− 8.35307
	Balloon ascensions at 55000 meters..	7.9667×10^{-11}	−11.90127*
	" " reduced to the sun	7.9667×10^{-11}	−11.90127
	Laboratory value reduced to the sun.	7.900×10^{-11}	−11.89763
	Laboratory value at the earth.....	7.900×10^{-11}	−11.89763*

Compare the * values.

As in the case of a, σ, we have $\frac{60\,\sigma}{A}$, in gram calories per square centimeter per minute, having about the same values in the laboratory, at 55000 meters in the atmosphere, and at the radiation layer in the solar gases.

Check on the Computations

We have now computed $\frac{60\,\sigma}{A}$ in two different ways, quite distinct from one another:

(1) The value of c from pairs (log c, α) in the (M. K. S.) system was reduced directly to $\frac{60\,\sigma}{A}$ by the factor $\frac{10 \times 60}{4.1518 \times 10^7}$ $= 0.000014336\ [-\ 5.15644]$.

(2) The data of the general law $P = \rho R T$ were reduced from the (M. K. S.) to the (C. G. S.) system; from these were computed K . N . k . h . c_1, c_2, and finally $a = \frac{6\,\alpha\,c_1}{{c_2}^4}$, $\sigma = \frac{c}{4}\,a$, $\frac{60\,\sigma}{A} = 60\,\frac{c}{4}\,a\,.\,\frac{1}{A}$.

TABLE 50

EXAMPLES OF THE AGREEMENT OF $60\sigma_A$ AS COMPUTED BY TWO DIFFERENT METHODS

UCCLE, SEPT. 13, 1911			HELIUM ON THE SUN			CADMIUM ON THE SUN		
z Meters	Direct	Indirect	z Kilomet.	Direct	Indirect	z Kilomet.	Direct	Indirect
80000..	−13.36644	−13.36644	10000	−15.12234	−15.12235	300	−15.52855	−15.52854
70000..	−11.04944	−11.04945	5000	−15.72819	−15.72819	200	−15.82711	−15.82711
60000..	−11.64544	−11.64546	1000	−15.70684	−15.70683	100	−15.77835	−15.77835
50000..	−10.03744	−10.03742	500	−15.73344	−15.73345	40	−15.77304	−15.77305
40000..	−10.18444	−10.18443	0	−15.76984	−15.76984	0	−15.84725	−15.84724
30000..	−10.29144	−10.29145	−500	−15.79714	−15.79715	−40	−15.91579	−15.91580
20000..	−10.51544	−10.51545	−1500	−15.83214	−15.83215	−100	−8.43548	−8.43547
10000..	−10.68544	−10.68542	−3000	−8.33440	−8.33442	−140	−8.44294	−8.44296
0000..	−10.72944	−10.72943	−5000	−8.34084	−8.34082	−200	−8.44737	−8.44736

The check is complete in the logarithms, and indeed this forms a convenient check upon the long intervening numerical

processes. It fully disproves *Very's* contention that the reduction factor should be,

$$\frac{1000 \times 60}{4.1851 \times 10^7} = 0.0014336 \; [-\, 3.15644].$$

His factor violates the law of dimensions, and leads to impossible conditions, incompatible with solar and terrestrial atmospheric physics. He has transferred the factor of the surface integral in the Poynting equation to the volume integral, which is erroneous.

Second Computation of the Wien-Planck Coefficients, Taking N = *constant* (6.062×10^{23}) and E_0 = *variable.*

The preceding Tables, 36 to 50, have been computed on the assumption that the kinetic energy of one molecule is a constant, $E_0 = 4.3769 \times 10^{-11}$, so that N and the other quantities dependent upon it become variables. The difficulty raised by this dilemma has been stated on page 140. We have in Tables 51 and 53 compiled the data by using N = constant (6.062×10^{23}) in the formulas for k, h, c_1, c_2, E_0, n, instead of the E_0 = constant, as in Table 34. Table 52 is for the solar elements, and Table 53 is for atmospheric air in three balloon ascensions. The heights z for each element are in part the same as those in the Tables 36–50, and by comparing the respective tables it can easily be seen what has been the effect of making this change. The values of $a = \frac{6 \alpha c_1}{c_2^4}$ are identical in both computations, so that the Stefan Law $J_0 = \sigma T^4$ remains unaltered in its values. The distribution in the spectrum becomes somewhat different, due to the changes in c_1 and c_2. It is remarkable that while c_1 is larger below the plane of the source of the solar radiation z_R, and smaller above it, *they nearly agree in value on that plane.* That is to say, the N = constant system is about equal to the E_0 = constant system for the critical strata in the solar atmosphere where the black radiation is generated. Similarly, c_2 is larger below and smaller above

the planes z_R, but nearly equal on that plane for the N = constant and the E_0 = constant systems. This is seen in Table 51, where the final values are as follows:

$\log c_1 = 8\pi c h$,	N = constant	− 13.83601
	E_0 = constant	− 13.84623
	Means	− 13.84112
Laboratory value		− 13.62338
$\log c_2 = \frac{c h}{k}$,	N = constant	0.89025
	E_0 = constant	0.87570
	Means	0.88298
Laboratory value		0.64181

TABLE 51

COMPARISON OF LOG c_1, LOG c_2, AT THE RADIATION LEVELS, AS COMPUTED WITH N=CONSTANT (HEAVY TYPE) AND WITH E_0=CONSTANT (LIGHT TYPE)

Element	Log $c_1 = 8\pi c h$		Means	Log $c_2 = \frac{c h}{k}$		Means
	Below	Above		Below	Above	
H_1=1.00	**−14.81319**	**−12.83545**	**−13.82432**	**−1.83162**	**1.90906**	**0.87034**
	−14.81319	−12.90829	−13.86074	−1.83162	1.92727	0.87945
H_2=2.00	**−13.99743**	**−12.82635**	**−12.41189***	**0.97706**	**1.88327**	**1.43017***
	−13.94548	−12.87856	−12.41202*	0.96467	1.89632	1.43050*
H =4	**−14.88971**	**−12.88834**	**−13.88903**	**−1.84879**	**1.96140**	**0.90510**
	−14.68828	−12.96015	−13.82422	−1.79843	1.97935	0.88889
C =12	**−14.76247**	**−12.85613**	**−13.80930**	**−1.78090**	**1.94005**	**0.86047**
	−14.76247	−12.94243	−13.85245	−1.78090	1.96162	0.87126
Ca=40	**−14.87619**	**−12.72973**	**−13.80296**	**−1.82750**	**1.95076**	**0.88913**
	−14.70853	−12.95001	−13.82927	−1.77764	1.96922	0.87343
Zn=65	**−14.89982**	**−12.84379**	**−13.87181**	**−1.82960**	**1.95123**	**0.89042**
	−14.78310	−12.96068	−13.87189	−1.80042	1.98045	0.89044
Cd=112	**−14.78527**	**−12.86050**	**−13.82289**	**−1.80370**	**1.93985**	**0.87178**
	−14.78527	−12.94107	−13.86317	−1.80370	1.95999	0.88185
Hg=198	**−14.84221**	**−12.80124**	**−13.82173**	**−1.81595**	**2.07311**	**0.94453**
	−14.77151	−12.87220	−13.82186	−1.79828	1.89085	0.84457

General mean for N =constant	**−13.83601**	General mean for N =constant	**0.89025**
" " " E_0 =constant	−13.84623	" " " E_0 =constant	0.87570
Average for both	−13.84112	Average for both	0.88298
Laboratory value	−13.62338	Laboratory value	0.64181

The general result is that the Wien-Planck formula,

$$\frac{8\pi E_\lambda}{c} = \frac{8\pi c h}{\lambda^5} \cdot \frac{1}{e^{\frac{ch}{k\lambda T}} - 1},$$

has reproduced the usual laboratory values of c_1, c_2, approximately, at the critical places of the generation of the black radiation in the solar atmospheres of the monatomic gases. For the diatomic hydrogen the values of log c_1 and log c_2 are

TABLE 52

SECOND COMPUTATION OF THE WIEN-PLANCK COEFFICIENTS, TAKING N = CONSTANT (6.062×10^{23}) AND E_0 = VARIABLE

Hydrogen (Monatomic, $H_1 = 1.00$)

z	k	h	c_1	c_2	E_0	n
26000...	4.9256×10^{-16}	8.6208×10^{-25}	6.5000×10^{-13}	52.506	2.3272×10^{-12}	1.1933×10^{16}
24000...	5.9027 "	1.0819×10^{-24}	8.1578 "	54.990	3.0988 "	1.8335 "
20000...	7.8382 "	1.5520 "	1.1701×10^{-12}	59.399	5.0554 "	3.8207 "
16000...	9.8098 "	2.0810 "	1.5691 "	63.641	7.5780 "	7.0117 "
14000...	1.0769×10^{-15}	2.3579 "	1.7778 "	65.687	9.0456 "	9.1445 "
10000...	1.2489 "	2.8762 "	2.1686 "	69.087	1.2364×10^{-11}	1.4613×10^{17}
6000...	1.4490 "	3.4737 "	2.6191 "	71.920	1.6192 "	2.1900 "
4000...	1.6005 "	3.8934 "	2.9355 "	72.980	1.8365 "	2.6454 "
2000...	1.8028 "	4.4488 "	3.3543 "	74.028	2.0796 "	3.1874 "
0...	2.0368 "	5.1160 "	3.8574 "	75.355	2.3528 "	3.8356 "
−2000...	2.3108 "	5.8831 "	4.4358 "	76.383	2.6656 "	4.6255 "
−4000...	2.6155 "	6.7940 "	5.1226 "	77.930	3.0134 "	5.5597 "
−6000...	2.9671 "	7.8345 "	5.9070 "	79.214	3.4141 "	6.7046 "
−8000...	3.3588 "	9.0800×10^{-24}	6.8462×10^{-12}	81.108	3.8596 "	8.0586 "
−10000...	3.8135 "	8.6264×10^{-26}	6.5134×10^{-14}	0.67861	4.3769 "	9.7324 "
−12000...	4.2966 "	1.0079×10^{-25}	7.5990 "	0.70370	4.9562 "	1.1727×10^{18}
−14000...	4.3882 "	1.0331 "	7.7890 "	0.70623	5.5789 "	1.4006 "

Hydrogen (Diatomic, $H_2 = 2.00$)

z	k	h	c_1	c_2	E_0	n
25000...	1.5898×10^{-19}	1.0563×10^{-27}	7.9645×10^{-16}	199.33	1.9073×10^{-17}	2.1450×10^{2}
20000...	1.6208×10^{-16}	4.2837×10^{-25}	3.2292×10^{-13}	79.287	2.3221×10^{-13}	2.4403×10^{12}
15000...	7.5557 "	1.8554×10^{-24}	1.3989×10^{-12}	73.670	2.3291×10^{-12}	7.1175×10^{14}
10000...	1.5397×10^{-15}	3.8151 "	2.8764 "	74.333	8.0910 "	1.4794×10^{16}
5000...	1.9279 "	4.9243 "	3.7129 "	76.627	1.7033×10^{-11}	9.5363 "
1000...	2.2183 "	5.6586 "	4.2665 "	76.527	2.5555 "	2.5879×10^{17}
0...	2.4254 "	6.2094 "	4.6818 "	76.772	2.7915 "	3.1220 "
−1000...	2.6588 "	6.8247 "	5.1456 "	77.003	3.0549 "	4.0161 "
−2000...	2.9075 "	7.4135 "	5.5896 "	76.493	3.3414 "	5.0075 "
−4000...	3.4901 "	8.8918 "	6.7043×10^{-12}	76.432	3.9997 "	6.1927 "
−6000...	4.1642 "	1.3185 "	9.9410×10^{-13}	9.4986	4.7884 "	1.1859 "
−8000...	4.7642 "	1.5241 "	1.1491×10^{-12}	9.5973	5.7169 "	1.8783 "
−10000...	4.8734 "	1.5609 "	1.1769 "	9.6086	6.7155 "	2.7928 "
−12000...	4.8734 "	1.5573 "	1.1742 "	9.5865	7.7197 "	3.9342 "
−14000...	4.8734 "	1.5454 "	1.1652 "	9.5130	8.7238 "	5.3160 "

TABLE 52—*Continued*

Helium, $m=4$

z	k	h	c_1	c_2	E_0	n
10000...	2.3627×10^{-17}	2.4814×10^{-26}	1.8710×10^{-14}	31.509	2.6225×10^{-14}	1.4275×10^{13}
5000...	8.3012×10^{-16}	1.7937×10^{-24}	1.3524×10^{-12}	64.923	4.7070×10^{-12}	3.4323×10^{16}
2000...	1.4058×10^{-15}	3.7343 "	2.8156 "	79.692	1.4234×10^{-11}	1.8050×10^{17}
1000...	1.6192 "	4.4438 "	3.4286 "	82.332	1.8459 "	2.6656 "
0...	2.0444 "	5.7778 "	4.3563 "	84.786	2.3628 "	3.8603 "
−1000...	2.6210 "	7.6713 "	5.7840 "	87.806	3.0215 "	5.5820 "
−2000...	3.3628 "	$\mathbf{1.0256\times10^{-23}}$	7.7328 "	**91.496**	3.8665 "	8.0802 "
−4000...	4.3720 "	1.0288×10^{-25}	7.7573 "	0.70598	6.1979 "	1.6400×10^{18}
−6000...	4.3720 "	1.0256 "	7.7327 "	0.70373	8.6926 "	2.7241 "
−7000...	4.3720 "	1.0135 "	7.7172 "	0.70233	9.9405 "	3.3310 "

Carbon, $m=12$

z	k	h	c_1	c_2	E_0	n
1000....	1.1494×10^{-15}	2.8513×10^{-24}	2.1499×10^{-12}	74.418	1.1121×10^{-11}	1.2464×10^{17}
800....	1.2552 "	3.1986 "	2.4116 "	76.448	1.3179 "	1.6080 "
600....	1.3713 "	3.5568 "	2.6817 "	77.650	1.5427 "	2.0364 "
400....	1.5490 "	4.0875 "	3.0819 "	79.164	1.7925 "	2.5506 "
200....	1.8016 "	4.8216 "	3.6354 "	80.288	2.0822 "	3.1932 "
0....	2.0899 "	5.7181 "	4.3113 "	82.080	2.4122 "	3.9819 "
−200....	2.4308 "	6.7633 "	5.0994 "	83.470	2.8021 "	4.9850 "
−400....	2.8198 "	8.0402 "	6.0621 "	85.540	3.2463 "	6.2164 "
−600....	3.2798 "	$\mathbf{9.5230\times10^{-24}}$	$\mathbf{7.1802\times10^{-12}}$	**87.106**	3.7708 "	7.7825 "
−800....	3.8135 "	7.6756×10^{-26}	5.7873×10^{-14}	0.60381	4.3769 "	9.7324× "
−1000....	4.4354 "	9.2620 "	6.9833 "	0.62646	5.0829 "	1.2179×10^{18}
−2000....	5.2888 "	1.1456×10^{-25}	8.6378 "	0.64984	9.4995 "	2.9039 "

Calcium, $m=40$

z	k	h	c_1	c_2	E_0	n
1000....	5.7870×10^{-18}	4.6618×10^{-27}	3.5148×10^{-15}	24.167	2.6041×10^{-15}	4.4671×10^{11}
800....	1.9029×10^{-16}	2.7974×10^{-25}	2.1092×10^{-13}	44.102	4.2815×10^{-13}	9.4174×10^{15}
600....	5.7680 "	1.1172×10^{-24}	8.4232 "	57.924	2.5602×10^{-12}	1.3771×10^{16}
400....	1.0112×10^{-15}	2.3329 "	1.7589×10^{-12}	69.212	7.1287 "	6.3981 "
200....	1.3173 "	3.4023 "	2.5652 "	77.482	1.3436×10^{-11}	1.6553×10^{17}
100....	1.5573 "	4.1731 "	3.1464 "	80.393	1.7519 "	2.4644 "
0....	1.9462 "	5.3485 "	4.0326 "	82.446	2.2493 "	3.5853 "
−100....	2.4990 "	7.1182 "	$\mathbf{5.3670\times10^{-12}}$	85.452	2.8809 "	5.1966 "
−200....	3.3512 "	$\mathbf{9.9730\times10^{-24}}$	7.5195×10^{-14}	**89.282**	3.8531 "	8.0497 "
−400....	4.8558 "	1.0880×10^{-25}	8.2035 "	0.67220	6.2630 "	1.6658×10^{18}
−600....	4.8558 "	1.0843 "	8.1754 "	0.66990	9.0452 "	2.8913 "
−800....	4.8558 "	1.0807 "	8.1480 "	0.66765	1.1822×10^{-10}	4.3203 "
−1000....	4.8558 "	1.0780 "	8.1280 "	0.66601	1.4580 "	5.9273 "
−2000....	4.8558 "	1.0742 "	8.0992 "	0.66365	2.8459 "	1.6134×10^{15}

TABLE 52—*Continued*

Zinc, $m = 65$

z	k	h	c_1	c_2	E_0	n
600.....	4.1704×10^{-17}	4.4253×10^{-26}	3.3365×10^{-14}	31.833	4.0037×10^{-14}	2.6926×10^{13}
400.....	5.2404×10^{-16}	1.0250×10^{-24}	7.7280×10^{-13}	58.677	2.4523×10^{-12}	1.2909×10^{16}
200.....	1.1361×10^{-15}	2.8737 "	2.1667×10^{-12}	75.882	1.0122×10^{-11}	1.0825×10^{17}
100.....	1.4277 "	3.8275 "	2.8859 "	80.427	1.6061 "	2.1636 "
0.....	2.0693 "	5.8203 "	4.3883 "	84.378	2.3947 "	3.9386 "
−100.....	3.1069 "	9.2562×10^{-24}	6.9790×10^{-12}	89.378	3.5768 "	7.1897 "
−200.....	4.6771 "	1.0531×10^{-25}	7.9400×10^{-14}	0.67546	5.3544 "	1.3169×10^{18}

Cadmium, $m = 112$

z	k	h	c_1	c_2	E_0	n
200.....	7.4130×10^{-16}	1.4297×10^{-24}	1.0780×10^{-12}	57.859	3.2243×10^{-12}	1.9470×10^{16}
100.....	1.3961×10^{-15}	3.4522 "	2.6029 "	74.177	7.9580 "	1.3862×10^{17}
80.....	1.4332 "	3.6483 "	2.7507 "	76.366	1.4188×10^{-11}	1.7963 "
60.....	1.5095 "	3.9150 "	2.9518 "	77.803	1.1020 "	2.2536 "
40.....	1.6601 "	4.3665 "	3.2922 "	78.906	1.2700 "	2.7946 "
20.....	1.8970 "	5.0824 "	3.8320 "	80.375	1.4588 "	3.4403 "
0.....	2.1827 "	5.9411 "	4.4795 "	81.658	1.6772 "	4.2413 "
−20.....	2.5116 "	6.9680 "	5.2537 "	83.420	1.9237 "	5.2101 "
−40.....	2.8843 "	8.1738 "	6.1629 "	85.016	2.2119 "	6.4231 "
−60.....	3.3145 "	9.6192×10^{-24}	7.2527×10^{-12}	87.066	2.5981 "	7.7193 "
−80.....	3.8135 "	8.0893×10^{-26}	6.0991×10^{-14}	0.63636	4.3769 "	9.7324 "
−100.....	4.3397 "	9.4570 "	7.1303 "	0.65375	5.0254 "	1.1974×10^{18}
−200.....	4.4926 "	9.8138 "	7.3995 "	0.65533	8.5872 "	2.6744 "

Mercury, $m = 198$

z	k	h	c_1	c_2	E_0	n
200.....	2.6278×10^{-17}	2.1953×10^{-26}	1.6552×10^{-14}	25.062	1.8161×10^{-15}	2.6013×10^{11}
100.....	8.1208×10^{-16}	1.4672×10^{-24}	1.1063×10^{-12}	54.203	4.8116×10^{-12}	3.5475×10^{16}
80.....	9.9204 "	1.9254 "	1.4517 "	58.225	7.3663 "	6.7196 "
60.....	1.1787×10^{-15}	2.4217 "	1.8259 "	61.637	1.0431×10^{-11}	1.1323×10^{17}
40.....	1.3882 "	2.9922 "	2.2561 "	64.662	1.4056 "	1.7711 "
20.....	1.6529 "	3.7052 "	2.7936 "	67.248	1.8347 "	2.6473 "
0.....	2.0469 "	4.7212 "	3.5597 "	69.196	2.3549 "	3.8409 "
−20.....	2.6258 "	6.2974 "	4.7481 "	71.948	3.0181 "	5.5729 "
−40.....	3.3720 "	8.3922×10^{-24}	6.3276×10^{-12}	118.33	3.8721 "	8.0980 "
−60.....	4.2269 "	9.2225×10^{-26}	6.9537×10^{-14}	0.65456	4.9454 "	1.1688×10^{18}
−80.....	4.3282 "	9.4758 "	7.1438 "	0.65680	6.1636 "	1.6263 "
−100.....	4.3282 "	9.4488 "	7.1243 "	0.65493	7.3885 "	2.1345 "

quite different from the means just computed. A study of Tables 52 and 53, above and below the radiation level z_R, and an intercomparison of the entire N = constant system with the E_0 = constant system, make it difficult to decide whether one is better than the other, since the divergence is about the same in opposite directions.

TABLE 53

SECOND COMPUTATION OF THE WIEN-PLANCK COEFFICIENTS TAKING N = CONSTANT (6.062×10^{23}) AND E_0 = VARIABLE

Balloon Ascension, Uccle, June 9, 1911

z	k	h	c_1	c_2	E_0	n
70000.....	2.2849×10^{-20}	7.0356×10^{-30}	5.3049×10^{-18}	9.23760	2.4145×10^{-21}	1.6448×10
60000.....	6.8192×10^{-18}	2.1060×10^{-28}	1.5879×10^{-16}	0.92648	2.6598×10^{-16}	4.2395×10^{13}
50000.....	3.6806×10^{-17}	9.1420 "	6.8928 "	0.74515	3.1470×10^{-14}	1.4747×10^{15}
40000.....	4.1715 "	1.0911×10^{-27}	8.2268 "	0.78470	1.0199 "	3.3560×10^{17}
30000.....	4.9893 "	1.2780 "	9.6358 "	0.76844	1.6389 "	1.0788×10^{18}
20000.....	7.9440 "	1.9870 "	1.4982×10^{-15}	0.75040	2.5665 "	3.2545 "
10000.....	1.1963×10^{-16}	2.9803 "	2.2471 "	0.74738	4.0538 "	1.0027×10^{19}
000.....	1.3598 "	3.4419 "	2.5955 "	0.75890	3.9417 "	2.5393 "

Balloon Ascension, Uccle, September 13, 1911

z	k	h	c_1	c_2	E_0	n
80000.....	1.9762×10^{-19}	2.9057×10^{-29}	2.1909×10^{-17}	4.41110	2.9648×10^{-19}	2.2120×10^{6}
70000.....	5.4412×10^{-18}	1.7195×10^{-28}	1.2965×10^{-16}	0.94805	3.6735×10^{-16}	9.0932×10^{13}
60000.....	1.4529×10^{-17}	4.0310 "	3.0393 "	0.83240	1.7435×10^{-15}	4.2047×10^{15}
50000.....	2.9574 "	7.6977 "	5.8039 "	0.78085	4.8800 "	5.2979×10^{16}
40000.....	4.2809 "	1.1260×10^{-25}	8.4898 "	0.82626	1.0661×10^{-14}	3.6261×10^{17}
30000.....	5.0749 "	1.3013 "	9.8118 "	0.76927	1.6976 "	1.1399 "
20000.....	8.0032 "	2.0114 "	1.5165×10^{-15}	0.75397	2.6380 "	3.3742×10^{18}
10000.....	1.1778×10^{-16}	2.9542 "	2.2274 "	0.75265	4.1275 "	1.0157×10^{19}
000.....	1.3606 "	3.4632 "	2.6111 "	0.76357	5.9699 "	2.5191 "

Balloon Ascension, Uccle, November 9, 1911

z	k	h	c_1	c_2	E_0	n
90000.....	7.0927×10^{-20}	9.7702×10^{-30}	7.3665×10^{-18}	4.1325	4.2489×10^{-19}	6.4684×10^{6}
80000.....	1.7830×10^{-18}	6.3581×10^{-29}	4.7939×10^{-17}	1.0698	1.0701×10^{-16}	5.2278×10^{12}
70000.....	6.1459 "	1.8166×10^{-28}	1.3696×10^{-16}	0.88672	6.4536 "	4.3616×10^{14}
60000.....	1.3574×10^{-17}	3.6568 "	2.7571 "	0.80816	2.0368×10^{-15}	7.3798×10^{15}
50000.....	2.7384 "	6.9577 "	5.2459 "	0.76223	4.9301 "	7.2954×10^{16}
40000.....	3.9570 "	1.0123×10^{-27}	7.6325 "	0.76747	1.0033×10^{-14}	3.7383×10^{17}
30000.....	4.7053 "	1.1774 "	8.8774 "	0.75609	1.5904 "	1.1619×10^{18}
20000.....	7.3330 "	1.7982 "	1.3559×10^{-15}	0.73568	2.4552 "	3.3843 "
10000.....	1.1528×10^{-16}	2.8129 "	2.1209 "	0.73202	3.8242 "	1.0098×10^{19}
000.....	1.3606 "	3.3585 "	2.5322 "	0.74048	5.6494 "	2.6326 "

Table 52 summarizes the following results for the N = constant system: The Boltzmann entropy coefficient diminishes with the height, without any per saltum change at the level of radiation z_R; the same is true of the kinetic energy of one molecule E_0, and for the number of molecules per unit volume n. On the other hand, the Planck Wirkungsquantum h,

and the Wien-Planck coefficients all change abruptly at the radiation levels z_R. In making up the mean change for any of the chemical elements it is sufficient to add to the lower term 0.461 (upper . . . lower) instead of taking the direct arithmetical mean. Table 53, for the earth's atmosphere, shows that there is a gradual diminution in the values of h, c_1, c_2, as well as in those for k, E_0, n. The intercomparison between the solar and the terrestrial elements can be readily studied, and applied in any theoretical discussions regarding the laws that govern the processes of radiation.

The foregoing data have been computed directly from the non-adiabatic thermodynamics of the atmospheres of the sun and the earth, and these represent the accomplished effects of radiation upon the gases per unit volume, as summarized in the temperature which is the general parameter. We shall now proceed to give the data relating to the electromagnetic energy more directly, in order to be able to examine the distribution of the radiant energy—that is, its concentration, or its departure from the normal value in the æther, outside the atmospheric gaseous media. It has been explained that the nearly perfect solar black radiation seems to arrive at the levels about 50000 meters above the sea level without much change in its conditions, and that there is a progressive concentration from that height down to the sea level.

NOTE.—The coefficient h is to be regarded as indicating the potential energy controlling the movements in $h\nu$, where ν is the frequency, while $k\tau$ is a general constant in a given atmosphere, and represents the corresponding kinetic energy of a single atom or electron.

CHAPTER V

The Elements of Black Radiation in the Atmospheres of the Sun and the Earth

The Concentrations of Black Radiation in Gaseous Media

We must distinguish clearly between the three stages of the physical conversion of the black body radiation as electromagnetic energy in the æther, outside the gaseous media, and the final effect as temperature of the gaseous media through the kinetic energy of the molecules. In any given volume of the gaseous media, (1) there is absorbed a certain amount of radiant energy, and this is not a variable amount so long as the temperature is steady; (2) this constant amount of energy per volume $\left[\frac{M.L^2}{T^2} \cdot \frac{1}{L^3}\right]$ is maintained by a flux of energy entering and leaving the volume in a constant flow per unit time, with the velocity of light, $\left[\frac{M.L^2}{T^2} \cdot \frac{1}{L^3} \cdot \frac{L}{T}\right] = \left[\frac{M}{T^3}\right]$, from the non-gaseous volumes; (3) there is a marked difference in the volume density of the constant æther-flux through it, and of the volume kinetic energy at different levels in the atmosphere as developed by the computations, having the nature of concentration, or convergence and divergence of the energy, away from the simple black radiation of the æther itself. These changes in concentration are expressions of the operation of many physical processes, measured in the bulk, and although these processes are not understood, it may be of importance to give the corresponding quantities which come immediately from the general formulas of radiation, as summarized by Planck in *Die Theorie der Wärmestrahlung*, Leipzig, 1913. The following computations depend entirely upon the volume density, $u = a\,T^4$, where T is the temperature of the volume, and the volume energy per

deg.[4] is taken from the thermodynamic computations, as in Table 22. The coefficient (c) of that table is identical with σ per unit volume; and $a = \frac{4\sigma}{c}$, this c being velocity of light. This transition from my thermodynamic notation to the Planck notation should cause no confusion. The solar radiation on entering the earth's atmosphere cannot be treated in the simple manner just indicated, because all of the electromagnetic radiant energy is not converted into molecular kinetic energy per unit volume, as determined by the steady temperature that is observed. Rather, the incoming radiation separates into three branches, (1) the absorbed part making temperature; (2) the scattered or reflected part that does not make temperature; (3) the free radiation of high-frequency that penetrates to the sea level, and is there converted into a lower frequency, to return with a further complex absorption and scattering to space. These processes have been indicated in the Treatise, also in Bulletin No. 4, O. M. A. The reader should bear the line of our argument in mind. Having computed $K_{10} = c\,T^{a}$ from the thermodynamic temperature conditions of the atmosphere, the temperature representing the final absorptions of all kinds, where (c) differs from the black body σ, and (a) is different from 4.00, we next passed from (a) to 4.00, retaining the same value of (c), as a variable σ, and computed $u = \frac{4}{c} \cdot \sigma\,T^{4}$, with the other dependent quantities. Our hypothesis consists in supposing that by changing the exponent from its local volume value (a) to the black body value 4.00, we thereby have added to the absorbed radiant energy, first, the part which was scattered before reaching the sea level, and secondly, the remaining free radiation of the heat current. Our procedure is justified by the fact that the thermodynamic data, at the earth and the sun, agree with the simplest interpretation of the bolometer ordinates, and with the interpretation of the pyrheliometric data, which closely identify it with the theory of scattering that is associated with the number of molecules per unit volume, or the local density. The difficulties inherent in these problems

make it necessary to take certain tentative steps in computation, which must be tested by comparison with the facts of observation. If the volume æther energy, $\sigma = \frac{c}{4} a$, of black solar radiation were to be utilized completely in the gaseous media to make temperature, without concentration into or leakage from the perfect mirror enclosure containing the given volume, this process of proceeding from temperature T, and K_{10}, first to σ, and then to $a = \frac{4\sigma}{c}$, would produce the primitive æther volume energy for each unit volume in the column, which is 3×10^{10} centimeters in length. As a matter of fact, the (a) thus computed in $u = a T^4$, does vary from the laboratory standard value, and this must signify that the radiant energy has concentration into or leakage from the gaseous volume, which does not act as if it were perfectly enclosed. The following tables exhibit the results of such computations. If the exact conditions that they represent in the atmospheres of the sun and the earth are understood, they are valuable as representing the departures or concentrations in atmospheres in reference to the standard values of the æther outside of gaseous atmospheres. It may be possible from them to make some further advances toward the fundamental laws of radiation within ordinary gaseous media, whether on the basis of dynamics, or electromagnetics, or the electrons in atoms and molecules.

The Poynting equation summarizes these leakage processes:

$$A = \frac{1}{4\pi} \int\int [E\,H]_n \, dS - \frac{1}{4\pi} \int\int\int \frac{\partial}{\partial t} \left(\tfrac{1}{2} H^2 + \tfrac{1}{2} E^2\right) d\tau.$$

At z_R, $T = 7654.^{0}5$, $\log T^4 = 15.53568$, $\log u = 1.40063$, $u = 2.5155 \times 10$.

It is difficult to extrapolate the values of u, as computed at the assigned heights, to the corresponding values at z_R, in order to compute the mean $u\ (z_R)$.

The ratio factor is obtained directly by dividing the value of u above z_R by the value of u below z_R.

TABLE 54. u = THE SPECIFIC VOLUME DENSITY OF BLACK RADIATION IN THE ATMOSPHERES OF THE SUN.

$$u = aT^4 = \frac{4\pi}{c}\,\mathbf{K} = \frac{8\pi}{c}\,\tilde{K} = \frac{4}{c}\,\sigma\,T^4 = 3p = \frac{3}{4}\,s\,T = \frac{3}{4\pi}\,I, \left[\frac{M}{L\,T^2} = \frac{M\,L^2}{T^2}\,\frac{1}{L^3}\right].$$

	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	
z 25000....	6.8948×10^{-5}	1.3420×10^{-16}							
20000....	2.0869×10^{-4}	4.4146×10^{-8}							
15000....	5.1620 "	5.5001×10^{-6}							
10000....	1.1727×10^{-3}	9.1292×10^{-5}	3.6963×10^{-8}						
5000....	2.1230 "	8.4170×10^{-4}	1.0154×10^{-4}						
1000....	2.6327 "	2.8104×10^{-3}	1.5797×10^{-3}	7.8782×10^{-9}	5.4203×10^{-10}				
800....	2.6526 "	2.8542 "	1.6498 "	1.1009×10^{-3}	1.8330×10^{-6}				
600....	2.6726 "	2.8979 "	1.7197 "	1.5031 "	3.6797×10^{-5}	1.4093×10^{-7}			
400....	2.6926 "	2.9416 "	1.7896 "	1.8053 "	2.4291×10^{-4}	4.0114×10^{-5}			
200....	2.7126 "	2.9853 "	1.8595 "	2.0023 "	9.8828 "	2.9668×10^{-4}	4.4178×10^{-5}	2.4930×10^{-8}	
100....	2.7226 "	3.0072 "	1.8945 "	2.0825 "	1.5476×10^{-3}	1.4172×10^{-3}	5.8929×10^{-4}	2.0262×10^{-4}	
80....	2.7246 "	3.0114 "	1.9014 "	2.0987 "	1.6375 "	1.5312 "	9.9657 "	4.9247 "	
60....	2.7266 "	3.0158 "	1.9084 "	2.1147 "	1.7275 "	1.6452 "	1.4855×10^{-3}	9.9541 "	
40....	2.7286 "	3.0202 "	1.9154 "	2.1307 "	1.8174 "	1.7592 "	1.8888 "	1.7397×10^{-3}	
20....	2.7306 "	3.0246 "	1.9224 "	2.1467 "	1.9074 "	1.8732 "	2.0853 "	2.9627 "	
Photos. 0....	2.7326 "	3.0290 "	1.9294 "	2.1627 "	1.9974 "	1.9872 "	2.2813 "	3.4898 "	2.4512×10^{-3}
−20....	2.7349 "	3.0337 "	1.9349 "	2.1842 "	2.0538 "	2.0828 "	2.4474 "	3.9677 "	
−40....	2.7371 "	3.0386 "	1.9404 "	2.2059 "	2.1103 "	2.1782 "	2.6495 "	4.5411×10^{-3}	
−60....	2.7393 "	3.0435 "	1.9459 "	2.2276 "	2.1668 "	2.2736 "	2.8221×10^{-3}	9.1048×10^{3}	
−80....	2.7415 "	3.0484 "	1.9514 "	2.2493 "	2.2233 "	2.3690 "	8.2790×10^{3}	2.0252×10^{4}	
−100....	2.7437 "	3.0533 "	1.9569 "	2.2710 "	2.2798 "	2.4644×10^{-3}	9.0045 "	4.2181 "	
−200....	2.7548 "	3.0775 "	1.9844 "	2.3793 "	2.6524×10^{-3}	8.7074×10^{3}	6.8687×10^{4}		
−400....	2.7772 "	3.1261 "	2.0393 "	2.5513 "	1.4067×10^{4}	8.9500×10^{4}			
−600....	2.7996 "	3.1747 "	2.0942 "	2.7956×10^{-3}	6.1840 "				
−800....	2.8220 "	3.2233 "	2.1491 "	9.6910×10^{3}	1.8228×10^{5}				
−1000....	2.8444 "	3.2719 "	2.2040 "	1.0031×10^{4}	4.2679 "				
−2000....	2.9606 "	3.6537	2.4734×10^{-3}	6.4657 "	6.2317×10^{6}				
−4000....	3.1395 "	4.3468×10^{-3}	1.6179×10^{4}						
−6000....	3.3733 "	2.7373	6.3174 "						
−8000....	3.5388×10^{-3}	3.6028							
−10000....	6.8268×10^{3}	6.3869							
−12000....	7.0357 "	11.2300							
−14000....	1.0492×10^{4}	18.7420							
z_R.....	−9000	−4500	−2250	−750	−225	−140	−80	−45	
Ratios.....	5.1838	.....	1.5287	2.8847	1.8856	2.8302	3.4088	4.9876	3.2442×10^{-7}
Diatomic hydrogen									1.5880 $\times10^{-3}$

TABLE 55

VALUES OF $u = aT^4$ IN THE EARTH'S ATMOSPHERE

1911	June 9	Sept. 13	Nov. 9	Means	T	log T	log T^4	log u	u	Factor
z 90000	—	—	4.1993×10^{-17}	—	0	—	—	—	—	
80000	—	2.8025×10^{-14}	6.0839×10^{-10}	—	20	1.30103	5.20412	−9.06906	1.1724×10^{-9}	
70000	*7.5693×10^{-20}	4.2735×10^{-9}	3.4542×10^{-8}	—	58	1.76343	7.05372	−8.91866	8.2920×10^{-8}	
60000	*6.3947×10^{-10}	1.6838×10^{-7}	4.1970×10^{-7}	2.9404×10^{-7}	90	1.95424	7.81696	−8.68190	4.8073 "	6.116
50000	*1.5326×10^{-7}	1.4842×10^{-6}	2.0925×10^{-6}	1.7884×10^{-6}	115	2.06070	8.24280	−6.10774	1.2816×10^{-6}	1.396
40000	9.9472×10^{-6}	1.0799×10^{-5}	1.1655×10^{-5}	1.1227×10^{-5}	167	2.22272	8.89088	−6.75582	5.6993 "	4.773
30000	4.1276×10^{-5}	4.4992 "	4.6771 "	4.4346 "	222	2.34635	9.38540	−5.25034	1.7796×10^{-5}	2.492
20000	6.6060 "	7.1132 "	7.4593 "	7.0595 "	219	2.34044	9.36176	−5.22670	1.6854 "	4.188
10000	1.2179×10^{-4}	1.3446×10^{-4}	1.1463×10^{-4}	1.2363×10^{-4}	227	2.35603	9.42412	−5.28906	1.9501 "	6.340
100	3.5788 "	3.6514 "	3.2108 "	3.4803 "	287	2.45788	9.83152	−5.69646	4.9712 "	7.001

*Omitted from the means. Computed with log $a = -15.86494$.

The balloon volume density is about 4.63 times that of the laboratory (a). The former concentrates by this factor, but it is compensated by lowering the exponent from 4.00 to 3.80.

The computations for $u = a\,T^4 = \frac{c}{4}.\,\sigma\,T^4$ are as follows: From Table 6 take the temperature T of the element at the height indicated, and compute log T^4; from Table 22 take log c in the (M. K. S.) system, and convert it into log σ in the (C. G. S.) system by the factor 10; multiply σ by $\frac{c}{4} = 1.3333 \times 10^{-10}$, $[\log \frac{c}{4} - 10.12494]$, to produce ($a$); finally, compute the volume density u. There are several remarks regarding the values of u in the solar atmosphere, Table 54, and in the terrestrial atmosphere, Table 55. Above the transition level z_R the exponent in $u = a\,T^4$ resulted at about the Stefan black body value, 4.00, as in Tables 18–22, so that the low-efficiency values of Table 54 are those which actually exist. The observed thermal efficiency of the inner corona during eclipses has been singularly low. Below z_R the exponent averages about 2.420, Table 24, so that the actual efficiency passes through the radiation levels with a simple gradient. The potential efficiency is, however, very great below z_R, as is seen from Table 54 by using the exponent 4.00. The imprisoned potential radiation changes abruptly by a factor whose mean value is,

$$\text{Factor of radiation} = 3.2442 \times 10^{-7} \text{ (monatomic)}$$
$$\text{“ “} = 1.5880 \times 10^{-3} \text{ (diatomic)}$$

This is not quite correct, because the values of u should be extrapolated to the exact heights z_R, and this it is difficult to do practically without additional computations.

The mean value of u on the photosphere is $u_P = 2.4512 \times 10^{-3}$.

In the earth's atmosphere the first section of Table 55 gives a few values computed from balloon ascensions and the mean values up to 60000 meters; in the second section the values are computed from the mean temperatures T, and log a =

$-$ 15.86494 from Table 26. It is noted that the balloon and the laboratory values coincide at about 50000 meters, but that the former concentrate sevenfold at the sea level.

The incoming solar radiation, therefore, undergoes a process of concentration in the successive volumes from 50000 meters to the sea level, until it becomes 4.63 times as dense per unit volume as the density computed directly from the laboratory value, from $\log a = -15.86494$ and the exponent 4.00. This result should not be misinterpreted. For with the concentration of the coefficient there is a diminution of the exponent, from 4.00 to 3.80, so that the effective $u_a = \frac{4\sigma}{c} T^{3.80}$ remains about the same as if there had been no concentration of the coefficient. Take an example from the isothermal region, for $T = 220°$ and the exponent 3.72, computing for both cases:

Black radiation, $u = [-15.86497] \times 220^{4.00} = 1.716 \times 10^{-5}$;

Concentrated coefficient with smaller exponent, $u = [-15.86497] \times 4.63 \times 220^{3.72} = 1.754 \times 10^{-5}$.

A very slight adjustment would equalize those results. Similar conditions prevail on the sun. Take the data from Table 24, for $T = 7655°$.

Nearly black radiation, $u = [-9.81409] \times \frac{4}{c} \times 7655^{4.053} = 4.795 \times 10^{-3}$;

Concentrated coefficient with smaller exponent, $u = [-2.24019] \times \frac{4}{c} \times 7655^{2.411} = 5.362 \times 10^{-3}$.

Here, again, small adjustments would produce an equality. This concentration at the sun is [6.42610], or conversely, the diminution of the coefficient is $[-7.57390] = 3.749 \times 10^{-7}$, while that given on Table 54, which is not quite accurate, is 3.2442×10^{-7}. This remarkable interplay between coefficient and exponent we have interpreted as connected with radiation in the solar atmosphere at the levels z_R, with further scattering in the upper levels, amounting to an equivalent 1.87 gr. cal./cm.^{2} min.

TABLE 56. p = THE TOTAL PRESSURE OF BLACK RADIATION PER UNIT VOLUME IN THE SUN'S GASES.

$$p = \frac{1}{3}u = \frac{1}{3}aT^4 = \frac{4}{3c}\sigma T^4 = \frac{sT}{4} = \frac{PVu}{2(U_1 - U_0)} = \frac{PU}{3KT} = \frac{4\pi}{3c}\mathbf{K} = \frac{I}{4\pi}, \left[\frac{ML^2}{T^2}\cdot\frac{1}{L^3}\right].$$

		H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	
z	25000....	2.2983×10^{-5}	4.4732×10^{-17}							
	20000....	6.9563 "	1.4715×10^{-9}							
	15000....	1.7207×10^{-4}	1.8334×10^{-6}							
	10000....	3.9089 "	3.0431×10^{-5}	1.2321×10^{-8}						
	5000....	7.0768 "	2.8057×10^{-4}	3.3848×10^{-5}						
	1000....	8.7756 "	9.3680×10^{-4}	5.2657×10^{-4}	2.6261×10^{-4}	1.8066×10^{-10}				
	800....	8.8424 "	9.5130 "	5.4989 "	3.6696 "	6.1099×10^{-7}				
	600....	8.9090 "	9.6590 "	5.5320 "	5.0103 "	1.2266×10^{-5}	4.6977×10^{-8}			
	400....	8.9756 "	9.8050 "	5.9651 "	6.0173 "	8.0970 "	1.3371×10^{-5}			
	200....	9.0422 "	9.9510 "	6.1982 "	6.6742 "	3.2943×10^{-4}	1.7611×10^{-4}	1.4726×10^{-5}	8.3098×10^{-9}	
	100....	9.0755 "	1.0024×10^{-3}	6.3148 "	6.9415 "	5.1471 "	4.7242 "	1.9643×10^{-4}	6.7540×10^{-5}	
	80....	9.0820 "	1.0037 "	6.3381 "	6.9951 "	5.4493 "	5.1041 "	3.3219 "	1.6416×10^{-4}	
	60....	9.0887 "	1.0052 "	6.3614 "	7.0485 "	5.7515 "	5.4841 "	4.9517 "	3.3181 "	
	40....	9.0954 "	1.0067 "	6.3847 "	7.1019 "	6.0537 "	5.8641 "	6.2960 "	5.7990 "	
	20....	9.1021 "	1.0082 "	6.4080 "	7.1553 "	6.3559 "	6.2441 "	6.9512 "	9.8755 "	
Photos.	0....	9.1088 "	1.0097 "	6.4313 "	7.2087 "	6.6581 "	6.6241 "	7.6045 "	1.1633×10^{-3}	8.1706×10^{-4}
	−20....	9.1164 "	1.0114 "	6.4497 "	7.2811 "	6.8462 "	6.9423 "	8.1580 "	1.3226 "	
	−40....	9.1238 "	1.0130 "	6.4680 "	7.3533 "	7.0345 "	7.2604 "	8.8318 "	1.5137×10^{-3}	
	−60....	9.1312 "	1.0146 "	6.4863 "	7.4255 "	7.2228 "	7.5785 "	9.4070×10^{-4}	3.0349×10^{-3}	
	−80....	9.1386 "	1.0162 "	6.5046 "	7.4977 "	7.4111 "	7.8966 "	2.7597×10^{3}	6.7507 "	
	−100....	9.1460 "	1.0178 "	6.5229 "	7.5699 "	7.5994 "	8.2147×10^{-4}	3.0015 "	1.4060×10^{4}	
	−200....	9.1832 "	1.0258 "	6.6145 "	7.9310 "	8.8422×10^{-4}	2.9025×10^{3}	2.2895×10^{4}		
	−400....	9.2577 "	1.0420 "	6.7975 "	8.5044 "	4.6889×10^{3}	3.1260×10^{4}			
	−600....	9.3322 "	1.0582 "	6.9805 "	9.3188×10^{-4}	2.0613×10^{4}				
	−800....	9.4067 "	1.0744 "	7.1635 "	3.2304×10^{3}	6.0763 "				
	−1000....	9.4812 "	1.0906 "	7.3465 "	3.3437 "	1.4226×10^{5}				
	−2000....	9.8687 "	1.2179 "	8.2444×10^{-4}	2.1553×10^{4}	2.0772×10^{6}				
	−4000....	1.0465×10^{-3}	1.4489×10^{-5}	5.3931×10^{3}						
	−6000....	1.1244 "	0.91243	2.1058×10^{4}						
	−8000....	1.1796×10^{-3}	1.2009							
	−10000....	2.2756×10^{3}	2.1290							
	−12000....	2.3453 "	3.7433							
	−14000....	3.4960 "	6.2474							
z_R.....		−9000	−4500	−2250	−750	−225	−140	−80	−45	
Ratios.....		5.1838	.....	1.5287	2.8845	1.8858	2.8301	3.4087	4.9876	3.2442×10^{-7}
Diatomic hydrogen.....										1.5880 $\times10^{-3}$

at the distance of the earth. Similarly, in the earth's atmosphere, we retained the coefficient unchanged, but by making the exponent 4.00 in place of that found in the pair value (log $c.\ \alpha$) we passed from the selective absorption J_a to the black J_0, which added the amount scattered and the free heat, since $J_a = 1.46$ and $J_0 = 3.95$ calories.

The pressure of the radiation energy per unit volume is taken $p = \frac{1}{3}u$, so that all the conditions that were described under u can be transferred to p. The factor of transition at z_R is the same; p (photosphere) $= 8.1707 \times 10^{-4}$.

TABLE 57

VALUES OF p IN THE EARTH'S ATMOSPHERE

1911	June 9	Sept. 13	Nov. 9	Means
z 90000...	—	—	1.3998×10^{-17}	—
80000...	—	9.3418×10^{-13}	2.0280×10^{-10}	—
70000...	*2.5231×10^{-20}	1.4245×10^{-9}	1.1514×10^{-8}	—
60000...	*2.1316×10^{-10}	5.6128×10^{-8}	1.3990×10^{-7}	9.8014×10^{-8}
50000...	*5.1086×10^{-8}	4.9474×10^{-7}	6.9750 "	5.9613×10^{-7}
40000...	3.3158×10^{-6}	3.5993×10^{-6}	3.8849×10^{-6}	3.7423×10^{-6}
30000...	1.3759×10^{-5}	1.4997×10^{-5}	1.5590×10^{-5}	1.4782×10^{-5}
20000...	2.2020 "	2.3711 "	2.4864 "	2.3532 "
10000...	4.0597 "	4.4819 "	3.8207 "	4.1210 "
100...	1.1929×10^{-4}	1.2171×10^{-4}	1.0703×10^{-4}	1.1601×10^{-4}

* Omitted from the means.

This pressure probably is due to the kinetic movements of the electrons in the volume as distinguished from P, the hydrostatic pressure in the gas due to the kinetic energy of the atoms and molecules. The formulas give numerous interrelations between the electronic and the molecular energies.

Nichols and Hull give the radiation pressure 7.01×10^{-5} $\frac{\text{erg.}}{\text{cm.}^3}$ or $\frac{\text{dynes}}{\text{cm.}^2}$.

If A = the coefficient of absorption, and I = constant emissive black energy, for all bodies by Kirchhoff's Law, then $E = A \,.\, I$ = coefficient of emission. $\left(\frac{E}{A} - E\right) = I\,(1 - A) =$

TABLE 58. I = THE ENERGY OF BLACK RADIATION = $\frac{E}{A}$.

$$I = \frac{4\pi}{3} u = \frac{16\pi^2}{3c} \mathbf{K} = \frac{32\pi^2}{3c} \tilde{K} = \frac{cF}{2} = \frac{c}{2}(F_e + F_m), \left[\frac{ML^2}{T^2} \cdot \frac{1}{L^3}\right].$$

		H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	
z	25000....	2.8881×10^{-4}	5.6213×10^{-16}							
	20000....	8.7416 "	1.8492×10^{-7}							
	15000....	2.1623×10^{-3}	2.3039×10^{-5}							
	10000....	4.9121 "	3.8240×10^{-4}	1.5483×10^{-7}						
	5000....	8.8930 "	3.5258×10^{-3}	4.2534×10^{-4}						
	1000....	1.1027×10^{-2}	1.1772×10^{-2}	6.6171×10^{-3}	3.3001×10^{-3}	2.2704×10^{-9}				
	800....	1.1110 "	1.1956 "	6.9102 "	4.6113 "	7.6778×10^{-6}				
	600....	1.1194 "	1.2139 "	7.2031 "	6.2961 "	1.5414×10^{-4}	5.9035×10^{-7}			
	400....	1.1278 "	1.2322 "	7.4960 "	7.5620 "	1.0175×10^{-3}	1.6803×10^{-4}			
	200....	1.1362 "	1.2505 "	7.7889 "	8.3870 "	4.1379 "	2.2130×10^{-3}	1.8505×10^{-4}	1.0442×10^{-7}	
	100....	1.1404 "	1.2597 "	7.9354 "	8.7229 "	6.4829 "	5.9364 "	2.4684×10^{-3}	8.4873×10^{-4}	
	80....	1.1413 "	1.2616 "	7.9646 "	8.7900 "	6.8596 "	6.4138 "	4.1745 "	2.0628×10^{-3}	
	60....	1.1422 "	1.2634 "	7.9939 "	8.8572 "	7.2364 "	6.8914 "	6.2224 "	4.1696 "	
	40....	1.1430 "	1.2652 "	8.0232 "	8.9244 "	7.6132 "	7.3690 "	7.9117 "	7.2872 "	
	20....	1.1438 "	1.2670 "	8.0525 "	8.9916 "	7.9900 "	7.8466 "	8.7352 "	1.2409×10^{-2}	
Photos.	0....	1.1446 "	1.2688 "	8.0818 "	9.0588 "	8.3668 "	8.3242 "	9.5560 "	1.4618 "	10.2675×10^{-3}
	−20....	1.1456 "	1.2709 "	8.1048 "	9.1495 "	8.6033 "	8.7238 "	1.0252×10^{-2}	1.6620 "	
	−40....	1.1465 "	1.2730 "	8.1278 "	9.2403 "	8.8399 "	9.1236 "	1.1098 "	1.9022×10^{-2}	
	−60....	1.1474 "	1.2750 "	8.1508 "	9.3311 "	9.0765 "	9.5234 "	1.1821×10^{-2}	3.8138×10^{4}	
	−80....	1.1483 "	1.2770 "	8.1738 "	9.4219 "	9.3131 "	9.9232 "	3.4679×10^{4}	8.4832 "	
	−100....	1.1492 "	1.2790 "	8.1968 "	9.5127 "	9.5497 "	1.0323×10^{-2}	3.7718 "	1.7669×10^{5}	
	−200....	1.1538 "	1.2893 "	8.3118 "	9.9665 "	1.1112×10^{-2}	3.6474×10^{4}	2.8771×10^{5}		
	−400....	1.1632 "	1.3096 "	8.5418 "	1.0687×10^{-2}	5.2515×10^{4}	3.0900×10^{5}			
	−600....	1.1726 "	1.3299 "	8.7718 "	1.1710×10^{-2}	2.5904×10^{5}				
	−800....	1.1820 "	1.3502 "	9.0018 "	4.0594×10^{4}	7.7150 "				
	−1000....	1.1914 "	1.3705 "	9.2318 "	4.2018 "	1.6684×10^{5}				
	−2000....	1.2402 "	1.5304 "	1.0360×10^{-2}	2.7084×10^{5}					
	−4000....	1.3151 "	1.8204×10^{-2}	6.7772×10^{4}						
	−6000....	1.4130 "	11.466	2.6463×10^{5}						
	−8000....	1.4823×10^{-2}	15.091							
	−10000....	2.8596×10^{4}	26.754							
	−12000....	2.9471 "	47.039							
	−14000....	4.3950 "	78.508							
z_R		−9000	−4500	−2250	−750	−225	−140	−80	−45	
Ratios		5.1838	.....	1.5287	2.8847	1.8856	2.8302	3.4088	4.9875	3.2442×10^{-7}
Diatomic hydrogen										1.5880×10^{-3}

the radiation transmitted. Hence,

$$I = E + I - A\,I \tag{148}$$

and the emission equals the absorption for any other gaseous medium. Since I can be computed from u in all thermodynamic atmospheres, it becomes useful in discussing the side of the general problem that pertains to the electromagnetic and the electron terms.

TABLE 59

VALUES OF I IN THE EARTH'S ATMOSPHERE

1911	June 9	Sept. 13	Nov. 9	Means
z 90000...	—	—	1.7590×10^{-16}	—
80000...	—	1.1739×10^{-13}	2.5484×10^{-9}	—
70000...	*3.1706×10^{-19}	1.7901×10^{-8}	1.4469×10^{-7}	—
60000...	*2.6786×10^{-9}	7.0533×10^{-7}	1.7580×10^{-6}	1.2633×10^{-6}
50000...	*6.4196×10^{-7}	6.2171×10^{-6}	8.7650 "	7.4911 "
40000...	4.1667×10^{-5}	4.5234×10^{-5}	4.8819×10^{-5}	4.3240×10^{-5}
30000...	1.7290×10^{-4}	1.8847×10^{-4}	1.9592×10^{-4}	1.8576×10^{-4}
20000...	2.7671 "	2.9795 "	3.1246 "	2.9571 "
10000...	5.1016 "	5.6321 "	4.8017 "	5.1785 "
100...	1.4991×10^{-3}	1.5295×10^{-3}	1.3450×10^{-3}	1.4579×10^{-3}

* Omitted from the means.

The energy of black radiation on the photosphere is about the same as in the earth's atmosphere at 40000 m. The energy at z_R goes through an abrupt change.

p = the dynamical pressure of the radiation enclosed in a perfectly reflecting vessel; F = the pressure exerted mechanically by the electromagnetic wave front. This is the Maxwell pressure of light, verified by Lebedew, Nichols, and Hull. These are related, $F = \frac{8\pi}{c}\,p$.

The equivalent light pressure in the sun to that in the earth's atmosphere is near the top of the respective gases. This pressure diminishes very rapidly on approaching the vanishing planes.

TABLE 60. F = THE MECHANICAL FORCE = $(Fe + Fm)$ OF RADIATION PRESSURE.

$$F = \frac{2}{c} I = \frac{8\pi}{3c} u = \frac{32\pi^2}{3c^2} \mathbf{K} = \frac{64\pi^2}{3c^2} \widetilde{K}, = \frac{8\pi}{c} P, \left[\frac{M L^2}{T^2} \frac{1}{L^3}\right].$$

	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	
z 25000....	1.9254×10^{-14}	3.7475×10^{-26}							
20000....	5.8277×10^{-14}	1.2328×10^{-17}							
15000....	1.4416×10^{-13}	1.5359×10^{-15}							
10000....	3.2748×10^{-13}	2.5494×10^{-14}	1.0322×10^{-17}						
5000....	5.9287×10^{-13}	2.3505×10^{-13}	2.8356×10^{-15}						
1000....	7.3518×10^{-13}	7.8482×10^{-13}	4.4114×10^{-13}	2.2000×10^{-13}	1.5136×10^{-19}				
800....	7.4078×10^{-13}	7.9702×10^{-13}	4.6067×10^{-13}	3.0743×10^{-13}	5.1186×10^{-16}				
600....	7.4636×10^{-13}	8.0923×10^{-13}	4.8020×10^{-13}	4.1974×10^{-13}	1.0276×10^{-14}	3.9356×10^{-17}			
400....	7.5194×10^{-13}	8.2144×10^{-13}	4.9973×10^{-13}	5.0414×10^{-13}	6.7833×10^{-13}	1.1202×10^{-14}			
200....	7.5752×10^{-13}	8.3365×10^{-13}	5.1926×10^{-13}	5.5914×10^{-13}	2.7598×10^{-13}	1.4754×10^{-13}	1.2337×10^{-14}	6.9617×10^{-18}	
100....	7.6031×10^{-13}	8.3976×10^{-13}	5.2903×10^{-13}	5.8153×10^{-13}	4.3220×10^{-13}	3.9575×10^{-13}	1.6456×10^{-13}	5.6583×10^{-14}	
80....	7.6086×10^{-13}	8.4098×10^{-13}	5.3099×10^{-13}	5.8599×10^{-13}	4.5731×10^{-13}	4.2759×10^{-13}	2.7830×10^{-13}	1.3753×10^{-13}	
60....	7.6142×10^{-13}	8.4220×10^{-13}	5.3294×10^{-13}	5.9047×10^{-13}	4.8243×10^{-13}	4.5943×10^{-13}	4.1483×10^{-13}	2.7798×10^{-13}	
40....	7.6198×10^{-13}	8.4342×10^{-13}	5.3489×10^{-13}	5.9495×10^{-13}	5.0755×10^{-13}	4.9127×10^{-13}	5.2745×10^{-13}	4.8581×10^{-13}	
20....	7.6254×10^{-13}	8.4464×10^{-13}	5.3684×10^{-13}	5.9943×10^{-13}	5.3267×10^{-13}	5.2311×10^{-13}	5.8234×10^{-13}	8.2730×10^{-13}	
Photos. 0....	7.6310×10^{-13}	8.4586×10^{-13}	5.3879×10^{-13}	6.0391×10^{-13}	5.5779×10^{-13}	5.5495×10^{-13}	6.3707×10^{-13}	9.7456×10^{-13}	6.8450×10^{-13}
−20....	7.6374×10^{-13}	8.4720×10^{-13}	5.4029×10^{-13}	6.1000×10^{-13}	5.7357×10^{-13}	5.8160×10^{-13}	6.8344×10^{-13}	1.1080×10^{-12}	
−40....	7.6436×10^{-13}	8.4856×10^{-13}	5.4179×10^{-13}	6.1605×10^{-13}	5.8934×10^{-13}	6.0825×10^{-13}	7.3990×10^{-13}	1.2681×10^{-12}	
−60....	7.6498×10^{-13}	8.4992×10^{-13}	5.4330×10^{-13}	6.2209×10^{-13}	6.0511×10^{-13}	6.3490×10^{-13}	7.8805×10^{-13}	2.5425×10^{-6}	
−80....	7.6560×10^{-13}	8.5128×10^{-13}	5.4481×10^{-13}	6.2813×10^{-13}	6.2088×10^{-13}	6.6155×10^{-13}	2.3120×10^{-6}	5.6555×10^{-13}	
−100....	7.6622×10^{-13}	8.5264×10^{-13}	5.4632×10^{-13}	6.3417×10^{-13}	6.3665×10^{-13}	6.8820×10^{-13}	2.5145×10^{-13}	1.1779×10^{-5}	
−200....	7.6934×10^{-13}	8.5942×10^{-13}	5.5384×10^{-13}	6.6443×10^{-13}	7.4076×10^{-13}	2.4316×10^{-6}	1.9181×10^{-5}		
−400....	7.7558×10^{-13}	8.7299×10^{-13}	5.6889×10^{-13}	6.9624×10^{-13}	3.9282×10^{-5}	2.5700×10^{-5}			
−600....	7.8182×10^{-13}	8.8656×10^{-13}	5.8394×10^{-13}	7.8070×10^{-13}	1.7269×10^{-13}				
−800....	7.8806×10^{-13}	9.0013×10^{-13}	5.9899×10^{-13}	2.7062×10^{-5}	5.0909×10^{-13}				
−1000....	7.9430×10^{-13}	9.1370×10^{-13}	6.1404×10^{-13}	2.8012×10^{-13}	1.1782×10^{-13}				
−2000....	8.2676×10^{-13}	1.0203×10^{-12}	6.9068×10^{-13}	1.8056×10^{-13}	1.7402×10^{-3}				
−4000....	8.7672×10^{-13}	1.2138×10^{-12}	4.5181×10^{-5}						
−6000....	9.4202×10^{-13}	7.6446×10^{-10}	1.7642×10^{-13}						
−8000....	9.8824×10^{-13}	1.0061×10^{-9}							
−10000....	1.9064×10^{-6}	1.7836×10^{-13}							
−12000....	1.9648×10^{-13}	3.1359×10^{-13}							
−14000....	2.9300×10^{-13}	5.2339×10^{-13}							
z_R.....	−9000	−4500	−2250	−750	−225	−140	−80	−45	
Ratios.....	5.1838	.....	1.5287	2.8847	1.8856	2.8302	3.4088	4.9876	3.2442×10^{-7}
Diatomic hydrogen.....									1.5880×10^{-3}

TABLE 61

VALUES OF F IN THE EARTH'S ATMOSPHERE

1911	June 9	Sept. 13	Nov. 9	Means
z 90000...	—	—	1.1727×10^{-26}	—
80000...	—	7.8260×10^{-24}	1.6990×10^{-19}	—
70000...	*2.1138×10^{-29}	1.1934×10^{-18}	9.6458×10^{-18}	—
60000...	*1.7858×10^{-19}	4.7022×10^{-17}	1.1720×10^{-16}	8.2111×10^{-17}
50000...	*4.2798×10^{-17}	4.1448×10^{-16}	5.8433 "	4.9941×10^{-16}
40000...	2.7778×10^{-15}	3.0157×10^{-15}	3.2546×10^{-15}	3.0160×10^{-15}
30000...	1.1527×10^{-14}	1.2564×10^{-14}	1.3061×10^{-14}	1.2617×10^{-14}
20000...	1.8448 "	1.9864 "	2.0830 "	1.9714 "
10000...	3.4011 "	3.7548 "	3.2011 "	3.4523 "
100...	9.9940 "	1.0197×10^{-13}	8.9664 "	9.7191 "

* Omitted from the means.

The Nichols and Hull value of p gives $F = \frac{8\pi}{c} \times 7.01 \times 10^{-5}$ $= 5.8727 \times 10^{-14}$ and this occurs near the top of the isothermal layer in the earth's atmosphere.

From the differential equation, $\left(\frac{dS}{dV}\right)_T = \frac{4}{3}\frac{u}{T} = \frac{4}{3}\cdot\frac{aT^4}{T} =$ $\frac{4}{3} a T^3$, it follows that the specific entropy per unit volume per degree is $s = \frac{4}{3} a T^3$. The ratio at z_R is 3.3012×10^{-7}, and this is equivalent to using a temperature 7690°.2. It should be noted that the structure of the spectrum depending upon $\frac{\partial s}{\partial u} = \frac{1}{T}$ (Planck, 135) refers to the pure æther enclosed with matter in an adiabatic volume, and it is not directly applicable to gaseous atmospheres open to space, wherein the term for work dW cannot be omitted, and $dW = P\,dv$ assumed negligible.

Planck has developed two cases for the relations $\frac{\partial L}{\partial K} = \frac{\partial s}{\partial u}$:

(149) I. (135), $\frac{\partial L}{\partial K} = \frac{\partial s}{\partial u} = \frac{1}{T}$; II. (81), $L = s.\frac{c}{4\pi} =$ $\frac{c\,a}{3\pi} T^3$, and (76)

TABLE 62. s = SPECIFIC ENTROPY OF BLACK RADIATION PER DEGREE.

$$s = \frac{4}{3}\frac{u}{T} = \frac{4}{3} a T^3 = \frac{4p}{T} = \frac{16\pi}{3cT}\mathbf{K} = \frac{I}{\pi T}, \left[\frac{M L^2}{T^2}, \frac{1}{L^3}, \frac{1}{\text{deg}}\right].$$

	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	
z 25000	2.7686×10^{-8}	2.2366×10^{-18}							
20000	5.5201×10^{-8}	6.1634×10^{-11}							
15000	1.2816×10^{-7}	3.5685×10^{-9}							
10000	2.3690×10^{-7}	3.4827×10^{-8}	6.6598×10^{-11}						
5000	3.7495×10^{-7}	1.9059×10^{-7}	3.5817×10^{-8}						
1000	4.5718×10^{-7}	4.8791×10^{-7}	2.7714×10^{-7}	1.6286×10^{-7}	2.4089×10^{-12}				
800	4.6081×10^{-7}	4.9562×10^{-7}	2.8848×10^{-7}	2.0969×10^{-7}	1.6293×10^{-9}				
600	4.6443×10^{-7}	5.0335×10^{-7}	2.9983×10^{-7}	2.6721×10^{-7}	1.6632×10^{-8}	2.9360×10^{-10}			
400	4.6805×10^{-7}	5.1108×10^{-7}	3.1118×10^{-7}	3.1199×10^{-7}	6.8908×10^{-7}	1.7142×10^{-8}			
200	4.7167×10^{-7}	5.1881×10^{-7}	3.2253×10^{-7}	3.4648×10^{-7}	1.9378×10^{-7}	1.1859×10^{-7}	2.0311×10^{-8}	6.0434×10^{-11}	
100	4.7348×10^{-7}	5.2268×10^{-7}	3.2821×10^{-7}	3.6060×10^{-7}	2.7514×10^{-7}	2.5194×10^{-7}	1.3784×10^{-7}	6.8393×10^{-8}	
80	4.7381×10^{-7}	5.2346×10^{-7}	3.2935×10^{-7}	3.6344×10^{-7}	2.8925×10^{-7}	2.7023×10^{-7}	2.0133×10^{-7}	1.3265×10^{-7}	
60	4.7417×10^{-7}	5.2423×10^{-7}	3.3049×10^{-7}	3.6626×10^{-7}	3.0335×10^{-7}	2.8853×10^{-7}	2.7132×10^{-7}	2.2495×10^{-7}	
40	4.7453×10^{-7}	5.2500×10^{-7}	3.3162×10^{-7}	3.6908×10^{-7}	3.1745×10^{-7}	3.0683×10^{-7}	3.2768×10^{-7}	3.4364×10^{-7}	
20	4.7493×10^{-7}	5.2577×10^{-7}	3.3275×10^{-7}	3.7190×10^{-7}	3.3155×10^{-7}	3.2513×10^{-7}	3.6156×10^{-7}	5.3378×10^{-7}	
Photos. 0	4.7529×10^{-7}	5.2654×10^{-7}	3.3388×10^{-7}	3.7472×10^{-7}	3.4565×10^{-7}	3.4343×10^{-7}	3.9383×10^{-7}	6.0666×10^{-7}	4.2500×10^{-7}
−20	4.7568×10^{-7}	5.2741×10^{-7}	3.3485×10^{-7}	3.7852×10^{-7}	3.5562×10^{-7}	3.6035×10^{-7}	4.2503×10^{-7}	6.9035×10^{-7}	
−40	4.7605×10^{-7}	5.2827×10^{-7}	3.3582×10^{-7}	3.8233×10^{-7}	3.6560×10^{-7}	3.7729×10^{-7}	4.6066×10^{-7}	7.9090×10^{-7}	
−60	4.7642×10^{-7}	5.2913×10^{-7}	3.3679×10^{-7}	3.8614×10^{-7}	3.7558×10^{-7}	3.9423×10^{-7}	4.9120×10^{-7}	1.5564	
−80	4.7679×10^{-7}	5.2999×10^{-7}	3.3776×10^{-7}	3.8995×10^{-7}	3.8556×10^{-7}	4.1117×10^{-7}	1.4426	2.8312	
−100	4.7716×10^{-7}	5.3085×10^{-7}	3.3873×10^{-7}	3.9376×10^{-7}	3.9554×10^{-7}	4.2811×10^{-7}	1.5551	4.9419	
−200	4.7903×10^{-7}	5.3515×10^{-7}	3.4358×10^{-7}	4.1280×10^{-7}	4.0142×10^{-7}	1.5078	7.1870		
−400	4.8275×10^{-7}	5.4374×10^{-7}	3.5328×10^{-7}	4.4321×10^{-7}	2.1812	9.1200			
−600	4.8647×10^{-7}	5.5233×10^{-7}	3.6298×10^{-7}	4.8630×10^{-7}	6.6393				
−800	4.9019×10^{-7}	5.6092×10^{-7}	3.7268×10^{-7}	1.6887	14.802				
−1000	4.9391×10^{-7}	5.6951×10^{-7}	3.8238×10^{-7}	1.7506	28.394				
−2000	5.1325×10^{-7}	6.3581×10^{-7}	4.3023×10^{-7}	7.1997	212.66				
−4000	5.4496×10^{-7}	7.5858×10^{-7}	2.2825						
−6000	5.8633×10^{-7}	4.7614×10^{-4}	6.3544						
−8000	6.1589×10^{-7}	6.0046×10^{-7}							
−10000	1.1895	9.2692×10^{-7}							
−12000	1.2199	1.4178×10^{-3}							
−14000	1.6506	2.0938×10^{-7}							
z_R	−9000	−4500	−2250	−750	−225	−140	−80	−45	
Ratios	5.1778	.....	1.8849	2.8797	1.8403	2.8392	3.4049	5.0816	3.301210^{-7}
Diatomic hydrogen									1.593110^{-3}

TABLE 63

VALUES OF s IN THE EARTH'S ATMOSPHERE

1911	June 9	Sept. 13	Nov. 9	Means
z 90000...	—	—	1.3997×10^{-26}	—
80000...	—	6.0859×10^{-15}	2.0279×10^{-11}	—
70000...	*5.0461×10^{-20}	1.2662×10^{-10}	6.5791×10^{-10}	—
60000...	*3.2793×10^{-11}	2.8063×10^{-9}	5.5959×10^{-9}	4.2011×10^{-9}
50000...	*3.5849×10^{-9}	1.7990×10^{-8}	2.3250×10^{-8}	2.0620×10^{-8}
40000...	8.1366×10^{-8}	8.6736 "	9.1948 "	8.6683 "
30000...	2.5130×10^{-7}	2.6901×10^{-7}	2.7679×10^{-7}	2.6570×10^{-7}
20000...	4.0890 "	4.3148 "	4.4559 "	4.2866 "
10000...	7.1883 "	7.6710 "	6.9126 "	7.2576 "
100...	1.6471×10^{-6}	1.6644×10^{-6}	1.5466×10^{-6}	1.6194×10^{-6}

* Omitted from the means.

(150) $$K = \frac{c\,a}{4\pi} \cdot T^4, \text{ whence } \frac{L}{K} = \frac{4}{3}\frac{1}{T}.$$

We have followed Case I.

The volume density of the energy $u = a\,T^4$ when multiplied by $\frac{c}{4\pi}$ gives the radiation energy, $K = \frac{c}{4\pi} \cdot a\,T^4$, as a flux in one direction. Similarly, the volume density of the entropy $s = \frac{4}{3}\,a\,T^3$ when multiplied by $\frac{c}{4\pi}$ gives the entropy radiation as a flux $L = \frac{c}{3\pi} \cdot a\,T^3$.

The Cases I and II differ by the factor $\frac{4}{3}$.

When the volume density of the electromagnetic intensity is multiplied by c, the velocity of propagation in the æther, 3×10^{10}, and divided by $\int d\,\Omega = 4\pi$, we have the radiation flux in one direction, kinetic energy per unit volume times velocity, which is equivalent to $\left[\frac{M\,L^2}{T^2} \cdot \frac{1}{L^2} \cdot \frac{1}{T}\right]$ the kinetic energy per unit area per unit time, as a flux across the unit surface, and this is equivalent to $\frac{M}{T^3}$. The transformation factor from (M. K. S.) to (C. G. S.) is 1000 for **K**, while it is 10 for u. There need be

TABLE 64. L = THE SPECIFIC INTENSITY OF THE ENTROPY RADIATION.

$$L = \frac{c}{4\pi} a T^3 = \frac{3c}{16\pi} s = \frac{\mathbf{K}}{T} = \frac{c}{4\pi} \cdot \frac{\rho}{m} \frac{U}{T}, \left[\frac{M L^2}{T^2} \cdot \frac{1}{L^3} \cdot \frac{L}{T} \cdot \frac{1}{\text{deg.}} = \frac{M}{T^3 \text{deg.}} \right].$$

	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	
z 25000	49.572	4.0046×10^{-9}							
20000	115.86	0.1104							
15000	229.47	6.3896							
10000	424.18	62.357	.11924						
5000	671.37	341.15	64.131						
1000	816.72	873.62	496.23	291.59	4.3133×10^{-3}				
800	822.80	887.46	516.66	375.45	2.9172				
600	828.88	901.29	536.97	478.44	29.779	5.3339×10^{-1}			
400	834.96	915.12	557.28	558.62	123.38	30.694			
200	841.04	928.95	577.59	620.39	346.96	212.33	36.368	0.1082	
100	844.08	935.87	587.70	645.61	492.65	451.11	246.81	122.46	
80	844.68	937.26	589.72	650.69	517.89	483.88	360.48	237.51	
60	845.29	938.64	591.74	655.75	543.14	516.66	485.80	402.78	
40	845.90	940.02	593.76	660.81	568.39	549.44	589.43	615.29	
20	846.51	941.40	595.78	665.87	593.64	582.22	647.38	955.74	
Photos. 0	847.12	942.78	597.80	670.93	618.89	615.00	708.75	1086.20	760.93
−20	847.84	944.32	599.53	677.75	636.74	645.31	761.05	1236.10	
−40	848.54	945.86	601.27	684.57	654.61	675.62	824.82	1416.10	
−60	849.24	947.40	603.01	691.39	672.48	705.93	879.50	2.7867×10^{9}	
−80	849.94	948.94	604.75	698.21	690.35	736.24	2.5831×10^{9}	5.0926×10^{9}	
−100	850.64	950.48	606.49	705.03	708.22	766.55	2.7846×10^{9}	8.8485×10^{9}	
−200	854.16	958.18	615.17	739.13	826.18	2.6977×10^{9}	1.2868×10^{10}		
−400	861.20	973.56	632.54	793.58	3.9056×10^{9}	1.6000×10^{10}			
−600	868.24	988.94	649.91	870.72	1.1888×10^{10}				
−800	875.28	1004.32	667.28	3.0236×10^{9}	2.6811×10^{10}				
−1000	882.32	1019.7	684.65	3.1345×10^{9}	5.0840×10^{10}				
−2000	918.98	1138.4	770.33	1.2891×10^{10}	3.8077×10^{11}				
−4000	975.77	1358.4	4.0869×10^{9}						
−6000	1049.8	8.5255×10^{5}	1.1378×10^{10}						
−8000	1102.8	1.0751×10^{6}							
−10000	2.1299×10^{9}	1.6597×10^{6}							
−12000	2.1842×10^{9}	2.5386×10^{6}							
−14000	2.9554×10^{9}	3.7491×10^{6}							
z_R	−9000	−4500	−2250	−750	−225	−140	−80	−45	
Ratios	5.1778	.....	1.8849	2.8797	1.8403	2.8392	3.4049	5.0816	3.3012×10^{-7}
Diatomic hydrogen									1.5931×10^{-8}

no confusion if volume energy and surface flux are carefully distinguished.

TABLE 65

VALUES OF L IN THE EARTH'S ATMOSPHERE

1911	June 9	Sept. 13	Nov. 9	Means
z 90000...	—	—	2.5063×10^{-8}	—
80000...	—	1.1151×10^{-5}	3.6310×10^{-2}	—
70000...	$*9.0352\times10^{-11}$	2.2672×10^{-1}	1.1780	—
60000...	$*5.8716\times10^{-2}$	5.0248	1.0020×10	7.5224
50000...	*6.4189	3.2212×10	4.1628 "	3.6920×10
40000...	1.4569×10^{2}	1.5530×10^{2}	1.6463×10^{2}	1.5521×10^{2}
30000...	4.4996 "	4.8167 "	4.9560 "	4.7574 "
20000...	7.3215 "	7.7258 "	7.9785 "	7.6753 "
10000...	1.2871×10^{3}	1.3735×10^{3}	1.2377×10^{3}	1.2994×10^{3}
100...	2.9492 "	2.9801 "	2.7692 "	2.8995 "

* Omitted from the means.

TABLE 67

VALUES OF K IN THE EARTH'S ATMOSPHERE

1911	June 9	Sept. 13	Nov. 9	Means
z 90000...	.....	.....	1.0025×10^{-7}	.....
80000...	.....	6.6903×10^{-5}	1.4524	.....
70000...	$*1.8070\times10^{-10}$	1.0202×10	8.2462×10	.....
60000...	*1.5267	4.0198×10^{2}	1.0020×10^{3}	7.0199×10^{2}
50000...	$*3.6587\times10^{2}$	3.5433×10^{3}	4.9954 "	4.2694×10^{3}
40000...	2.3747×10^{4}	2.5781×10^{4}	2.7823×10^{4}	2.5784×10^{4}
30000...	9.8540 "	1.0741×10^{5}	1.1166×10^{5}	1.0392×10^{5}
20000...	1.5771×10^{5}	1.6981 "	1.7808 "	1.6853 "
10000...	1.4538 "	3.2098 "	2.7366 "	2.4634 "
100...	4.2720 "	4.3585 "	7.6653 "	5.4319 "

* Omitted from the means.

Table 69 gives the total inner kinetic energy per unit volume, and it is the same as Table 36. We have confined the computations to the monatomic elements, and so have avoided the question of the potential energy arising from atoms and molecules of higher complexity. The interrelation of the potential and the kinetic energies in respect of Maxwell's First and Second Electromagnetic Laws still constitutes a problem beyond the range

TABLE 66. **K** = THE UNPOLARIZED SPECIFIC INTENSITY OF RADIATION.

$$\mathbf{K} = 2\tilde{K} = \frac{c}{4\pi}u = \frac{c}{4\pi}aT^4 = \frac{\sigma}{\pi}T^4 = \frac{3c}{4\pi}p = \frac{3c}{16\pi^2}I = \frac{3c}{16\pi}sT = LT, \left[\frac{ML^2}{T^2}\cdot\frac{1}{L^3}\cdot\frac{L}{T}\right].$$

	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	
z 25000	1.6460×10^{5}	3.2037×10^{-7}							
20000	4.9821 "	1.0539×10^{2}							
15000	1.2323×10^{6}	1.3131×10^{4}							
10000	2.7966 "	2.1794×10^{5}	8.8240×10						
5000	4.7301 "	2.0094×10^{6}	2.4242×10^{5}						
1000	6.1561 "	6.7093 "	3.7713×10^{6}	1.8808×10^{6}	1.2940				
800	6.2297 "	6.8136 "	3.9381 "	2.6281 "	4.3758×10^{3}				
600	6.3032 "	6.9180 "	4.1051 "	3.5883 "	8.7848×10^{4}	3.3645×10^{2}			
400	6.3767 "	7.0224 "	4.2721 "	4.3099 "	5.7990×10^{5}	9.5764×10^{4}			
200	6.4502 "	7.1268 "	4.4391 "	4.7800 "	2.3595×10^{6}	1.2613×10^{6}	1.0547×10^{5}	5.9513×10	
100	6.4870 "	7.1790 "	4.5226 "	4.9715 "	3.6948 "	3.3833 "	1.4068×10^{6}	4.8371×10^{5}	
80	6.4943 "	7.1896 "	4.5393 "	5.0097 "	3.9096 "	3.6454 "	2.3792 "	1.1757×10^{6}	
60	6.5017 "	7.2000 "	4.5560 "	5.0480 "	4.1243 "	3.9176 "	3.5463 "	2.3764 "	
40	6.5090 "	7.2104 "	4.5727 "	5.0863 "	4.3390 "	4.1898 "	4.5091 "	4.1532 "	
20	6.5164 "	7.2208 "	4.5894 "	5.1246 "	4.5537 "	4.4720 "	4.9784 "	7.0725 "	
Photos. 0	6.5237 "	7.2312 "	4.6061 "	5.1629 "	4.7684 "	4.7442 "	5.4463 "	8.3314 "	5.8518×10^{6}
−20	6.5292 "	7.2427 "	4.6192 "	5.2147 "	4.9033 "	4.9721 "	5.8426 "	9.4722 "	
−40	6.5345 "	7.2543 "	4.6323 "	5.2664 "	5.0382 "	5.1999 "	6.3253 "	$\mathbf{1.0841\times10^{7}}$	
−60	6.5398 "	7.2659 "	4.6454 "	5.3181 "	5.1730 "	5.4277 "	$\mathbf{6.7372\times10^{6}}$	2.1736×10^{13}	
−80	6.5451 "	7.2775 "	4.6585 "	5.3698 "	5.3078 "	5.6555 "	1.9765×10^{13}	4.8349 "	
−100	6.5504 "	7.2891 "	4.6716 "	5.4215 "	5.4426 "	$\mathbf{5.8833\times10^{6}}$	2.1497 "	1.0070×10^{14}	
−200	6.5771 "	7.3470 "	4.7371 "	5.6801 "	$\mathbf{6.3327\times10^{6}}$	2.0787×10^{13}	1.6392×10^{14}		
−400	6.6304 "	7.4630 "	4.8682 "	6.0907 "	3.3582×10^{13}	2.1880×10^{14}			
−600	6.6837 "	7.5790 "	4.9993 "	$\mathbf{6.6740\times10^{6}}$	1.4763×10^{14}				
−800	6.7370 "	7.6950 "	5.1304 "	2.3136×10^{13}	4.3517 "				
−1000	6.7903 "	7.8110 "	5.2615 "	2.3947 "	1.0189×10^{15}				
−2000	7.0678 "	8.7224 "	$\mathbf{5.9046\times10^{6}}$	1.5436×10^{14}	1.4877×10^{16}				
−4000	7.4950 "	1.0377×10^{7}	3.8625×10^{13}						
−6000	8.0532 "	6.5347×10^{9}	1.5082×10^{14}						
−8000	$\mathbf{8.4483\times10^{6}}$	8.6010 "							
−10000	1.6298×10^{13}	1.5248×10^{10}							
−12000	1.6797 "	2.6809 "							
−14000	2.5048 "	4.4743 "							
z_R	−9000	−4500	−2250	−750	−225	−140	−80	−45	
Ratios	5.1838	.....	1.5287	2.8847	1.8856	2.8302	3.4088	4.9876	3.2442×10^{-7}
Diatomic Hydrogen									1.5880×10^{-3}

TABLE 68. $(U_1 - U_0)$ = TOTAL VOLUME INNER KINETIC ENERGY.

$$(U_1 - U_0) = \frac{1}{2} m q^2 = H V = \frac{3}{2} P V = \frac{3}{2} K T = N E_0 = \frac{P V u}{2 p} = \frac{3}{2} k N T = C v m T = C v \rho V T, \left[M . \frac{L_2}{T^2} \right].$$

	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	
z 25000	1.6468×10^{12}	1.1567×10^{7}							
20000	3.0647 "	1.4077×10^{11}							
15000	5.0190 "	1.4119×10^{12}							
10000	7.4953 "	4.8938 "	1.5898×10^{10}						
5000	1.0474×10^{13}	1.0326×10^{13}	2.8534×10^{12}						
1000	1.3835 "	1.5491 "	1.1190×10^{13}	6.7416×10^{12}	1.5786×10^{9}				
800	1.3919 "	1.5778 "	1.1815 "	7.9893 "	2.5955×10^{11}				
600	1.4005 "	1.6064 "	1.2442 "	9.3520 "	1.5520×10^{12}	2.4272×10^{10}			
400	1.4091 "	1.6350 "	1.3069 "	1.0866×10^{13}	4.3215 "	1.4867×10^{12}			
200	1.4177 "	1.6636 "	1.3696 "	1.2622 "	8.1452 "	6.1367 "	1.9546×10^{12}	1.3142×10^{10}	
100	1.4220 "	1.6779 "	1.4010 "	1.3623 "	1.0620×10^{13}	9.7370 "	7.2362 "	2.9169×10^{12}	
80	1.4229 "	1.6806 "	1.4071 "	1.3823 "	1.1223 "	1.0693×10^{13}	8.6008 "	4.4655 "	
60	1.4238 "	1.6835 "	1.4134 "	1.4023 "	1.1826 "	1.1649 "	1.0020×10^{13}	6.3527 "	
40	1.4246 "	1.6864 "	1.4197 "	1.4223 "	1.2429 "	1.2606 "	1.1548 "	8.5206 "	
20	1.4255 "	1.6893 "	1.4260 "	1.4423 "	1.3032 "	1.3562 "	1.3264 "	1.1122×10^{13}	
Photos. 0	1.4263 "	1.6922 "	1.4323 "	1.4623 "	1.3635 "	1.4518 "	1.5251 "	1.4276 "	1.4726×10^{13}
−20	1.4281 "	1.6955 "	1.4402 "	1.4861 "	1.4400 "	1.5952 "	1.7493 "	1.8296 "	
−40	1.4300 "	1.6987 "	1.4482 "	1.5097 "	1.5166 "	1.7385 "	2.0113 "	**2.3473** "	
−60	1.4319 "	1.7019 "	1.4562 "	1.5333 "	1.5932 "	1.8818 "	**2.3087** "	2.9979 "	
−80	1.4338 "	1.7051 "	1.4642 "	1.5569 "	1.6698 "	2.0251 "	2.6535 "	3.7364 "	
−100	1.4357 "	1.7083 "	1.4722 "	1.5805 "	1.7464 "	**2.1684** "	3.0464 "	4.4790 "	
−200	1.4450 "	1.7243 "	1.5120 "	1.6986 "	**2.3358** "	3.2461 "	5.2056 "		
−400	1.4641 "	1.7562 "	1.5919 "	1.9679 "	3.7958 "	5.9570 "			
−600	1.4831 "	1.7881 "	1.6718 "	**2.2859** "	5.4833 "				
−800	1.5021 "	1.8200 "	1.7517 "	2.6533 "	6.3872 "				
−1000	1.5211 "	1.8519 "	1.8316 "	3.0813 "	8.8488 "				
−2000	1.6159 "	2.0256 "	**2.3438** "	5.7586 "	1.7252×10^{14}				
−4000	1.8267 "	**2.4247** "	3.7572 "						
−6000	2.0696 "	2.9023 "	5.2695 "						
−8000	**2.3397** "	3.4656 "							
−10000	2.6533 "	4.0711 "							
−12000	3.0045 "	4.6798 "							
−14000	3.3820 "	5.2885 "							

of this research. The data here presented, from the thermodynamic side, must be taken into the account, especially at the solar source of radiation.

TABLE 69

VALUES OF $(U_1 - U_0)$ IN THE EARTH'S ATMOSPHERE

1911	June 9	Sept. 13	Nov. 9	Means
90000.....	.....	.....	2.5798×10^5	.....
80000.....	.....	3.1797×10^6	6.4850×10^7	.....
70000.....	*4.1555×10^4	2.2265×10^8	3.9119×10^8	.....
60000.....	*1.6122×10^8	1.0568×10^9	1.2344×10^9	1.1456×10^9
50000.....	*1.6014×10	2.9581 "	2.9880 "	2.9731 "
40000.....	6.1829 "	6.4619 "	6.0909 "	6.2452 "
30000.....	9.9355 "	1.0291×10^{10}	9.6396 "	9.9554 "
20000.....	1.5560×10^9	1.5992 "	1.4883×10^{10}	1.5478×10^{10}
10000.....	2.4574 "	2.5023 "	2.3182 "	2.4593 "
100.....	3.5843 "	3.6189 "	3.4247 "	3.5426 "

*Omitted from the means.

P is the thermodynamic hydrostatic pressure due to the kinetic energy of the molecules per unit volume, and p is the kinetic energy of the electrons per unit volume, $\left[\frac{M L^2}{T^2} \cdot \frac{1}{L^3}\right]$, so that their ratio is a simple number. Within the solar isothermal layer the ratio is not far from the velocity of light, as if $p \times c = P$, so that electrons having the velocity of light have the effect of molecules in producing volume pressure. This value of the ratio P/p is found near the bottom of the earth's atmosphere. Below the plane of solar radiation the ratio drops, and p approaches P in value.

Certain Data in the Radiation Terms Computed Directly from the Formula $u = a T^4$ and Its Dependents for Black Radiation

It will be convenient to give some examples of the values of these radiation terms as derived from the standard value of $\log a = -15.86494$. We have selected certain temperatures for the computation in the atmospheres of the sun and the earth.

TABLE 70

$\frac{P}{p}$ = the thermodynamic pressure of kinetic energy / the pressure of black radiation *in vacuo*. $\frac{P}{p} = \frac{3KT}{U} = \frac{3KT}{Vmu} = \frac{2(U_1 - U_0)}{Vu}$; $U = Vmu$

	H_1 1.00	H_2 2.00	He 4	C 12	Ca 40	Zn 65	Cd 112	Hg 198	
z 25000	1.1523×10^{9}	60.971							
20000	1.8512 "	2.5672×10^{7}							
15000	2.5686 "	6.0281×10^{8}							
10000	3.0815 "	2.6834×10^{9}	2.0256×10^{7}						
5000	3.8209 "	3.8596 "	3.1820×10^{9}						
1000	5.8754 "	4.7062 "	6.2294 "	3.5188×10^{9}	4.2923×10^{6}				
800	6.0214 "	4.9509 "	5.8743 "	3.8500 "	4.3996×10^{8}				
600	6.1673 "	5.1955 "	7.5194 "	4.1801 "	1.9162×10^{9}	1.5298×10^{7}			
400	6.3132 "	5.4401 "	8.1645 "	5.0651 "	3.7553 "	1.5784×10^{9}			
200	6.4591 "	5.6847 "	8.8096 "	6.6413 "	4.5011 "	4.1480 "	2.8420×10^{9}	1.8664×10^{7}	
100	6.5321 "	5.8070 "	9.1322 "	7.7623 "	5.5793 "	4.9041 "	5.6159 "	1.6849×10^{9}	
80	6.5466 "	5.8333 "	9.1967 "	7.9864 "	6.0785 "	5.8219 "	5.1146 "	2.0102 "	
60	6.5612 "	5.8578 "	9.2612 "	8.2106 "	6.5775 "	6.7396 "	5.0267 "	2.3732 "	
40	6.5758 "	5.8823 "	9.3257 "	8.4348 "	7.0765 "	7.6573 "	5.6370 "	2.8619 "	
20	6.5904 "	5.9068 "	9.3902 "	8.6590 "	7.5755 "	8.5750 "	7.2197 "	3.2714 "	
Photos. 0	6.6050 "	5.9293 "	9.4547 "	8.8832 "	8.0745 "	9.4927 "	9.3545 "	5.1836 "	7.8722×10^{9}
−20	6.6210 "	5.9608 "	9.5711 "	9.1690 "	9.0862×10^{9}	1.1768×10^{10}	1.2286×10^{10}	8.4784 "	
−40	6.6373 "	5.9922 "	9.6875 "	9.4550 "	1.0098×10^{10}	1.4044 "	1.6086 "	1.3810×10^{10}	
−60	6.6537 "	6.0236 "	9.8039 "	9.7410 "	1.1110 "	1.6319 "	2.1320×10^{10}	1.2698×10^{4}	
−80	6.6700 "	6.0550 "	9.9203 "	1.0027×10^{10}	1.2121 "	1.8595 "	1.0291×10^{4}	9.8992×10^{3}	
−100	6.6863 "	6.0864 "	1.0037×10^{10}	1.0313 "	1.3133 "	2.0870×10^{10}	1.3365 "	7.4777 "	
−200	6.7676 "	6.2435 "	1.0620 "	1.1742×10^{10}	2.3353×10^{10}	1.6196×10^{4}	6.6870×10^{3}		
−400	6.9300 "	6.5575 "	1.1795 "	1.5820 "	1.4834×10^{4}	6.7610×10^{3}			
−600	7.0924 "	6.8715 "	1.2965 "	2.0995×10^{10}	8.4584×10^{3}				
−800	7.2548 "	7.1855 "	1.4135 "	8.7912×10^{3}	5.6039 "				
−1000	7.4172 "	7.4995 "	1.5305 "	1.2343×10^{4}	4.0545 "				
−2000	8.3292 "	9.1590 "	2.5263×10^{10}	9.1430×10^{3}	1.4736 "				
−4000	1.0673×10^{10}	1.4347×10^{10}	1.2565×10^{4}						
−6000	1.3571 "	4.2458×10^{7}	7.4965×10^{3}						
−8000	1.7577×10^{10}	5.9609 "							
−10000	1.2480×10^{4}	5.8709 "							
−12000	1.6522 "	5.4089 "							
−14000	1.4894 "	4.9488 "							
z_R	−9000	−4500	−2250	−750	−225	−140	−80	−45	
Ratios	1.4084	.....	2.0106	2.3879	1.5743	1.2886	2.0717	1.0876	1.6899×10^{8}
Diatomic hydrogen									7.7793×10^{2}

TABLE 71

VALUES OF $\frac{P}{p}$ IN THE EARTH'S ATMOSPHERE

1911	June 9	Sept. 13	Nov. 9	Means
90000.....	.....	.....	1.3090×10^{5}	.....
80000.....	.....	9.7693×10^{5}	1.8391×10^{6}	.....
70000.....	*1.1260×10^{6}	1.5633×10^{7}	1.6298×10^{7}	.....
60000.....	*3.5267×10^{7}	8.7074×10^{7}	7.1630 "	7.9352×10^{7}
50000.....	*7.6247×10^{8}	3.4838×10^{8}	3.0639×10^{8}	3.2739×10^{8}
40000.....	6.8820 "	7.1593 "	6.4362 "	6.8258 "
30000.....	8.5668 "	8.6018 "	7.9017 "	8.3568 "
20000.....	2.5289×10^{9}	2.5027×10^{9}	2.2279×10^{9}	2.4198×10^{9}
10000.....	6.6751 "	6.2360 "	6.7222 "	6.5444 "
100.....	8.3906 "	8.2370 "	9.2638 "	8.6305 "

* Omitted from the means.

TABLE 72

THE NORMAL VALUES OF THE BLACK RADIATION TERMS

Radiation Terms	SOLAR ATMOSPHERE		TERRESTRIAL ATMOSPHERE		
	Radiation Layer, z_R	Photosphere	Sea Level	20000 m.	40000 m.
Temperature, T...	7655°	7687°	286°.3	219°.5	166°.0
Volume density, u.	2.5162×10	2.5585×10	4.9229×10^{-5}	1.7008×10^{-5}	5.5639×10^{-6}
Volume pressure, p	8.3872	8.5282	1.6410×10^{-5}	5.6694×10^{-6}	1.8546×10^{-6}
Energy of radiation, I.....	1.0540×10^{2}	1.0717×10^{2}	2.0621×10^{-4}	3.8261×10^{-5}	2.3306×10^{-5}
Radiant pressure, F	7.0265×10^{-9}	7.1447×10^{-9}	1.3747×10^{-14}	4.7497×10^{-15}	1.5502×10^{-15}
Specific entropy, s.	4.3825×10^{-3}	4.4376×10^{-3}	2.2926×10^{-7}	1.0332×10^{-7}	4.4689×10^{-8}
Entropy radiation, L.....	7.8470×10^{6}	7.9437×10^{6}	4.1050×10^{2}	1.8499×10^{2}	8.0017×10
Intensity of radiation, K.....	6.0069×10^{10}	6.1079×10^{10}	1.1753×10^{5}	4.0604×10^{4}	1.3283×10^{4}
Kinetic energy, $(U_1 - U_0)$.....	2.5350×10^{13}	1.1697×10^{13}	1.2327×10^{9}	5.3894×10^{8}	2.1728×10^{8}
Ratio of pressures, $\frac{P}{p}$.....	3.0102×10^{6}	7.2252×10^{5}	6.0837×10^{10}	1.0020×10^{10}	1.3227×10^{9}
Pressure, P.....	25247850	6161900	998337	56808	24531
Density, $\frac{1}{m}\rho$.....	0.0000014889	0.000000629	0.0012152	0.00015823	0.00001696
Efficiency, $m\,R$....	2207720000	1277040000	2870300	1636900	872613

Mean values of the entire system were taken for P, ρ, R, T, and the standard $\log a = -15.86494$ in Table 26. These values should be compared with those given on Tables 54–71,

in order to understand the departures that occur in the thermodynamic computed values. It should be noted that it is the variation in the coefficient and exponent of $u = a\,T^{\alpha}$ that is primarily responsible for these changes.

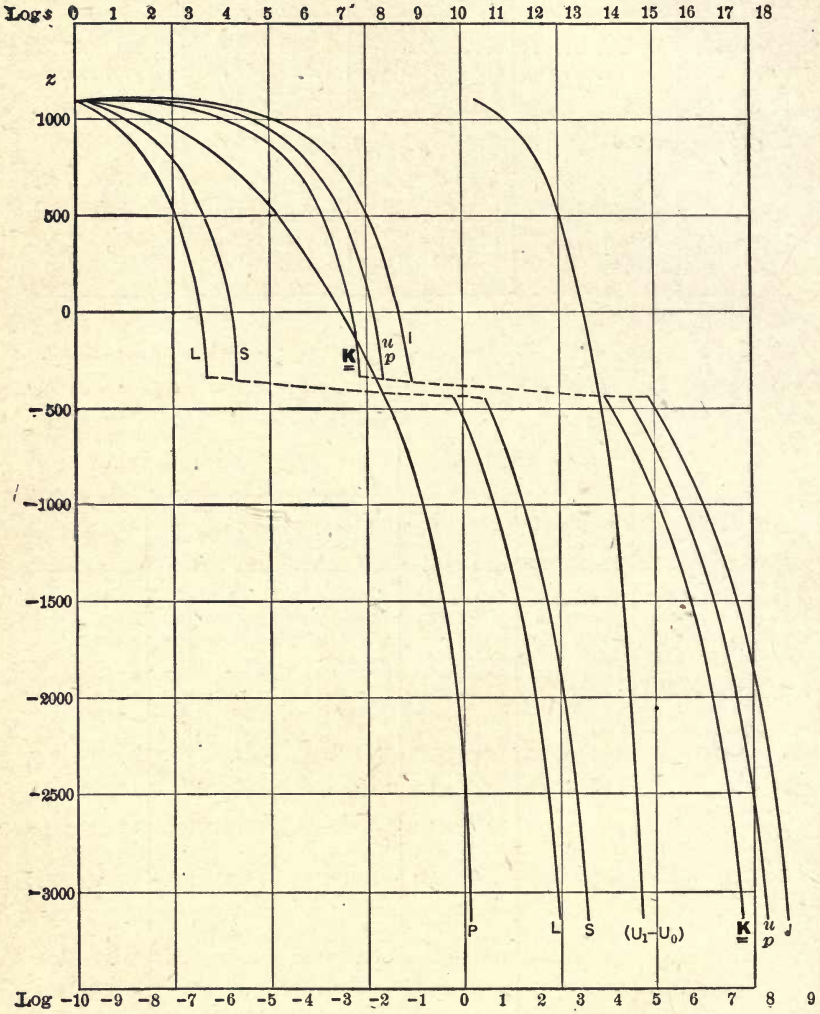


FIG. 21. The Great Discontinuity in the Radiation Data Occurring at the z_R Levels. Calcium Vapor.

P and $(U_1 - U_0)$ have no discontinuity at the z_R level; u, p, **K**, J, s, L, all suffer a per saltum change at that level where the radiation is generated.

The Discontinuity at the Levels Where the Solar Radiation is Generated

In order to illustrate the discontinuity in several terms on the levels where the solar radiation originates, a diagram, Fig. 21, has been constructed for Calcium Vapor on a composite scale, such that the ordinates are in kilometers from the photosphere and the abscissas are the logarithm's characteristics. The hydrostatic pressure P and the inner kinetic energy $(U_1 - U_0)$ run out smoothly across this level z_R, but the density u, pressure p, intensity $\mathbf{K}$, energy I, entropy s, and entropy radiation L, all suffer the marked discontinuity which has been computed and illustrated in the tables. It is seen that while the thermodynamic terms P, ρ, R do not go through such a discontinuity, all the terms involved in the generation of the radiation have been affected by this sudden variation, so that the physical processes must be different. The thermodynamic terms are probably confined to the kinetic and the potential energies of the molecules, while the radiation terms apparently are to be referred to some change in the structural motions of the electrons as free electric charges.

CHAPTER VI

Reconciliation of the Data Derived from the Pyrheliometer and the Bolometer

The Poynting Equation

THE resolution of the problem of the intensity of the solar radiation outside of the earth's atmosphere depends upon the interpretation that ought to be placed upon the two sides of the Poynting surface-flux, volume-density equation,

$$(66)\quad \frac{1}{4\pi}\int\int (V.EH)_n\, dS = \frac{1}{4\pi}\int\int\int \frac{\partial}{\partial t}\left(\frac{1}{2}H^2+\frac{1}{2}E^2\right) d\tau + A.$$

The surface flux is equal to the volume changes plus the waste. The observations with pyrheliometers and bolometers are commonly recorded in terms of the surface-flux, density per æther-volume times the velocity of propagation, $\frac{M\,L^2}{T^2}\cdot\frac{1}{L^3}\cdot\frac{L}{T}=\frac{M}{T^3}$, so that conversions from the (M. K. S.) to the (C. G. S.) system require the factor 1000. On the other hand, the thermodynamic computations are properly conducted in the volume-density form, with dimension $\frac{M\,L^2}{T^2}\cdot\frac{1}{L^3}=\frac{M}{L\,T^2}$, so that the conversion factor is 10. As already stated, Mr. Very has made the erroneous criticism that the conversion factor for the volume-density data of thermodynamics should be 1000 instead of 10, thus transferring the surface-flux factor 1000 to the volume-density, and so violating the law of dimensions. While the data of the two sides of the equation must be equivalent, no confusion can be permitted regarding the conversion of computations from one system of units to another.

The fundamental difficulty with this problem in practise is that the solar radiation in the electromagnetic form of energy

is transformed in the earth's atmosphere into equivalent thermodynamic forms, but these are so different in their manifestations that it is not easy fully to identify them. The pyrheliometer produces data which are very unlike the data derived from the bolometer, and each of these differs in form from the thermodynamic effects as recorded by the atmospheric conditions. In fact, it is still impractical to trace these transformations in detail, and this cannot be done until the physics of the transformation of energy in gases has been fully established. At the present time it is necessary to determine the total effects, first as a series of summations, and then proceed to the details so far as it is possible to do so.

It should be noted that Mr. Abbot, and many other observers, are approaching the general problem from too narrow a point of view, and are trying to reconcile these conflicting results, as heretofore indicated, by an overemphasis upon the complete sufficiency of the pyrheliometer to solve the problem. They are obliged to assume that the sun does not radiate like a black body at its high temperature, 7655°, in order to account for the bolometer discrepancy, thus debasing its 4.00 calories down to about 2.00 calories. Nor do they take into account the positive thermodynamic evidence that 4.00 calories are expended in the earth's atmosphere. Mr. Abbot's first assumption that the sun does not radiate at black body efficiency, at a temperature of 7655°, is disproved by the solar thermodynamics, summarized in the preceding chapters; his second assumption, that the terrestrial thermodynamics, as computed, does not require consideration is based only upon Mr. Very's erroneous statements regarding the conversion factor from (M. K. S.) to (C. G. S.), thus placing himself in conflict with the law of dimensions, as well as with the uniform practice of meteorologists and mathematical physicists.

Mr. Abbot summarizes his evidence regarding the Intensity of the Solar Radiation outside the atmosphere, Smithsonian Miscellaneous Collections, Volume 65, Number 4, 1915, and repeats his opinion that this intensity is 1.93 calories per sq. cm. per minute. He arrives at this conclusion by the original

Langley Method of discussing the Bouguer Formula of depletion, as if the extrapolation of the graph (sec z, log I) could be extended from sec $z = 1.00$, in the zenith, to sec $z = 0.00$, on the outside of the earth's atmosphere. The last step is non-mathematical, and as a physical process it is entirely unlike those preceding it (sec z_1 . I_1), (sec z_2 . I_2), etc., so that it is incompetent to integrate from the station to the top of the atmosphere through the numerous unknown physical processes involved. The formula $I = I_0\, p^{\sec z}$ is competent to determine p *at the station*, but not to trace out the varying values of p in the higher strata; it is competent to determine the *ratio* I/I_0 *at the station*, but not to discover the constituent physical terms that make up I_0 as a thermodynamic effect, from the parameter T to the other terms of the general law, $P = \rho\, R\, T$ in its non-adiabatic free atmosphere application. He supposes that by repeating this imperfect extrapolation at many stations, and under many surface conditions, the resulting agreement among them becomes proof that the physical result is complete. This is accompanied by a general evasion of many other facts which are clearly in contradiction to his pyrheliometer result. On the other hand, we shall be able to point out that there are great physical tracts in thermodynamics which are never recorded by the pyrheliometer, as compared with the full effect of the solar radiation, and thus establish our thesis that this apparatus registers only a portion of the incoming solar energy.

His " New Evidence" consists of pyrheliometer data, registered automatically up to the height of about 22000 meters in balloon ascensions, and this is a welcome and important addition to our available data. These data will be accepted as substantially correct, and adopted into our reconstructed system. Mr. Fowle has published the results of his investigation into the percentage of absorption and scattering by dry air and aqueous vapor, as determined from the bolometer graphs at Washington, D. C., Mt. Wilson, and Mt.Whitney. These will be incorporated into the system without any discussion. We shall show that there are three decisive thermodynamic factors which must still be taken into the account, in order to reconcile the pyrheliometer

data with the bolometer data, and both of these with the thermodynamic data of the terrestrial and the solar radiation. It is probable that the scheme here explained is capable of great refinement regarding the physical processes in atmospheres, and that these will lead to important contributions to the mathematical analysis of the theory of radiation and the underlying constitution of matter. It should, also, be noted that the new high-level pyrheliometer data fit in with the curve which had already been adopted to represent this term from the sea level to the top of the atmosphere. The new data, therefore, serve in no respect to modify the original problem, which is to explain the broad discrepancy between the result by the pyrheliometer, 2.00 calories, and the result by the bolometer 4.00 calories, without denying the approximate efficiency of the sun as a black radiator at the temperature of 7655° and 5.85 calories.

The Cause of the Change from the Total Solar Radiation 5.85 Calories to the Effective Radiation 3.98 Calories Received on the Outer Strata of the Earth's Atmosphere

It has been shown from solar thermodynamics that the radiation originates as black, and at an intensity equivalent to 5.85 calories at the distance of the earth. Of this amount it is found from the terrestrial thermodynamics that 3.98 calories are received on the outer strata of the earth's atmosphere and that about 1.40 calories reach the sea level. The former of these depletions is solar and the latter is terrestrial. In order to explain the change from the *true* solar to the *effective* solar radiation, we shall assume the data of Table 55, Vol. III, Smithsonian Institution, which contain the relative intensity of the radiation for several different wave lengths, from $\lambda = 0.323\ \mu$ to $\lambda = 2.097\ \mu$, as measured by the bolometer along a solar radius in terms of 1.000 at the center of the disk and 0.000 at the limb. The slit of the bolometer is set to a given wave length, and the disk of the sun in drifting across it produces a relative intensity curve with maximum at the center. Fig. 22 contains the set of relative curves, the dotted line corresponding with $0.470\ \mu$, which

is the common maximum ordinate of the system. The large dots show the limits of the cosine-ordinates, which express approximately the law of extinction for the shortest wave length. The coefficients of intensity are taken as the ordinates and the

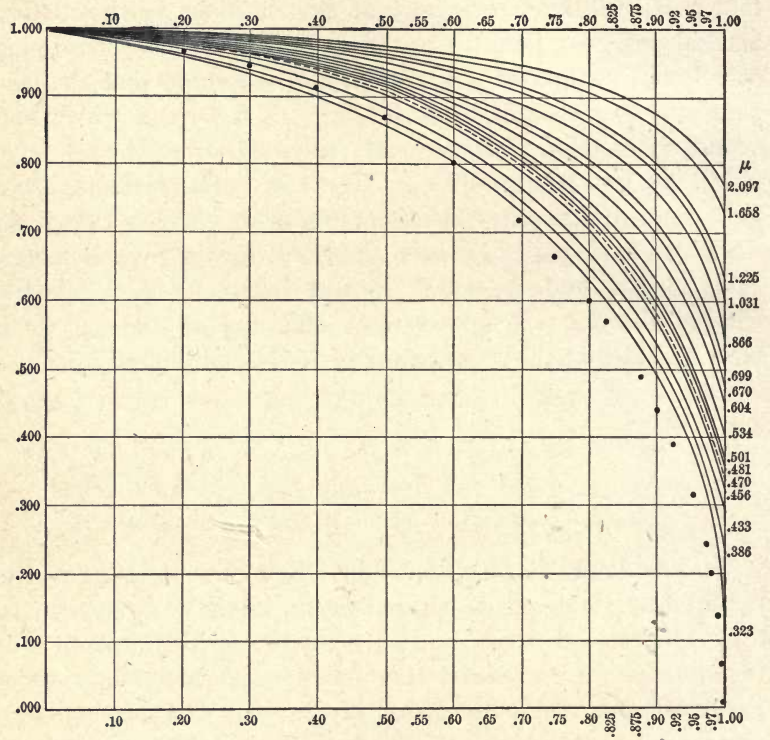


FIG. 22. The Loss of Radiation Between the Center and the Limb of the Disk of the Sun. (Abbot's Table 53, Vol. III).

parts of the radius as the abscissas. The phenomenon of the diminution of the brightness of the sun's disk from the center to the limb is commonly attributed to scattering, and the figure shows how this is to be distributed in the spectrum. The short waves are depleted more than the long waves, and they both are increasingly absorbed in some function of the path-length from points below the surface to points of escape in the direction of the earth. This complex subject will be resumed in Chapter VII.

TABLE 73. The Effective Radiation from One Hemisphere of the Sun.

I. Relative intensity of the radiation along a radius (Abbot's Table 55).

λ.....ρ	0.000	0.200	0.400	0.550	0.650	0.750	0.825	0.875	0.920	0.950	1.000	The last column was scaled from a diagram, made from those preceding, by extending the curves.
0.323 μ	1.000	.960	.897	.835	.775	.690	.660	.530	.452	.382	.010	
.386	1.000	.980	.926	.856	.792	.710	.633	.554	.483	.418	.240	
.433	1.000	.978	.927	.866	.806	.729	.647	.583	.510	.450	.280	
.456	1.000	.986	.942	.885	.831	.756	.681	.616	.538	.471	.330	
.481	1.000	.987	.944	.891	.840	.771	.701	.638	.566	.499	.350	
.501	1.000	.985	.945	.895	.845	.777	.711	.650	.583	.517	.380	
.534	1.000	.987	.950	.902	.856	.792	.728	.672	.605	.548	.400	
.604	1.000	.989	.957	.913	.872	.816	.761	.710	.649	.594	.430	
.670	1.000	.991	.961	.924	.887	.838	.786	.740	.680	.629	.470	
.699	1.000	.990	.963	.926	.890	.841	.793	.748	.691	.637	.500	
.866	1.000	.992	.969	.939	.911	.871	.830	.792	.744	.699	.540	
1.031	1.000	.998	.977	.951	.925	.889	.851	.816	.772	.730	.600	
1.225	1.000	.995	.976	.953	.932	.901	.865	.834	.794	.757	.640	
1.655	1.000	.997	.982	.966	.950	.928	.901	.877	.847	.816	.730	
2.097	1.000	.997	.986	.971	.956	.936	.915	.892	.866	.838	.780	

II. Relative intensity of the radiation for a hemisphere.

Complete	.628	1.885	2.983	3.770	4.398	4.949	5.338	5.640	5.871	6.123	Sums 41.585	Factor
0.323 μ	.615	1.715	2.583	3.035	3.224	3.341	3.176	2.769	2.448	1.206	24.112	0.580
.386	.622	1.728	2.658	3.106	3.303	3.326	3.165	2.927	2.648	2.014	24.497	0.589
.433	.621	1.728	2.678	3.152	3.378	3.405	3.256	3.085	2.818	2.235	26.351	0.633
.456	.624	1.738	2.726	3.235	3.492	3.558	3.464	3.254	2.965	2.455	27.511	0.661
.481	.624	1.740	2.738	3.265	3.545	3.642	3.576	3.395	3.129	2.602	28.256	0.679
.501	.624	1.739	2.744	3.280	3.567	3.682	3.635	3.480	3.229	2.749	28.729	0.690
.534	.624	1.743	2.762	3.314	3.624	3.761	3.737	3.604	3.388	2.902	29.459	0.709
.604	.625	1.745	2.789	3.367	3.712	3.905	3.929	3.835	3.652	3.135	30.694	0.738
.670	.625	1.749	2.813	3.416	3.795	4.019	4.073	4.004	3.846	3.368	31.708	0.762
.699	.625	1.750	2.819	3.423	3.809	4.043	4.116	4.061	3.898	3.484	32.028	0.770
.866	.625	1.753	2.846	3.487	3.919	4.213	4.329	4.332	4.239	3.796	33.539	0.806
1.031	.627	1.759	2.876	3.536	3.989	4.306	4.457	4.478	4.409	4.072	34.509	0.830
1.225	.627	1.758	2.879	3.555	3.993	4.370	4.537	4.614	4.556	4.280	35.169	0.485
1.655	.627	1.751	2.905	3.612	4.130	4.528	4.745	4.862	4.885	4.733	36.778	0.884
2.097	.627	1.736	2.920	3.634	4.161	4.583	4.826	4.958	5.002	4.954	37.401	0.899

In order to determine the effective intensity of the solar radiation towards the earth, it is necessary to integrate the parts contributed by each wave length over one hemisphere of the sun. To do this the data of Table 73, Section II, are computed as follows: Take the mean value of the intensity for two successive points, and multiply this by $2\pi \times \rho_{12}$, where ρ_{12} is the mean radius of the two points. Example:

Line 0.323 μ, $\rho_1 = 0.650$ $I_1 = 0.775$

$\rho_2 = 0.750$ $I_2 = 0.690$

$\rho_{12} = 0.700$ $I_{12} = 0.733$

$$2\pi \rho_{12} I_{12} = 6.283 \times 0.700 \times 0.733 = 3.224.$$

Section II contains the products for each group, so that the horizontal sums give relative numbers of the effective intensity of the several spectrum lines over the hemisphere. Had there been no depletion, the same hemisphere would have given an intensity 1.000 in each zone, with the total 41.585, while the line 0.323 μ gives 24.112, so that the factor of efficiency is 0.580 for this line. The last column in II gives the efficiency factor for each of the selected lines in the spectrum. Whatever may be the original solar radiation, the hemispherical action of the sun's atmosphere is to reduce the lines in these proportions.

We have shown that the solar radiation originates at $T = 7655°$, as *black radiation*, and the coordinates as computed by the Wien-Planck Formula appear in Table 79. They can be compared there with the energy spectrum curve for 6950°, 5810°, 5450°, as well as with the bolometer ordinates at Washington, Mt. Wilson, Mt. Whitney, and the total as extrapolated by Abbot to the outer limit of the atmosphere. The ordinates are given in relative numbers, but for intercomparison they are homogeneous, and for reduction to calories the factor 1/20.9 is sufficient.

We will interpolate for the curves 7655°, 6950°, total bolometer, the ordinates corresponding with the wave lengths herein adopted. Those for the 7655°-curve will be multiplied by the factors of efficiency, *and the product is to be compared with the ordinates for the* 6950° *and the total curves.*

TABLE 74

COMPARISON OF THE BLACK SOLAR INTENSITY OF RADIATION AT 7655° WITH THE EFFECTIVE, THE THERMODYNAMIC INTENSITY AT 6950°, AND THE EXTRAPOLATED BOLOMETER TOTAL.

λ	T 7655°	Factor	Effective at Sun	T 6950°	Total Bolometer
0.323 μ	9.790	0.580	5.678	5.406	1.500
0.386	10.552	0.589	6.215	6.348	3.690
0.433	10.294	0.633	6.516	6.514	5.472
0.456	9.884	0.661	6.533	6.454	6.051
0.481	9.411	0.679	6.390	6.268	6.056
0.501	9.031	0.690	6.231	6.116	6.052
0.534	8.337	0.709	5.911	5.946	5.768
0.604	6.898	0.738	5.091	4.959	4.994
0.670	5.721	0.762	4.359	4.217	4.070
0.699	5.188	0.770	3.995	3.664	3.664
0.866	3.191	0.806	2.572	2.511	2.403
1.031	1.994	0.830	1.655	1.603	1.536
1.225	1.199	0.845	1.013	0.986	0.985
1.655	0.459	0.884	0.406	0.389	0.466
2.097	0.204	0.899	0.183	0.176	0.211

A comparison of the effective radiation at the sun, as computed from black radiation at 7655°, with the black radiation at 6950°, *shows that they are practically identical throughout the spectrum.* This means that the solar radiation, which is equivalent to 5.85 calories at its strata of origin below the photosphere, is effective at the earth to the amount 3.98 calories, and that about 1.87 calories have been lost by the hemispherical scattering from the disk and by absorption in the superincumbent strata.

By comparison of these curves with the Abbot ordinates as determined by bolometer observations, and extrapolated to the

limit of the earth's atmosphere, it appears that there is substantial agreement from $\lambda = 0.500\ \mu$ to the end of the spectrum in the long waves, but that there is a great depletion at the end of the spectrum in the short waves. This is in accordance with the data in Table 83 and Fig. 26. The depletion in the earth's atmosphere can be discussed under the thermodynamic terms, which agree with the bolometer in requiring 3.98 calories to fall upon the earth's atmosphere. *Of this the pyrheliometer measures only 1.40 calories at the sea level, extrapolated to 1.940 calories on the vanishing plane. This is only half the effective solar constant and it is only one-third the true solar constant.* The question as to whether the sun radiates as a black body seems to be settled in the affirmative. The physical laws regarding the processes of absorption and transmission in each atmosphere remain to be studied, but it is evident that we have obtained the P, ρ, R, T, in all the strata concerned in the sun, and throughout those at the earth, so that the laws of transmission, refraction, and numerous other functions can be approached with suitable material for their discussion.

It should also be remembered that *radiation from an isothermal layer, if sufficiently thick, must be black radiation.* It has been found that the solar gases all pass through such deep isothermal strata, and we should, therefore, expect the solar radiation to be black. Furthermore, it is found from the terrestrial thermodynamics that about 3.98 calories of black radiation are actually consumed in the earth's atmosphere.

Summary of the Terrestrial Thermodynamic Data

It will be convenient at this point to summarize the results of the thermodynamic data as computed, and reported in Bulletin No. 4, O. M. A., 1914, or in the Meteorological Treatise, 1915. These summaries are compiled for the balloon ascensions, Uccle, June 9, September 13, and November 9, 1911, from Tables 12–26 of the bulletin. The sums were taken for each ascension through the 5000-meter intervals indicated in Table 75 under $z_1 - z_0$; the means of these sums in each stratum for the three ascensions were taken in the (M. K. S.) system of the computations, and

these were converted into the equivalent gr. cal./cm.^{2} min. for the following discussions; finally, the successive sums Σ were computed from the vanishing plane to the bottom of the respective strata. The first and second columns under each element refer in the first to the strata-means, and in the second to the general sum from the top of the atmosphere to the bottom of the several strata. The third column under $\Sigma - (Q_1 - Q_0)$, $\Sigma - (U_1 - U_0)$, $\Sigma\, g\,(z_1 - z_0)$, has been computed by subtracting 1.5804 from the second column, for reasons that will be stated.

In order to understand the relations of the thermodynamic terms to one another, a résumé of the non-adiabatic formulas is given:

Résumé of the Thermodynamic Formulas

$$(151)\quad -\frac{P_1 - P_0}{\rho_{10}} = +(W_1 - W_0) - R_{10}(T_a - T_0),$$
$$-Cp_{10}(T_a - T_0) = (R_{10} - Cp_{10} - R_{10})(T_a - T_0).$$
$$-\frac{P_1 - P_0}{\rho_{10}} = +(Q_1 - Q_0) + g(z_1 - z_0),$$
$$-Cp_{10}(T_a - T_0) = (Cp_a - Cp_{10} - Cp_a)(T_a - T_0).$$

$$(152)\quad -R_{10}(T_a - T_0) = -\frac{P_1 - P_0}{\rho_{10}} - (W_1 - W_0),$$
$$-R_{10}(T_a - T_0) = (-Cp_{10} - R_{10} + Cp_{10})(T_a - T_0).$$
$$-R_{10}(T_a - T_0) = +g(z_1 - z_0) + (U_1 - U_0),$$
$$-R_{10}(T_a - T_0) = (-Cp_a + Cp_a - R_{10})(T_a - T_0).$$

$$(153)\quad +(W_1 - W_0) = -\frac{P_1 - P_0}{\rho_{10}} + R_{10}(T_a - T_0),$$
$$(R_{10} - Cp_{10})(T_a - T_0) = (-Cp_{10} + R_{10})(T_a - T_0).$$
$$+(W_1 - W_0) = -\frac{P_1 - P_0}{\rho_{10}} - (U_1 - U_0) - g(z_1 - z_0),$$
$$(R_{10} - Cp_{10})(T_a - T_0) = (-Cp_{10} - Cp_a + R_{10} + Cp_a)(T_a - T_0).$$

$$(154)\quad -(U_1 - U_0) = +R_{10}(T_a - T_0) + g(z_1 - z_0),$$
$$-(Cp_a - R_{10})(T_a - T_0) = (R_{10} - Cp_a)(T_a - T_0).$$
$$-(U_1 - U_0) = +\frac{P_1 - P_0}{\rho_{10}} + (W_1 - W_0) + g(z_1 - z_0),$$
$$-(Cp_a - R_{10})(T_a - T_0) = (Cp_{10} + R_{10} - Cp_{10} - Cp_a)(T_a - T_0).$$

$$(155)\quad -(Q_1 - Q_0) = -(W_1 - W_0) - (U_1 - U_0),$$
$$-(Cp_a - Cp_{10})(T_a - T_0) = (-R_{10} + Cp_{10} - Cp_a + R_{10})(T_a - T_0).$$

$$-(Q_1 - Q_0) = g(z_1 - z_0) + \frac{P_1 - P_0}{\rho_{10}},$$
$$-(Cp_a - Cp_{10})(T_a - T_0) = (-Cp_a + Cp_{10})(T_a - T_0).$$
$$-(Q_1 - Q_0) = g(z_1 - z_0) + R_{10}(T_a - T_0) - (W_1 - W_0),$$
$$-(Cp_a - Cp_{10})(T_a - T_0) = (-Cp_a + R_{10} - R_{10} + Cp_{10})(T_a - T_0).$$

(156)
$$g(z_1 - z_0) = -(Q_1 - Q_0) + (W_1 - W_0) - R_{10}(T_a - T_0),$$
$$-Cp_a(T_a - T_0) = (-Cp_a + Cp_{10} + R_{10} - Cp_{10} - R_{10})(T_a - T_0).$$
$$g(z_1 - z_0) = -(Q_1 - Q_0) - \frac{P_1 - P_0}{\rho_{10}},$$
$$-Cp_a(T_a - T_0) = (-Cp_a + Cp_{10} - Cp_{10})(T_a - T_0).$$
$$g(z_1 - z_0) = -(W_1 - W_0) - (U_1 - U_0) - \frac{P_1 - P_0}{\rho_{10}},$$
$$-Cp_a(T_a - T_0) = (-R_{10} + Cp_{10} - Cp_a + R_{10} - Cp_{10})(T_a - T_0).$$
$$g(z_1 - z_0) = -(U_1 - U_0) - R_{10}(T_a - T_0),$$
$$-Cp_a(T_a - T_0) = (-Cp_a + R_{10} - R_{10})(T_a - T_0).$$

These formulas are easily verified from Fig. 23.

Special attention may be directed to the following facts:

1. The boundary $8\,A = \Sigma\, g(z_1 - z_0)$ limits the adiabatic system, while $4\,B\,A = \Sigma - \frac{P_1 - P_0}{\rho_{10}}$ limits the non-adiabatic system, and the difference between them is $\Sigma - (Q_1 - Q_0) = 0\,4\,C\,A = 4\,B\,A\,8$, counted in the proper algebraic directions.

2. The work $\Sigma + (W_1 - W_0)$ is the area $0\,3\,A = 4\,C\,A\,7$.

3. The efficiency $\Sigma - R_{10}(T_a - T_0) = 3\,A\,B\,4 = 7\,A\,8$.

4. The $\Sigma - \frac{P_1 - P_0}{\rho_{10}} = \Sigma - (Q_1 - Q_0) = \frac{1}{2}\,\Sigma\, g(z_1 - z_0)$.

5. Hence, the adiabatic area is formed from the non-adiabatic area by simply adding and subtracting $Cp_a(T_a - T_0)$, as in formula (4). It follows that the differences for the strata, $(W_1 - W_0)$, $(U_1 - U_0)$, $(Q_1 - Q_0)$, are the same in both systems, and that we can transfer the area $4\,D\,A\,8$ to coincide with the area $0\,A\,D\,4$, and this is convenient for several reasons.

6. The total hydrostatic energy $\Sigma = \frac{P_1 - P_0}{\rho_{10}}$ measures the effect of the solar radiation, after it has been transformed from the electromagnetic form into the thermodynamic form, and it

can be measured by the sum of the free heat as well, $\Sigma - (Q_1 - Q_0)$. The sum for Abbot's value of the solar constant 1.94 calories is found on the hydrostatic curve at about 18000 meters above the sea level, on the free heat curve at 40000 meters, but on the black radiation curve at 7000 meters. Mr. Abbot has taken my

TABLE 75

SUMMARY OF THE TERRESTRIAL THERMODYNAMICS

Uccle, June 9, Sept. 13, Nov. 9, 1911, Data from Bulletin No. 4

$z_1 - z_0$	$\Sigma\ (W_1 - W_0)$		$\Sigma - \frac{P_1 - P_0}{\rho_{10}}$		$\Sigma - (Q_1 - Q_0)$		
70–66	0.0184	0.0184	0.0254	0.0254	0.6623	0.6623	−0.9181
65–61	0.0321	0.0505	0.0439	0.0693	0.6441	1.3064	−0.2740
60–56	0.0572	0.1077	0.0780	0.1473	0.6118	1.9182	0.3278
55–51	0.0925	0.2002	0.1279	0.2752	0.5625	2.4807	0.9003
50–46	0.1386	0.3388	0.1926	0.4678	0.4990	2.9797	1.3993
45–41	0.1584	0.4972	0.2521	0.7199	0.4694	3.4491	1.8687
40–36	0.1410	0.6382	0.1988	0.9187	0.4955	3.9446	2.3642
35–31	0.1623	0.8005	0.2283	1.1470	0.4669	4.4115	2.8311
30–26	0.1993	0.9998	0.2804	1.4274	0.4157	4.8272	3.2468
25–21	0.2521	1.2519	0.3546	1.7820	0.3426	5.1698	3.5894
20–16	0.3183	1.5702	0.4496	2.2314	0.2467	5.4165	3.8361
15–11	0.3985	1.9687	0.5657	2.7971	0.1374	5.5539	3.9735
10– 6	0.4563	2.4250	0.6259	3.4230	0.0784	5.6323	4.0519
5– 1	0.4691	2.8941	0.6591	4.0821	0.0302	5.6625	4.0821
	2.8941		4.0821		5.6625		

$z_1 - z_0$	$\Sigma - (U_1 - U_0)$			$\Sigma\ g\ (z_1 - z_0)$			
70–66	0.6807	0.6807	−0.8997	0.6879	0.6879	−0.8925	
65–61	0.6762	1.3569	−0.2235	0.6890	1.3769	−0.2035	
60–56	0.6679	2.0248	0.4444	0.6901	2.0670	0.4866	
55–51	0.6551	2.6799	1.0995	0.6912	2.7582	1.1778	
50–46	0.6362	3.3161	1.7357	0.6924	3.4506	1.8702	
45–41	0.6278	3.9439	2.3635	0.6934	4.1440	2.5636	
40–36	0.6364	4.5803	2.9999	0.6945	4.8385	3.2581	
35–31	0.6292	5.2095	3.6291	0.6956	5.5341	3.9537	
30–26	0.6152	5.8247	4.2443	0.6967	6.2308	4.6504	
25–21	0.5991	6.4238	4.8434	0.6978	6.9286	5.3482	
20–16	0.5647	6.9885	5.4081	0.6989	7.6275	6.0371	
15–11	0.5379	7.5264	5.9460	0.7000	8.3275	6.7471	
10– 6	0.5243	8.0507	6.4703	0.7011	9.0286	7.4482	
5– 1	0.4996	8.5503	6.9699	0.6882	9.7168	8.1364	
	8.5503			9.7168			

TABLE 75—*Continued*

$z_1 - z_0$	$\Sigma R_{10}(T_a - T_0)$		$\Sigma \Delta J_0$		$\Sigma \Delta J_a$		$S - J_a$
70–66	0.0070	0.0072	0.00039	0.00039	0.00002	0.00002	
65–61	0.0188	0.0200	0.00070	0.00109	0.00008	0.00010	3.990
60–56	0.0396	0.0422	0.00211	0.00320	0.00029	0.00039	3.990
55–51	0.0750	0.0783	0.00577	0.00897	0.00098	0.00137	3.989
50–46	0.1290	0.1345	0.01282	0.02179	0.00531	0.00668	3.983
45–41	0.2227	0.2001	0.08993	0.11172	0.00922	0.01590	3.974
40–36	0.2805	0.2582	0.22285	0.33457	0.02343	0.03933	3.951
35–31	0.3465	0.3246	0.12940	0.46397	0.02665	0.06598	3.934
30–26	0.4276	0.4061	0.13468	0.59865	0.04446	0.11044	3.880
25–21	0.5301	0.5048	0.13975	0.73840	0.06792	0.17836	3.812
20–16	0.6612	0.6390	0.16625	0.90465	0.10836	0.28672	3.703
15–11	0.8284	0.8011	0.50154	1.40619	0.19539	0.48211	3.508
10– 6	0.9980	0.9779	1.15080	2.55699	0.40720	0.88931	3.101
5– 1	1.1880	1.1665	1.38090	3.93789	0.56672	1.45603	2.534
			3.93789		1.45603		

statements regarding the summation along the $\Sigma - (Q_1 - Q_0)$ curve as admission that the pyrheliometer measures the true solar constant, because this curve has its half-sum at that height, where the atmosphere has already a very low density. On the other hand, the half-sum for the hydrostatic curve is at 18000 meters, and for the black radiation it is at 8000 meters. These complex summations must be carefully studied before their final interpretation is permitted. The transformation between the electromagnetic radiation and the thermodynamic volume-effects involves some processes and terms that the pyrheliometer does not register, and it is on this account that Mr. Abbot's value of the solar constant is erroneous. The purpose of the foregoing summary is to indicate that we can pass from a non-adiabatic system, whose boundary curve is $Cp_{10}(T_a - T_0)$, into a strictly adiabatic system, whose boundary is $Cp_a(T_a - T_0) = -g(z_1 - z_0)$. *The Boyle-Gay Lussac Law in free atmospheres refers to the non-adiabatic* (*P. ρ. R. T.*), *which become adiabatic when such impressed forces as gravitation are superposed upon them.* The mirror-enclosure, within which the radiation formulas are usually developed, does not exist in free atmospheres, but in these gases the enclosing walls are removed and these thermody-

TABLE 76

SUMMARY OF THE ABSORBED AND THE BLACK BODY RADIATIONS AS COMPUTED WITHIN THE TERRESTRIAL ATMOSPHERE

I. ΣJ_a = the total radiation absorbed $= \Sigma (c_1 T_1^{a1} - c_0 T_0^{a0}) \frac{\text{gr. cal.}}{\text{cm.}^2 \text{ min.}}$

z Meters	Uccle 3	Omahar 3	Europe 6	U. S. 7	Tropics 6	Means	$S - J_a$	
90000....	—	—	—	—	—	—	3.980	Data from
80	—	—	—	—	—	—	3.980	Tables 32–36,
70	—	—	—	—	—	—	3.980	Bulletin No. 4
60	—	—	—	—	—	—	3.980	
50000....	0.001	—	—	—	—	0.001	3.979	
45	0.007	—	—	—	—	0.007	3.973	
40	0.016	—	—	—	—	0.016	3.964	
35	0.039	—	—	—	—	0.039	3.941	
30000....	0.065	—	—	—	—	0.065	3.915	
25	0.110	—	—	0.110	—	0.110	3.870	
20	0.177	0.178	—	0.175	—	0.177	3.803	
15	0.286	0.293	0.274	0.278	0.280	0.282	3.698	
10000....	0.481	0.488	0.460	0.483	0.516	0.486	3.494	
8	0.623	0.615	0.609	0.615	0.676	0.628	3.352	
6	0.784	0.820	0.788	0.785	0.852	0.806	3.174	
4	0.982	1.029	0.974	0.999	1.023	1.001	2.979	Mt. Whitney
2	1.217	1.325	1.196	1.225	1.220	1.237	2.743	Mt. Wilson
000	1.455	1.624	1.407	1.429	1.465	1.476	2.504	Washington, D. C.

II. ΣJ_0 = the total black body radiation $= \Sigma (c_1 T_1^4 - c_0 T_0^4) \frac{\text{gr. cal.}}{\text{cm.}^2 \text{ min.}}$

z Meters	Uccle 3	Omaha 3	Europe 6	U. S. 7	Tropics 6	Means	
90000....	—	—	—	—	—	—	
80	—	—	—	—	—	—	
70	—	—	—	—	—	—	
60	0.003	—	—	—	—	0.003	
50000....	0.014	—	—	—	—	0.014	
45	0.032	—	—	—	—	0.032	
40	0.117	—	—	—	—	0.117	As the result of all the available thermodynamic data the value 3.98 has been adopted for the surface summations of the intensity of the solar radiation.
35	0.345	—	—	—	—	0.345	
30000....	0.478	—	—	—	—	0.478	
25	0.618	—	—	0.618	—	0.618	
20	0.760	0.760	—	0.766	—	0.762	
15	0.937	0.962	0.915	0.887	0.940	0.928	
10000....	1.352	1.272	1.259	1.331	1.635	1.370	
8	1.714	1.498	1.673	1.704	2.101	1.738	
6	2.178	2.010	2.164	2.200	2.599	2.230	
4	2.675	2.408	2.655	2.780	3.062	2.716	
2	3.274	2.967	3.210	3.391	3.647	3.298	
000	3.808	3.609	3.743	3.946	4.338	3.909	

namic curves take their place. The mirror-wall condition is reached only near the bottom of the earth's atmosphere, or in the adiabatic strata of the solar gases, and it is on this account

that the practical transformation problem has been so difficult. *The main result that we wish to note follows:* THE SOLAR RADIATION ORIGINATES AS BLACK RADIATION AT AN EQUIVALENT OF 5.85 CALORIES; IT HAS BEEN FOUND TO BE EQUIVALENT TO ABOUT 3.98 CALORIES IN THE EARTH'S ATMOSPHERE; THE EVIDENCE IS THAT IT IS STILL BLACK RADIATION AT THE LEVELS 50000–60000 METERS, AS INDICATED IN THE TABLES 54–70.

These data are taken from Tables 32–36, Bul. No. 4, O. M. A., and they summarize the amount of the absorbed radiation ΣJ_a, and the black body radiation ΣJ_0, down to the levels indicated on the column of z in succession. From a consideration of all the available data, taking account of the season of the year of the 25 balloon ascensions as computed, and the latitude, it has been thought proper to assume $\Sigma J_0 = 3.980$ as the effective solar radiation at the distance of the earth on the vanishing plane of the atmosphere. The column $(S - J_a)$ shows the progress of the depletion to the sea level, and it is closely in agreement with the bolometer results at the stations occupied. Compare Meteorological Treatise, pages 380, 389.

The Thermodynamic Data in the 1000-Meter Levels

In Table 75 and Fig. 23 the thermodynamic data were summarized in the 5000-meter strata, and in the deep strata from the top of the atmosphere down to the levels indicated. In Table 77 and Fig. 24 the data are retained in the 1000-meter strata for $+\overline{(W_1 - W_0)}$, $-\dfrac{\overline{P_1 - P_0}}{\rho_{10}}$, $-\overline{(Q_1 - Q_0)}$ and $-\overline{(U_1 - U_0)}$. The summations for the black radiation $\Sigma \Delta J_0$ and for selective radiation $\Sigma \Delta J_a$ are repeated on Fig. 24. There are two points to be indicated in the construction of Figs. 23 and 24. In the balloon ascensions, as extended to 70000, 80000, 90000 meters, it is evident that the summations may be carried too high above the effective strata, into an asymptotic region, to represent properly the true solar radiation. The incoming column of the electromagnetic radiation may be represented as contained between parallel planes on a fixed scale; at some levels this begins

to transform when in contact with the gaseous media, and it is this effective height that is to be determined. On Fig. 23 the area $0\,4\,C\,A = \Sigma - (Q_1 - Q_0)$, and this is also equal to the area $4\,B\,A\,8$, whenever the curves $4\,B\,A$ and $4\,C\,A$ coincide at the

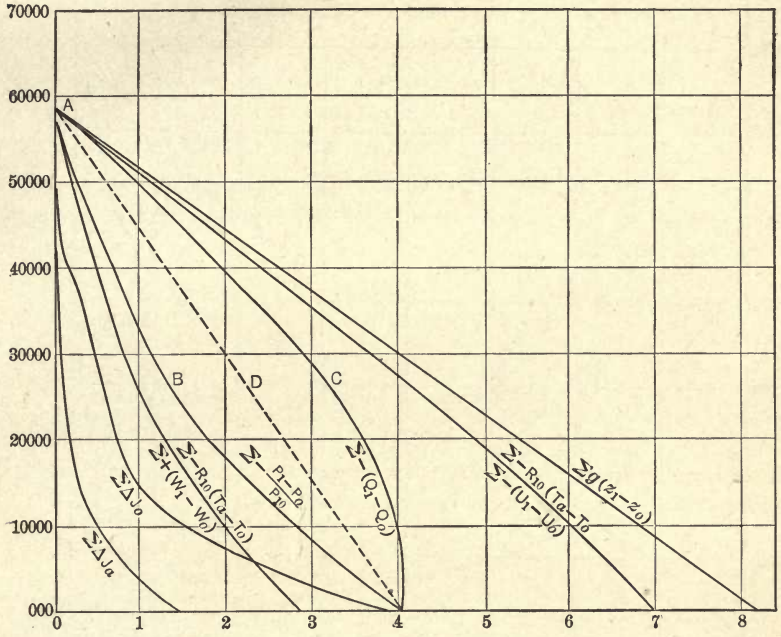


FIG. 23. Summary of the Terrestrial Thermodynamic Data.

points 4 and A. If the area $4\,C\,A\,8$ is extended too high the points 4 and A diverge along the two axes, and this is due to the arbitrary addition of $+ Cp_a$ and $- Cp_a$ to the non-adiabatic formula, which is required to produce the adiabatic formula. Hence, by making the curves $4\,B\,A$ and $4\,C\,A$ coincide at 4 and A, the effective height of the atmosphere as a thermodynamic medium is determined. In this case it is about 60000 meters above the sea level. My first computations for Huron, September 1, 1910, assumed this height to be 50000 meters; but the second computations extend it to 60000 meters, with a very rarefied coronal strata of an asymptotic character continuing up to 90000 meters. The thermal contents of this coronal

stratum are insignificant, although it can be clearly developed in the computations.

Having thus shown that the base 0—4 is equal to the base 4—8, the first being strictly non-adiabatic, and the second the

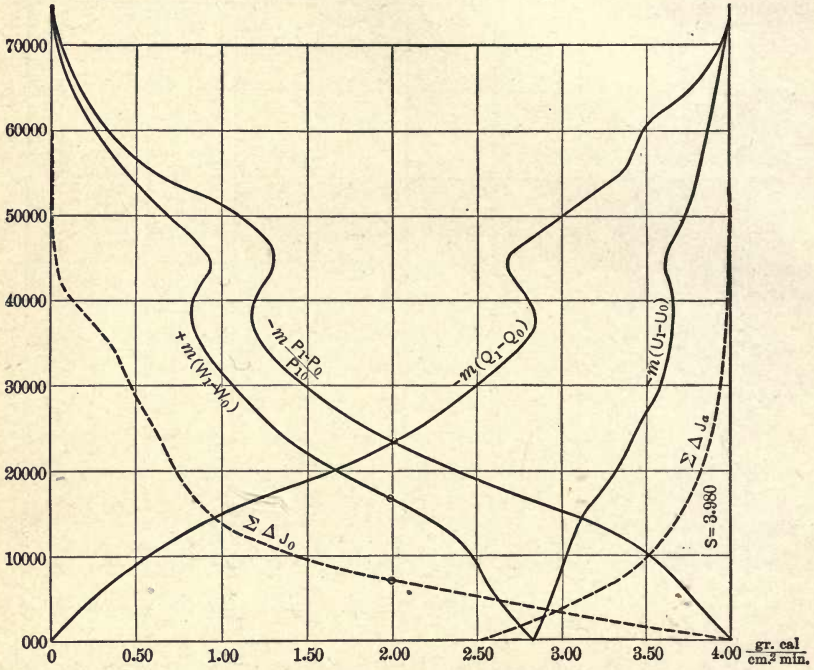


FIG. 24. The Thermodynamic Data in the Several Strata Having a Depth of 1000 Meters.

addition $(Q_1 - Q_0)$ required to transform it into the adiabatic system $g\,(z_1 - z_0)$ existing in the earth's atmosphere, it becomes proper to superpose the two systems upon each other when dealing with the individual strata. *Furthermore, since the incoming solar radiation has one fixed value from the top to the bottom of the earth's atmosphere, 3.98 calories, it follows that this must be the value in every stratum, in whatever manner it may become subdivided by the physical processes of the transformation.* Since, by Poynting's Law, the summation in every stratum must be the same for the radiation-flux and for the thermodynamic volume-density of the energy, we shall adopt the same vertical

boundary for the two systems, namely, an ordinate at 3.98 calories, the effective value of the intensity of the solar radiation at the vanishing plane of the earth's atmosphere. Since the gravitation energy in the 1000-meter strata is 9806, (M. K. S.) system, this is equivalent to 0.1405 gr. cal./cm.^{2} min., and it would be a proper scale for use with the thermodynamic data. But it is more instructive to adopt the full summation value 3.98, to which the factor of reduction is 28.74—that is, m, the molecular weight. At the same time the small asymptotic region above 60000 meters has been omitted. It may become possible to resume the computations on these high levels, adapting them more closely to a definite normal height for the atmosphere at its coronal base levels. At present we are concerned with detecting the several elements into which the solar radiation is transformed within the earth's atmosphere.

TABLE 77

THE THERMODYNAMIC DATA IN THE 1000-METER LEVELS

Height, in Meters z	EXTERNAL WORK		HYDROS. PRESSURE		FREE HEAT		INNER ENERGY	
	(W_1-W_0)	$m\,(W_1-W_0)$	$-\frac{P_1-P_0}{\rho_{10}}$	$-m\frac{P_1-P_0}{\rho_{10}}$	$-(Q_1-Q_0)$	$-m\,(Q_1-Q_0)$	$-(U_1-U_0)$	$-m\,(U_1-U_0)$
70000....	0.0030	0.0849	0.0041	0.1478	0.1334	3.9233	0.1363	3.9180
65000....	0.0050	0.1441	0.0056	0.1580	0.1307	3.7554	0.1357	3.8997
60000....	0.0090	0.2577	0.0124	0.3557	0.1207	3.4669	0.1344	3.8619
55000....	0.0156	0.4486	0.0197	0.5656	0.1169	3.3594	0.1321	3.7970
50000....	0.0238	0.6848	0.0378	1.0861	0.1052	3.0232	0.1293	3.7166
45000....	0.0323	0.9294	0.0453	1.3006	0.0932	2.6786	0.1255	3.6071
40000....	0.0286	0.8231	0.0408	1.1734	0.0979	2.8128	0.1265	3.6360
35000....	0.0300	0.8628	0.0422	1.2134	0.0968	2.7815	0.1268	3.6443
30000....	0.0367	1.0534	0.0515	1.4804	0.0858	2.4663	0.1245	3.5772
25000....	0.0459	1.3177	0.0645	1.8531	0.0750	2.1552	0.1208	3.4726
20000....	0.0579	1.6630	0.0815	2.3408	0.0581	1.6686	0.1158	3.3275
15000....	0.0730	2.0975	0.1031	2.9624	0.0357	1.0270	0.1087	3.1246
10000....	0.0870	2.4992	0.1219	3.5030	0.0192	0.5525	0.1062	3.0518
5000....	0.0901	2.5879	0.1304	3.7476	0.0099	0.2869	0.1027	2.9523
000....	0.0991	2.8476	0.1392	4.0013	0.0000	0.0000	0.1007	2.8953

The first column under the respective thermodynamic terms is the mean value for the three balloon ascensions reduced to gr. cal./cm.^{2} min. This column is multiplied by $m = 28.74$, the adopted molecular weight of the atmosphere. There is a margin of inaccuracy in the lowest stratum, between the sea level and the height of the station Uccle, which is 100 meters, because it

is not exactly known how well the adiabatic assumption which has been adopted for this stratum actually holds true. In our final result we have taken $m(W_1 - W_0) = -m(U_1 - U_0) = 2.82$. When the gas is in contact with the surface of the earth it is probable that the thermodynamic terms undergo a small modification, which should be more carefully examined. An especial research will be required to determine the mean values of all these terms in the first 1000 meters above the surface of the stations, where they are to be applied.

Résumé of the Preceding Results

When the results of the preceding investigations have been collected together, it appears that there are twelve lines of computation and observation, more or less independent of one another, which converge upon 3.98 gr. cal./cm.^{2} min. as the equivalent of the effective intensity of the solar radiation on the vanishing plane of the earth's atmosphere.

1. Solar thermodynamics, Table 28, Table 73..... = 3.90
2. Free heat, $\Sigma - (Q_1 - Q_0)$, Table 75..... = 4.08
3. Hydrostatic pressure, $\Sigma - \frac{P_1 - P_0}{\rho_{10}}$, Table 75..... = 4.08
4. Inner energy and external work, $\Sigma[-(U_1 - U_0) - (W_1 - W_0)]$, Table 73..... = 4.08
5. Black radiation, $\Sigma \Delta J_0$, Table 75..... = 3.94
6. Free heat in the stratum, $-m(Q_1 - Q_0)$, Table 77.. = 3.92
7. Hydrostatic pressure in stratum, $-m\frac{P_1 - P_0}{\rho_{10}}$, Table 77..... = 4.00
8. Inner energy in stratum, $-m(U_1 - U_0)$, Table 77.. = 3.92
9. External work and gas efficiency, $m(W_1 - W_0) + R_{10}(T_a - T_0)$, Table 77..... = 4.02
10. Kinetic + potential/energy + absorbed radiation $(I + II + \Sigma \Delta J_0)$..... = 3.90
11. Total radiation by Table 79, I + II + III + IV + V + VI..... = 3.98
12. The data of the pyrheliometer, Table 83, Fig. 26.... = 3.98

Adopted mean of the twelve processes..... = 3.98

The concurrence of so extensive a series of methods of discussion in fixing the solar radiation at the flux-intensity of 3.98 gr. cal./cm.^{2} min. at the distance of the earth *justifies the rejection of Abbot's imperfect method of extrapolation by the Bouguer formula—that is, those results from the pyrheliometer which reach only one-half the solar constant reduced to the distance of the earth, and only one-third of the true solar constant at its strata of origin below the photosphere.*

1. True solar intensity of radiation	5.85	calories
2. Effective solar intensity at the distance of the earth	3.98	"
3. Effective intensity by the bolometer (indicated)	3.98	"
4. Effective intensity by thermodynamics	3.98	"
5. Extrapolated intensity by the pyrheliometer	1.95	"
6. Intensity at the sea level by the pyrheliometer	1.50	"

The Potential Energy of the Solar Radiation in the Sun's Atmosphere

The primary fact regarding the electromagnetic solar radiation is that half of its energy is kinetic or magnetic, and half of its energy is potential or electric. This holds true in the æther, in the space between the atmosphere of the sun and the atmosphere of the earth, but it is not true in the gaseous media of either of these atmospheres. The reason that the kinetic and the potential energies are not equally divided in gaseous media, as they are in the pure æther, is that gases have *three degrees of freedom, while the æther has only two degrees of freedom.* For instance, Planck in the "Theorie der Wärmestrahlung," 1913, discusses his theory of the Wirkungsquantum as the effect of electromagnetic oscillations, such that two degrees of freedom are sufficient to account for the energy that is in operation. The equilibrium of electromagnetic energy is always due to a magnetic induction B and an electric displacement D, which are related by (44) to (59). Heaviside gives several illustrations of these energies, when an electric charge is suddenly moved or suddenly stopped, as in collisions, with the conse-

quent plane electromagnetic wave projected with the velocity of light v, as determined by the induction coefficients of the magnetic field μ and the electric field κ, in the formula $\mu \kappa v^2 = 1$. For oscillators there are *only two variables* $(f . \psi)$, where $f =$ the moment of the electric charge = the product of the electric charge times its polar distance $= e_+ a$; while $\psi = L\dot{f}$, the impulse or acceleration, so that,

$$\int\int df . d\psi = h = \text{constant},$$

which is the quantum hypothesis. On the other hand, the energy of a moved molecule has *three variables,* $\dot{x} . \dot{y} . \dot{z}$, *and temperature depends upon the translation and collisions of the molecules only, and not upon the inner structure of the atoms.* Let $U =$ the total energy of N oscillators in the unit mass, and $\frac{h\nu}{2}$ the mean energy of the oscillators in one elementary region, having a frequency ν for the electromagnetic oscillations. Thence, Planck deduces his formula,

$$(157) \quad U = N . \frac{h\nu}{2} \frac{1 + e^{-\frac{h\nu}{kT}}}{1 - e^{-\frac{h\nu}{kT}}} = N . h\nu \left(\frac{1}{2} + \frac{1}{e^{\frac{h\nu}{kT}} - 1} \right)$$

which is (87). If $T = 0$, no temperature, we have,

$$(158) \qquad U = N . \frac{h\nu}{2} \text{ (and not 0)},$$

so that the oscillators send out the mean energy $\frac{h\nu}{2}$ even when the temperature $T = 0$. *In the inner part of the atom there is energy independent of the temperature.* If $T = \infty$, $U_1 = k N T$, which is proportional to T and independent of h, that is, of the nature of the oscillators, namely, the electron-structure of the atoms and the monatomic molecules themselves. From (28) we have the total inner kinetic energy in the volume V for the total thermodynamic inner energy,

$$(159)\quad (U_1 - U_0) = \frac{3}{2} k N T = C_v m T = C_v \rho T V = \frac{3}{2} P V = \frac{3}{2} K T = H V,$$

where U_0 is some initial value due to the integration. For the unit volume $V = 1$, and $U_0 = 0$, we have,

$$(160)\quad U_2 = \frac{3}{2} k N T \text{ for thermodynamic collisions,}$$

$$(161)\quad U_1 = k N T \text{ for electromagnetic oscillations.}$$

Hence, U_1 (for oscillators) $= \frac{2}{3} U_2$ (for monatomic molecules).

The energy of electromagnetic radiation relative to the energy of monatomic molecular temperature collisions stands in the relation of 3 to 2. For diatomic and triatomic molecules the ratio is different.

It will be remembered that the solar thermodynamic computations were confined to monatomic elements, in which it was assumed that the inner energy is wholly kinetic and that the potential energy is negligible. Under these circumstances the equivalent solar intensity of radiation in the strata of origin z_R at certain distances below the photosphere amounts to 5.85 calories. On its arrival at the outer strata of the earth's atmosphere it has been found to be reduced to 3.98 calories. These ratios are nearly the same,

$$\frac{5.85}{3.98} = \frac{3}{2} = \frac{\text{solar intensity (electromagnetic)}}{\text{solar intensity (thermodynamic)}}.$$

Since the solar radiation, originating in the planes z_R, passes through the deep strata of the other chemical elements which are not monatomic, we may assume that one-third of the original radiation energy in the electromagnetic form has gone over into building the potential energy in the molecules and atoms of higher complexity. This, it appears, is a very complete account of the first great depletion of the energy of the solar radiation as recorded by the bolometer, and illustrated in Fig. 25. The spectrum of the bolometer shows that the long waves, 1.50 μ to 2.50 μ, emanate from the high temperature 7655°,

and succeed in passing through the gases of both the solar and the terrestrial atmospheres without depletion, while the shorter waves 0.0 μ to 1.50 μ undergo losses such that their equivalent temperature is 6950° and their efficiency 3.98 calories. The entire history of these depletions of the short wave lengths in the solar atmosphere will prove to be of value in molecular physics. We conclude that the potential energy of the gases in the outer levels of the sun's atmosphere is built up by the average expenditure of 1.87 gr. cal./cm.^{2} min., as a flux which is equivalent by Poynting's Law to the same amount of volume-density. This account of the potential energy is in conformity with the depletion by scattering already mentioned.

The Potential Energy of the Solar Radiation in the Earth's Atmosphere

There is yet another loss of the solar radiation due to the transformation of a portion of it in building up the potential energy of the atoms and the molecules in the earth's atmosphere. At the vanishing plane of the earth's atmosphere the electromagnetic energy is divided equally between the kinetic (magnetic) and the potential (electric) forms. On the other hand, the gases of the atmosphere are distributed in another ratio, such that,

$$\text{the total inner energy, } U = 1.641 = Cv\,\rho\,T$$
$$\text{the kinetic energy, } H = 1.000 = \frac{3}{2}P$$
$$\text{the potential energy, } J = 0.641 = U - H$$

This result is derived from the general formulas of Table 2.

$$(162)\quad \frac{U}{H} = \frac{Cv\,\rho\,T}{\frac{3}{2}P} = \frac{2}{3}\frac{Cv\,\rho\,T}{R\,\rho\,T} = \frac{2}{3}\frac{Cv}{R} = 1.641.$$

$$(163)\quad U = H + J, \text{ and } J = U - H = 0.641.$$

Examples of this fundamental relation are given in the Meteorological Treatise, Table 97, and in the following Table 78:

TABLE 78

THE TOTAL INNER ENERGY U, KINETIC ENERGY H, AND POTENTIAL ENERGY J

KINETIC ENERGY $H = \frac{3}{2} P$				TOTAL ENERGY $U = Cv\,pT$			$J = U - H$
1911	June 9	Sept. 13	Nov. 9	June 9	Sept. 13	Nov. 9	
z							
70000..	3.971×10^{-21}	3.340×10^{-3}	0.02815	6.515×10^{-21}	5.481×10^{-3}	0.4618	
65000..	7.629×10^{-8}	0.07220	0.2450	1.252×10^{-7}	0.11847	0.4019	
60000..	1.128×10^{-3}	0.9229	1.5031	1.849×10^{-3}	1.2027	2.4661	The constant ratio $\frac{U}{H} = 1.641$ is satisfied, and it is a check on the series of computations.
55000..	0.1835	5.006	7.4326	0.3011	8.215	12.194	
50000..	5.843	25.794	32.056	9.588	42.424	52.591	
45000..	75.692	111.76	120.12	124.21	183.39	197.07	
40000..	342.28	386.56	375.06	561.07	634.31	615.34	
35000..	809.43	898.54	863.76	1328.4	1474.4	1417.3	
30000..	1768.0	1935.1	1847.9	2901.5	3174.5	3032.0	
25000..	3821.6	4133.0	3899.0	6271.4	6782.0	6397.2	
20000..	8353.0	8901.0	8309.2	13708.	14607.	13635.	
15000..	18424.	19455.	17826.	30236.	31927.	29251.	
10000..	40649.	41924.	38528.	66705.	68803.	63226.	
5000..	82007.	82710.	79260.	134570.	136050.	130065.	
100..	150140.	150730.	148720.	246390.	246780.	244060.	

The kinetic energy is used in building up the temperature of the gases of the atmosphere, which is due to the transportation and collision of the molecules; the potential energy is used in storing up the static energy, which is held in the structural configuration of the atoms, the molecules, and the electrons, and the ions of which they may be composed. This change of the potential (electric) energy of the electromagnetic solar radiation into 0.641 parts of the gaseous potential energy is shown on Figs. 25, 26, where the region I is the kinetic energy H as determined by the pyrheliometer, while the region II is the potential energy J, which is 0.641 H. It is supposed that the transformation from J of the electromagnetic field to J of the gaseous field, that is, from 1.000 to 0.641, may be somewhat gradual in the high coronal strata, as indicated by the dotted line. If this is actually the case, there is evidently a great production of free electric charges in region VI, where the electricity, which manifests itself as auroral discharges and magnetic disturbances of the earth's normal field, is to be located in the first instance.

The Third Thermodynamic Depletion

It yet remains to be noted that there is another thermodynamic term which must be built up out of the solar radiation before an equilibrium can be obtained. That is the transformation of $U_v = Cv\,\rho\,T$, the total inner energy which is dependent upon the specific heat at constant volume Cv_{10}, into the specific heat at constant pressure Cp_a upon which the gravity effect depends,

$$U_p = g\,(z_1 - z_0) = -\,Cp_a\,(T_a - T_0). \tag{164}$$

Since $Cp_a = Cv_{10} + R_{10}$, this is equivalent to the addition of the term R_{10}, whose values may be studied in the formula,

$$+\,R_{10}\,(T_a - T_0) = +\,\frac{P_1 - P_0}{\rho_{10}} + (W_1 - W_0) = -\,(Q_1 - Q_0) - g\,(z_1 - z_0). \tag{165}$$

The region $-\,R_{10}\,(T_a - T_0)$ is indicated as III on Fig. 26, in conformity with the conditions already explained. Compare Table 75.

The Scattered and the Absorbed Radiation

The incoming radiation suffers two other important depletions, (1) by scattering or reflection on the molecules, and dust contents of the atmosphere, whereby no influence is effected upon the temperature, the energy being lost to space; and (2) by absorption, which is generally a potential effect rather than a kinetic effect upon the translation of the molecules. The region of scattered radiation is numbered IV on Figs. 25, 26, and one should distinguish between the low-level scattering and the high-level scattering, the former probably pertaining to the wave lengths 0.35 μ to 0.80 μ, and the latter to the shorter wave lengths 0.00 μ to 0.35 μ. There are two regions of absorption, the low-level marked V, and the high-level included in III, the latter above the isothermal levels wherein there is no absorption, and the former below the isothermal strata, extending through the convection region to the sea level.

The foregoing analysis admits that,

(1) The pyrheliometer measures the kinetic energy as depleted by the scattering and the absorption in the lower levels, that is, I, IV, V; but it denies that,

(2) The pyrheliometer measures the potential energy of region II, the thermodynamic term R of region III, the high-level scattering of region VI, and the high-level absorption of region VI, as well as the ionization of the same region VI. The boundaries within VI are still a subject of research.

The Pyrheliometer and Bolometer Observations

We shall now collect together the accepted data of the observations derived from the pyrheliometer and the bolometer, in order to learn to what extent they conform to the foregoing analysis of the thermodynamic data. It will be remembered that there are two distinct methods of discussing the pyrheliometer observations heretofore employed:

(1) The Langley-Abbot method, in which only the kinetic energy, as it is effective at the instrument, is accounted for; and,

(2) The Bigelow method, which is competent to include the potential energy with the kinetic energy, at least to some extent. The former method of extrapolation results in a series of values that are inconsistent at the several stations when taken in sufficient detail. The latter harmonizes them in a form which is simple for intercomparisons. Neither of these pyrheliometer methods succeeds in registering all the terms which are demanded by the bolometer and by the thermodynamics of the earth's atmosphere. This is in the Meteorological Treatise.

(3) A third method of reducing the pyrheliometer observations to about 3.98 calories is illustrated in Table 83, and all our data have finally been recomputed by this method, since it comprises the terms I, II, III, IV, V, VI, and requires no extrapolation beyond sec $z = 1$, thus being complete on the level of observation.

The general view is that while the electromagnetic radiation has the kinetic and the potential energies in equal parts, and the atmosphere has 1.00 part kinetic and 0.641 part potential, the complex silver disk of the pyrheliometer turns 1.00 part

kinetic energy into temperature, but conceals 1.00 part potential energy within the atoms and molecules. Similarly, the bolometer thread of metal conceals 1.00 part potential energy, but gives relative ordinates of kinetic energy in correct ratio to the surviving spectrum lines.

The Depletion of the Solar Radiation from 5.58 Calories in the Isothermal Layers of the Sun to 1.50 Calories at the Sea Level of the Earth

We are now in possession of the data which are necessary in order to describe with considerable clearness the course of depletion of the original solar black body radiation, or the solar constant, which amounts to about 5.85 gram calories per square centimeter per minute, down to about the 1.50 calories that are ordinarily received at the sea level of the earth's atmosphere. This assumes that the quantities of the radiation at the several levels in the sun's and the earth's atmospheres have been corrected for instrumental and observational defects, and all reduced to the scale of the earth's mean distance from the sun. It will not be necessary to recapitulate in this place the results of the research in the earth's atmosphere, as these may be found in the Treatise on Atmospheric Circulation and Radiation 1915, and in Bulletins No. 3, 1912, No. 4, 1914, of the Argentine Meteorological Office. These are sufficiently summarized in Tables 75, 76, and 77, and Figures 23 and 24, and they will be briefly explained. In Table 79 the wave lengths in the spectrum are taken for the differences 0.05 μ, up to 2.50 μ, and wherever the bolometer ordinates of the four central columns are lacking in Abbot's observations the necessary interpolations have been added to fill out the columns uniformly, so that the general sums may become homogeneous and comparable. The columns under 7655°, 6950°, 5810°, 5450°, are computed from the Wien-Planck Formula, using the constants as follows, $c_1 = 5.575 \times 10^{-15}$, $c_2 = 1.4455$:

$$(166) \qquad J_\lambda = \frac{5.575 \times 10^{-15}}{\lambda^5 \left(e^{\frac{1.4455}{\lambda T}} - 1\right)} = \frac{5.575 \times 10^5}{\mu^5 \left(10^{\frac{6277.4}{\mu T}} - 1\right)}$$

Since $\lambda = 10^4\mu$, c_1 (C. G. S.) is multiplied by 10^{20} for μ, and c_2 (C. G. S.) is multiplied by 10^4 for μ.

The column 7655° corresponds with the solar constant, 5.85 calories reduced to its equivalent at the distance of the

TABLE 79

SUMMARY OF THE DATA OF THE SOLAR RADIATION, SHOWING THE DEPLETION FROM THE SOLAR CONSTANT AT 5.85 CALORIES TO 1.50 CALORIES AT THE SEA LEVEL

Wave Length	THERMODYNAMICS		BOLOMETER				PYRHELIOMETER	
	7655°	6950°	Total	Mt. Whitney	Mt. Wilson	Washington	5810°	5450°
0.00μ	0.000	0.000	—	—	—	—	0.000	0.000
0.05	0.300	0.100	—	—	—	—	0.020	0.010
0.10	0.775	0.375	—	—	—	—	0.050	0.020
0.15	1.380	0.700	—	—	—	—	0.090	0.040
0.20	2.998	1.154	—	—	—	—	0.150	0.067
0.25	6.490	3.018	—	—	—	—	0.590	0.307
0.30	9.194	4.854	0.440	—	—	—	1.319	0.721
0.35	10.490	6.054	2.700	1.800	1.660	—	1.883	1.179
0.40	10.594	6.544	4.350	3.410	3.150	2.400	2.348	1.559
0.45	9.992	6.499	6.050	4.930	4.840	3.870	2.609	1.809
0.50	9.052	6.127	6.060	5.450	5.200	4.270	2.685	1.931
0.55	8.000	5.594	5.630	5.180	4.930	4.170	2.631	1.947
0.60	6.969	5.004	5.050	4.720	4.500	3.840	2.495	1.893
0.65	6.078	4.442	4.350	4.100	3.970	3.450	2.305	1.778
0.70	5.186	3.879	3.650	3.490	3.440	3.060	2.114	1.662
0.75	4.512	3.419	3.160	3.050	3.000	2.680	1.917	1.526
0.80	3.838	2.958	2.670	2.600	2.570	2.310	1.720	1.389
0.85	3.348	2.604	2.470	2.410	2.390	2.160	1.547	1.261
0.90	2.858	2.250	2.260	2.220	2.200	2.010	1.374	1.133
0.95	2.505	1.987	1.960	1.930	1.910	1.760	1.234	1.024
1.00	2.152	1.724	1.660	1.630	1.620	1.500	1.094	0.915
1.05	1.897	1.529	1.460	1.440	1.420	1.380	0.984	0.827
1.10	1.641	1.333	1.260	1.240	1.220	1.200	0.873	0.739
1.15	1.455	1.187	1.130	1.120	1.110	1.100	0.786	0.668
1.20	1.268	1.041	1.030	1.020	1.010	1.000	0.698	0.598
1.25	1.131	0.932	0.940	0.930	0.920	1.000	0.630	0.542
1.30	0.993	0.823	0.900	0.880	0.870	0.910	0.562	0.486
1.35	0.890	0.740	0.780	0.770	0.750	0.820	0.510	0.442
1.40	0.786	0.657	0.710	0.700	0.690	0.740	0.457	0.397
1.45	0.709	0.594	0.670	0.660	0.650	0.680	0.415	0.362
1.50	0.631	0.530	0.620	0.610	0.600	0.620	0.373	0.327
1.55	0.571	0.480	0.570	0.560	0.560	0.570	0.341	0.299
1.60	0.511	0.431	0.530	0.510	0.510	0.530	0.308	0.271
1.65	0.464	0.393	0.470	0.460	0.450	0.480	0.232	0.249
1.70	0.417	0.355	0.430	0.420	0.410	0.440	0.256	0.226
1.75	0.381	0.325	0.380	0.370	0.360	0.400	0.236	0.208
1.80	0.344	0.294	0.350	0.340	0.330	0.350	0.215	0.190
1.85	0.315	0.270	0.320	0.310	0.300	0.320	0.198	0.176
1.90	0.286	0.245	0.290	0.280	0.270	0.290	0.180	0.161
1.95	0.263	0.226	0.260	0.260	0.250	0.260	0.166	0.149
2.00	0.240	0.207	0.240	0.230	0.230	0.240	0.152	0.137
2.05	0.222	0.191	0.230	0.220	0.210	0.220	0.141	0.127
2.10	0.203	0.175	0.210	0.200	0.190	0.180	0.130	0.117
2.15	0.188	0.162	0.200	0.190	0.180	0.170	0.121	0.109
2.20	0.172	0.148	0.180	0.170	0.160	0.150	0.111	0.101
2.25	0.160	0.158	0.160	0.150	0.140	0.130	0.104	0.094
2.30	0.148	0.127	0.140	0.130	0.120	0.110	0.096	0.087
2.35	0.138	0.119	0.120	0.110	0.100	0.090	0.090	0.081
2.40	0.127	0.110	0.100	0.090	0.080	0.070	0.083	0.075
2.45	0.118	0.103	0.070	0.060	0.050	0.040	0.078	0.070
2.50	0.109	0.095	0.040	0.030	0.030	0.020	0.072	0.065
Sums	123.474	83.266	67.250	61.380	59.550	51.990	39.773	30.551

TABLE 79—*Continued*

Factor to reduce to gr. cal./cm.^{2} min. = 20.9

Reduced.....	5.91	3.98	3.22	2.94	2.85	2.49	1.90	1.46
Thermo-dynamics....	5.85	3.98	3.23	2.96	2.87	2.47	1.94	1.50
Log T.....	3.88395	3.84198	—	—	—	—	3.76418	3.73640
Log T^4.....	15.53580	15.36792	—	—	—	—	15.05672	14.94560
Log $\left(\frac{R}{D}\right)^2 \frac{60}{A}$..	−15.23180	−15.23180	—	—	—	—	−15.23180	−15.23186
Log J_0.....	0.76760	0.59970	—	—	—	—	0.28852	0.17740
J_0.....	5.856	3.978	—	—	—	—	1.943	1.505

earth, and it is derived from thermodynamics; the column 6950° is the effective radiation on the outermost stratum of the earth's atmosphere, and it is derived also from thermodynamics, as indicated; the column under 5810° corresponds with Abbot's extrapolated value of the solar constant 1.94 calories (compare Abbot's "Sun," p. 298); the column 5450° is equivalent to about 1.50 calories, the full amount measured by the pyrheliometer on the sea level. The four columns marked Total, Mt. Whitney, Mt. Wilson, Washington, are derived from Abbot's bolometer ordinates, Vol. II, Annals Smithsonian Institution, or the *Astrophysical Journal*, October, 1911, the method of reducing the original arbitrary units to the probable calories being given in Bulletin No. 3, p. 83. The values at the several wave lengths have been made as homogeneous as practicable in Table 79 and the corresponding curves are plotted on Fig. 25, where their mutual relations can be examined. The sums of the several columns are taken, and as they are comparable there should be a common factor to reduce the areas which are equivalent to the curves to the corresponding calories. Such a factor has been found to be 20.90. The factor used in Bul. No. 4, page 84, is 20.72, where the same table in part may be found. The lower portion of Table 79 contains the reductions in calories at the distance of the earth. Under "reduced" values the sums of the columns divided by the factor 20.9 are given; under "thermodynamics" are found the corresponding values, as deduced from the several independent discussions by thermodynamics, from bolometer observations, and from pyrheliometer reductions. It is readily seen that the

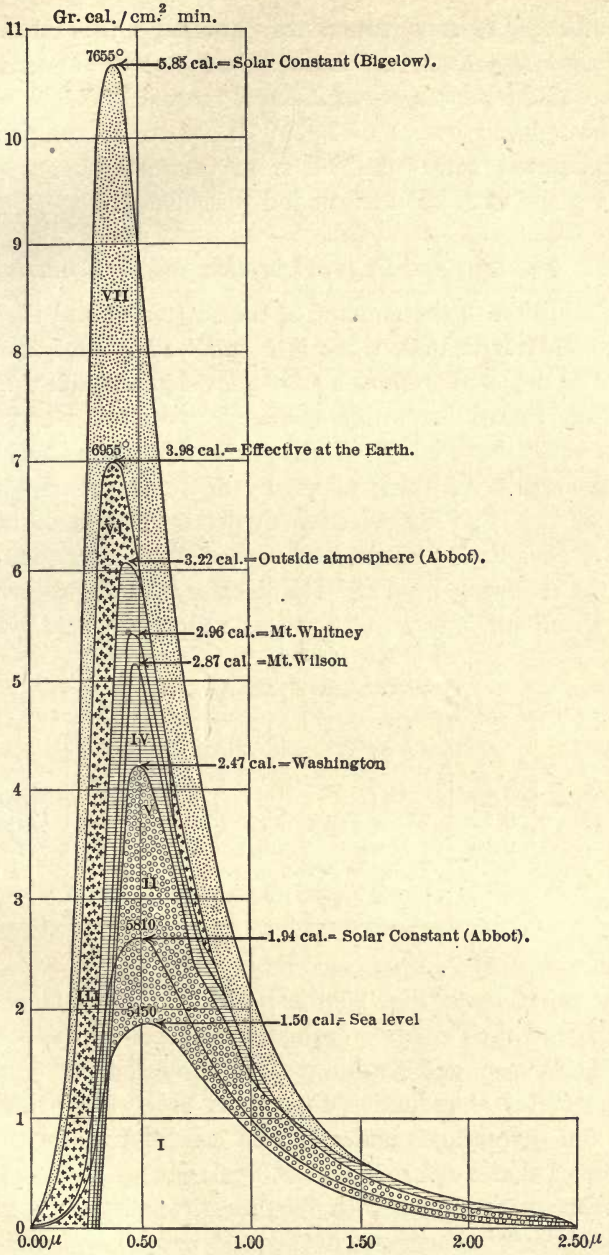


FIG. 25. Summary of the Data Derived from Thermodynamics, Bolometers, and Pyrheliometers.

data of Table 79 constitute a nearly homogeneous scheme, from the solar constant in the isothermal layers of the sun to the bottom of the earth's atmosphere at the sea level.

The reduced values of Table 79 are to be compared with the computed values derived from thermodynamics and the observations with bolometers and pyrheliometers.

The Line and Band Absorption and the Scattering

In addition to the amount of the scattering and the absorption already indicated, there is a small additional amount recorded as depleted areas of a triangular shape within the smooth spectrum curves heretofore assumed. Scattering and absorption modify the original effective solar spectrum, 3.98 calories on the earth's vanishing plane, by depressing many ordinates more or less, and by selective depletion at certain lines and bands. Mr. F. E. Fowle has studied these spectrum regions with the following results. The lines and bands, together with their absorbent, oxygen or aqueous vapor, are here given.

THE LINE AND BAND ABSORBENTS

Line	μ	Absorbent	Line	μ	Absorbent
B.....	0.69	Oxygen	Φ.....	1.13	Water Vapor
a.....	0.72	Water Vapor	ψ.....	1.42	Water Vapor
A.....	0.76	Oxygen	Ω.....	1.89	Water Vapor
—.....	0.81	Water Vapor	ω_1.....	2.01	?
ρ. σ. τ.....	0.93	Water Vapor	ω_2.....	2.05	?

Astrophysical Journal, December, 1915.

By referring to Fig. 25 the position of these lines can be located, and their relative ordinates for the stations, Mt. Whitney, Mt. Wilson, and Washington, can be calculated from Table 79, the factor of reduction to calories being 20.9. Mr. Fowle gives the percentages under certain conditions, and the corresponding calories referred to 1.930 calories for the temperature spectrum corresponding with 5800°. It is believed that this reference curve is unconfirmed in thermodynamics, and while adopting Mr. Fowle's relative ordinates as percentages, we reduce them

to the ordinates of Fig. 25, at least approximately, according to the bolometer ordinates of the station, Mt. Whitney 2.96 calories, Mt. Wilson 2.87, Washington 2.47, as given in Table 80.

TABLE 80

MEAN DEPLETIONS DUE TO SCATTERING AND ABSORPTIONS IN PERCENTAGES AND IN CALORIES

I. Reduced to Calories Observed by the Bolometer

Station	Precipitable Water in Cm.	Dry Air		Aqueous Vapor	
		Scatt.	Absd.	Scatt.	Absd.
Mount Whitney 4420 meters	0.00	7.3%	0.5%	—%	—%
	0.11	.....	.....	0.5	4.1
	0.25	.....	.....	0.5	5.2
	0.50	.....	.....	1.0	6.2
		7.3	0.5	0.7	5.2
Mount Wilson 1720 meters	0.00	7.8	0.5	—	—
	0.33	.....	.....	1.0	5.7
	0.50	.....	.....	1.6	6.2
	1.00	.....	.....	2.1	7.8
	2.00	.....	.....	4.7	9.3
		7.8	0.5	2.4	7.3
Washington Sea Level	0.00	9.3	0.5	—	—
	0.50	.....	.....	4.1	6.2
	1.80	.....	.....	13.5	7.8
	2.40	.....	.....	19.7	8.3
		9.3	0.5	12.4	7.4

Reduced to Calories	gr. cal. / cm.² min.	Dry Air		Aqueous Vapor		Total	
		Scatt.	Absd.	Scatt.	Absd.	Scatt.	Absd.
Mt. Whitney.....	2.96	0.216	0.015	0.021	0.154	0.237	0.169
Mt. Wilson.....	2.87	0.224	0.014	0.069	0.209	0.293	0.223
Washington.....	2.47	0.230	0.012	0.306	0.182	0.536	0.194

II. Reduced to the Calories Observed by the Pyrheliometer

Mt. Whitney....	1.72	0.126	0.009	0.012	0.089	0.138	0.098
Mt. Wilson.....	1.64	0.128	0.008	0.039	0.120	0.167	0.128
Washington.....	1.50	0.140	0.007	0.186	0.111	0.326	0.118

In Section I the percentages of the scattering and the absorption, as measured from the bolometer ordinates, have been reduced to gr. cal./cm.^{2} min.; in Section II the same percentages have been applied to the mean values of the radiation intensity observed by the pyrheliometer. These mean values give a general idea of the values to be expected, and they must be applied to the observations by the bolometer and the pyrheliometer, respectively, in such a way that both series shall give the same value of the effective solar radiation, which should be alike in all strata from the sea level to the vanishing plane of the earth's atmosphere.

We may now confirm the foregoing analysis of the thermodynamic data and the observed data of radiation as given, by comparing these with the data of the bolometer as contained in Table 79 and Figs. 25, 26. The several regions from I to VII have been marked on them respectively.

I. Kinetic energy up to the curve 5450°, equal to 1.50 calories.

II. Potential energy between the curve 5450° and the bolometer curve for Washington, 2.47 − 1.50 = 0.97 calorie as compared with 0.95 calorie on Table 79.

III. The specific heat term, $(Cp - Cv) = R$, which is made up from the absorption of the short wave ordinates. This amounts on the sea level to 1.19 − 0.19 = 1.00 calorie. The summation of the short wave ordinates from 0.00 μ to 0.38 μ is about 20.231; if this is divided by the factor 20.9 the result is 0.98 calorie. The absorption of the short wave ordinates is necessary in order to change the specific heats of the atmosphere from that of the inner energy Cv, $U_u = Cv\,\rho\,T$, into that of the gravitation energy U_p, $-g\,(z_1 - z_0) = Cp\,(T_a - T_0)$.

IV. The reflected or scattered energy.

V. The absorbed energy used in molecular and atomic forms. The detailed distribution of these energies in the lowest strata will require more study. On the Washington level $U = H + J + s = 2.62$ and on Table 79, 2.49; on the Mt. Wilson level $U = 2.82$ and on Table 79, 2.85; on the Mt. Whitney level $U = 2.98$ and on Table 79, 2.94. These points are indi-

cated on Fig. 26. Compare the results from thermodynamics.

VI. The transition region from the electromagnetic energy to the thermodynamic energy of ionization may provisionally

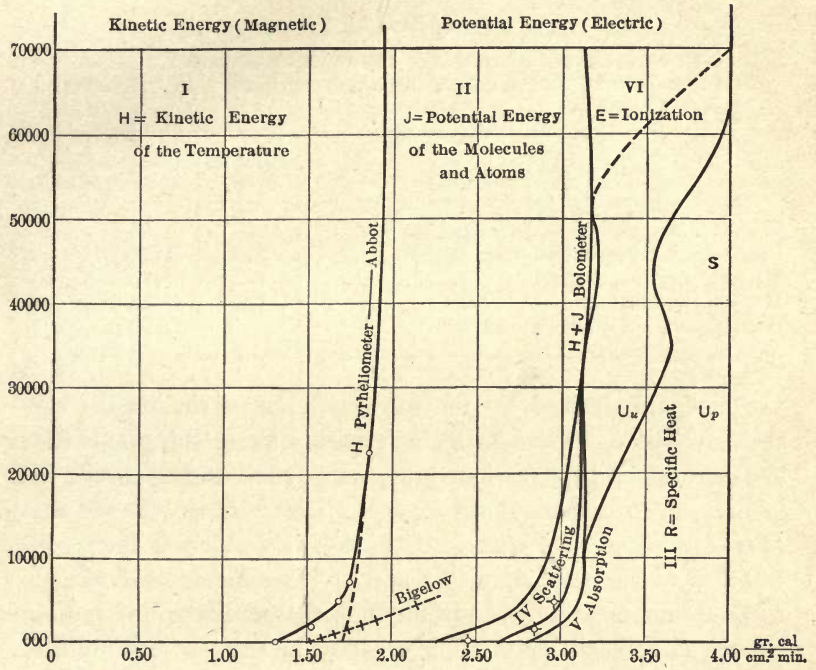


FIG. 26. The Seven Components Which Make up the Intensity of the Solar Radiation at the Earth, 3.98 gr. cal./cm.² min.

be assigned to the high-level absorptions along the slope of the ordinates from the wave length 0.30 μ to 0.50 μ.

VII. The great depletion from the total solar intensity, 5.85 calories, to the effective intensity at the earth, 3.98 calories, making the difference 1.87 calories, is contained in the area between the 7655° and the 6950° black body curves.

Direct Readings of the Pyrheliometers at Great Heights

In a paper, " New Evidence on the Intensity of Solar Radiation Outside the Atmosphere," 1915, Mr. Abbot summarizes the results of his direct readings of pyrheliometers on balloons

up to about 22000 meters. Taking these in connection with the pyrheliometer readings in the zenith at Mt. Whitney, Mt. Wilson, Washington, D. C., we have the following data:

TABLE 81

SUMMARY OF THE DIRECT READINGS OF THE PYRHELIOMETERS WHEN REDUCED TO THE ZENITH, AND WITHOUT ANY EXTRAPOLATION TO THE SUPPOSED SOLAR CONSTANT.

Location of the Pyrheliometer	Height in Meters	Zenith reading gr. cal./cm.^{2} min.
Abbot balloon ascension, July 11, 1914.....	22000	1.84
Peppler balloon ascension, Oct. 19, 1913.....	7500	1.76
Mt. Whitney, maximum.....	4420	1.72
Mt. Wilson, maximum.....	1780	1.64 (modified)
Washington, D.C., or sea level.....	0	1.58 (modified)

The data derived by the self-registering pyrheliometer carried on balloons serve to fix new points along the graph that has heretofore been assigned to the pyrheliometer curves, as in Fig. 26, so that *the main features of the problem have not been in the least modified by their addition.* The problem is the reconciliation of the pyrheliometer and the bolometer observations without making the assumption that the sun does not radiate energy as a black body. The mean value of I at Mt. Wilson is 1.53, the maximum being 1.64, and the mean value at Washington is 1.33, the maximum being about 1.58 calories.

Summary of the Pyrheliometer Results as Reduced by Bigelow's Method

The result of the discussion of the pyrheliometer observations at different stations, by Bigelow's method of reduction, is collected in Table 37, Bulletin No. 4, O. M. A., of which the following is an extract.

The height z is in meters, and the free heat received by the pyrheliometer is I_a, after the depletions, due to aqueous vapor, dust, and other matter near the station, have been eliminated. It may be noted that by the Bigelow method of reduction the pyrheliometer is made to record the kinetic energy of region

TABLE 82

SUMMARY OF THE AMOUNTS OF THE SOLAR RADIATION AS DETERMINED BY PYRHELIOMETER OBSERVATIONS

Station	z	Ia	
La Confianza	4483	2.142	gr. cal.
Mt. Whitney	4420	2.138	cm.² min.
La Quiaca	3465	2.005	
Humahuaca	2939	1.930	
Maimará	2384	1.852	These are plotted on Fig. 26 as crosses x x x x x while the Abbot data appear on the graph which is marked "Pyrheliometer."
Mt. Wilson	1780	1.771	
Jujuy	1302	1.705	
Bassour	1160	1.675	
Mt. Weather	526	1.601	
Cordoba	438	1.592	
Pilar	340	1.572	
Potsdam	89	1.538	
Washington	34	1.529	
Sea level	0	1.525	

I and in part the potential energy of region II. It is apparent that the course of the line of crosses on Fig. 26 is seeking the line $(H + J)$ which bounds the potential region. It is to be hoped that suitable observations can be made in balloon ascensions, which will enable this idea to be verified. Abbot's observation is along the line H, while Bigelow's reduction has a very different vertical gradient from Abbot's, that is, along the dotted line.

Computation of the Pyrheliometer Data, Leading to 3.98 gr. cal./ cm.² min. as the Intensity of the Solar Radiation at the Earth

The results of our computation are contained in Table 83 and Fig. 26. The data of Tables 60–69, Bulletin No. 4, Oficina Meteorológica Argentina, have been adopted, except for the stations Cordoba and La Quiaca, where the individual observations were computed throughout the years 1912, 1913, 1914, and 1915, and the general means of the entire series are placed in Table 83. This table, therefore, represents the average condition at each station after the local and periodic action has been eliminated. The method applied to the daily observations is equally valid.

We have exhibited the following components, into which the solar radiation divides itself in the earth's atmosphere:

1. H = the kinetic energy of the translation of the molecules, measured by the pyrheliometer as *temperature. It is evident that this instrument is a means of registering only kinetic energy,* and that the various forms of the potential energy in the atmosphere do not come into its competency. The bolometer, on the other hand, by means of its *relative ordinates,* does indicate some portions of this potential energy in its results, though in a confused fashion. The thermodynamics has the power to classify clearly the distribution of the various forms of the potential energy in all the strata of the atmosphere.

TABLE 83

COMPUTATION OF THE PYRHELIOMETER DATA TO INCLUDE THE POTENTIAL ENERGY, THE SPECIFIC HEAT, AND THE IONIZATION

Name of the Station	z Height, in Meters	ϕ Latitude	e Vapor Press.	k Coefficient Scattering	H Kinetic Energy	J Potential Energy	$H+J$ Inner Energy	s Scattered Energy	a Absorbed Energy	ΔU_e Vapor Effect	R Specific Heat	S Solar Radiation	$(a+R)$ Absorption and Spec. Heat
La Confianza...	4485	−20°	1.7	.072	1.741	1.116	2.857	.205	.080	−.004	0.840	3.988	0.920
Mt. Whitney...	4420	+37	2.1	.088	1.668	1.069	2.737	.241	.120	+.001	0.880	3.979	1.000
La Quiaca.....	3462	−22	3.3	.103	1.662	1.065	2.727	.281	.080	−.008	0.900	3.980	0.980
Mt. Wilson.....	1727	+34	5.8	.119	1.534	.983	2.517	.300	.125	+.010	1.020	3.972	1.145
Bassour.....	1160	+36	7.8	.172	1.386	.889	2.275	.392	.155	+.010	1.145	3.977	1.300
Mt. Weather...	526	+39	8.5	.178	1.322	.847	2.169	.386	.255	+.006	1.175	3.991	1.430
Cordoba.....	438	−31	9.4	.161	1.446	.927	2.373	.382	.102	−.005	1.125	3.977	1.230
Washington....	34	+39	4.7	.201	1.329	.852	2.181	.438	.177	−.004	1.188	3.980	1.365

z = the height in meters; ϕ = the latitude of the station.

e = the vapor pressure in millimeters. Unless this is reduced to a mean value for the station there will be an annual variation. The successive mean e_0 are 2, 2, 4, 5, 7, 8, 10, 5 mm. for these stations. The coefficient 0.012 was determined as the equivalent in calories for 1 mm. of e. Hence $\Delta e = 0.012\ (e - e_0)$.

k = the coefficient of scattering. $s = k\ (H+J)$.

$H = I_1$, the kinetic energy. This is the value I_1 of the zenith intensity of the radiation, observed by the pyrheliometer, and reduced by the Bouguer Formula $I = I_0\, p^{\sec z}$. No extrapolation is made beyond sec $z = 1$.

J = the potential energy, $J = 0.641\ H$.

$H + J = U$ the total inner energy after its losses by scattering and absorption.

$(a+R)$ = the specific heat effect plus the absorption. R is computed from thermodynamic data as in Table 75, and elsewhere, and an approximate value has been assigned for each station. If the mean solar constant at the earth is 3.980 calories, then we have $a = S - (H+J+s+\Delta e+R)$. The absorption in Table 83 has been obtained in this manner and it is in harmony with Fowle's results.

2. J = the potential energy in the atmosphere, corresponding with the kinetic energy through the formula, $J = 0.641\,H$.

3. s = the scattering, or reflection of the radiation on the air contents, without affecting the temperature as such on the levels below that of the reflection. If the coefficient is k as obtained from $k = (1.00 - p)$, where p is computed from the pyrheliometer observations, then $s = k\,(H + J)$ the part which is not transmitted.

a = the true absorption, as shown by the band and line depletions of the ordinates of the several stations. It is computed indirectly from the adopted mean intensity of the solar radiation,

$S = 3.980$ calories, by the formula,

(167) $$a = S - (H + J + k\,(H + J) + \Delta e + R).$$

Δe = the vapor pressure correction to a mean value of e_0, with a coefficient 0.012,

(168) $$\Delta e = 0.012\,(e - e_0).$$

R = the specific heat energy,

(169) $$R_{10}\,(T_a - T_0) = (Cp_a - Cv_{10})\,(T_a - T_0),$$

which is necessary to change the inner energy at the specific heat of constant volume U_v into the inner energy at the specific heat of constant pressure U_p. The Boyle-Gay Lussac Law in the free gas leads to U_v, where there is no gravitation, but to U_p where there is gravitation, as by formulas, (1) to (10).

Thus, we have, generally,

(170) $$U_v = Cv\,\rho\,T = (Cp - R)\,\rho\,T$$

(171) $$U_p = Cp\,\rho\,T = (Cv + R)\,\rho\,T = -g\,\rho\,z$$

It is U_v which is depleted by scattering, absorption, and ionization, so that we have,

(172) $$U_v = H + J + s + a + \Delta e + E \text{ (ionization.)}$$

To obtain the stratum-intensity of the radiation.

(173) $$S = U_v + R\,\rho\,T,$$

in conformity with final equilibrium against the impressed force of gravitation.

It should be noted that we have arrived at the solar radiation S, and by Table 83 find it to be 3.980 *calories on each stratum. In this process there has been no extrapolation by the Bouguer*

formula beyond its legitimate mathematical function in the zenith, so that all difficulties on that score are fully obviated.

E = the ionization, in the great region marked VI, which lies between $(H + J) + (s + a)$ and U_v.

There are two points to be noted in respect of the ionization: (1) Since the potential energy of the electromagnetic radiation is 1.00 on a scale of 2.00, and the atmospheric potential energy is 0.641, it follows that there is a change of 0.359 on the same scale. Hence, we have,

$$E = 0.359 \times \frac{1}{2} \times 3.980 = 0.714 \frac{\text{gr. cal.}}{\text{cm.}^2 \text{ min.}}$$

the amount of the solar radiation which is probably transformed into electric energy of the free ions in the upper levels, as produced by the action of the short waves upon the atmospheric atoms and molecules.

(2) In Fig. 26, the transition between the electromagnetic potential energy and the thermodynamic potential energy is represented as abrupt or discontinuous. This is probably not the case in nature, and it is likely that a gradual change occurs in the coronal region, between 90000 and 50000 meters, while the process is intensified along the upper branch of the isothermal layer, 50000 to 37000 meters, where the temperature is rapidly changing its value. *If the production of ions is to be associated with rapid changes of temperature in atmospheres*, we have five conspicuous regions where ionization occurs:

1. At the bottom of the solar isothermal layer.
2. At the top " " " " "
3. At the top " the terrestrial isothermal layer.
4. At the bottom " " " " "
5. At the bottom of the atmosphere and in the earth. These are the regions where electric charges are known to accumulate and to manifest themselves, as solar static electric fields, as currents of electricity inducing variations in the normal magnetic field, as auroras in the high level electricity in the earth's atmosphere, thunderstorm energy in the convectional region, and at the surface of the earth itself.

Synchronism of the Solar and the Terrestrial Variations during the Interval 1900–1915 in Argentina

It is of interest to extend the computation of the intensity of the radiation to the annual values, in order to compare the variations with the other series of annual variations derived from various solar and terrestrial sets of data. This can be done immediately for the years 1903–1916, and the record can be carried back to 1883 by means of the observations at Montpellier, Geneva, and Warsaw. Table 84 contains the summary of the annual values. It may be recalled that the computation here employed is the third attempt to make the pyrheliometer data conform to the requirements of solar and terrestrial thermodynamics, which sustain the bolometer data in requiring about 4.00 calories for the effective solar radiation at the earth, in place of the 2.00 calories derived from the pyrheliometer as interpreted by the Langley-Abbot method of discussing the Bouguer formula of depletion. The first method introduced a functional correction to I_0 due to the vapor pressure effect,

$$(174) \qquad I = (I_c - 0.0214\, e)\ p^{\sec z},\ (I_o = I_c - 0.0214\, e).$$

The second method was an attempt to reconcile the very discordant results derived from stations at different altitudes, and consisted in making p_w a function of the vapor pressure,

$$(175) \qquad I = I_0\,(p_w - f\,e)^{\sec z}$$

where each station has an equation for $f\,(e)$, Bulletin No. 4, O. M. A., Chapter 5; Treatise, Chapter VII. Both of these are methods of extrapolation, but the third method, described above, completely rejects every attempt to extrapolate beyond $\sec z = 1$, and it *succeeds in reducing each station on its own level to the solar intensity independently of other stations, and without making any use of certain bolometer factors*, which are excessively complex, and impractical of attainment under ordinary conditions of pyrheliometer observations.

In transferring these results to Fig. 27, the value 3.914 for Mt. Whitney, 1910, seems to be rather too small, due to the

TABLE 84

SUMMARY OF THE MEAN ANNUAL VALUES AS COMPUTED BY THE THERMODYNAMIC METHOD

Station	z	1903	1904	1905	1906	1907	1908	1909	1910	1911	1912	1913	1914	1915	1916
La Confianza	4485	—	—	—	—	—	—	—	—	—	—	3.978	—	—	—
Mt. Whitney	4420	—	—	—	—	—	—	3.981	3.914	—	—	4.017	—	—	—
La Quiaca	3465	—	—	—	—	—	—	—	—	—	3.983	4.016	4.021	4.024	—
Mt. Wilson	1780	—	—	3.960	3.969	—	3.925	3.957	3.961	3.975	4.006	—	—	—	—
Bassour	1160	—	—	—	—	—	—	—	—	3.960	3.950	—	—	—	—
Mt. Weather	526	—	—	—	—	—	3.957	3.950	—	3.945	3.790	—	—	—	—
Cordoba	438	—	—	—	—	—	—	—	—	—	3.973	4.000	3.983	3.956	3.920
Washington	34	3.989	3.979	3.950	3.980	3.971	3.939	3.954	3.982	—	—	—	—	—	—
Means		3.989	3.979	3.955	3.975	3.971	3.940	3.960	3.952	3.960	3.978	4.003	4.002	3.990	3.920

In computing this table an attempt was made to eliminate the effects of dust and vapor in the lower strata. Since this is so important a term another computation has been made, and the results are indicated in the last chapter.

limited number of the observations, but it is retained; the value for Mt. Weather, 1912, 3.790, seems to have been unduly depressed by the dust from the Volcano Katmai. There are local conditions of this sort which tend to distort the annual means, and as they cannot be eliminated from the normal in any simple manner, it becomes necessary to utilize a larger number of stations for pyrheliometer records in various parts of the world. These should be made and reduced on an accepted uniform plan, which conforms to the requirements of thermodynamics as well as of homogeneous statistics. The annual values in Table 84 produce a curve which is similar in its general features to the one in Table 93 of the Treatise, though it here presents an additional minor maximum for the year 1911.

We shall next proceed to compare these results with other easily accessible data, as given in Table 85 and Fig. 27. Similar tables of comparison have been prepared for Argentina, from 1870 to 1916, and they present quite the same general facts of synchronism throughout this interval.

It is necessary to receive prompt reports of the frequencies of the sun-spots and the prominences in order to utilize them in any forecast service for the use of the public.

There are several facts which appear quite distinctly.

(1) The sun-spot frequencies, the prominence numbers, the solar radiation, the amplitudes of the horizontal magnetic force, the precipitation, temperature, vapor pressure, and barometric pressure all agree in conforming to the long period of 11 years.

(2) They all conform with certain local irregularities in respect of maxima and minima in the subordinate 3.75-year period.

(3) The sun-spots, prominences, magnetic field, and barometric pressure all move together in a *direct* synchronism, while the radiation, the precipitation, the temperature in part, and the vapor pressure move in an *inverse* synchronism.

(4) Taking the entire set of curves as attempting to express a fundamental law of solar activity, it is thought that a minor crest occurs at the maximum of the 11-year period, that a

TABLE 85

SUMMARY OF THE SOLAR AND TERRESTRIAL OBSERVATIONS SHOWING A SYNCHRONISM IN THE VARIATIONS IN THE 11-YEAR AND THE 3.75-YEAR PERIODS

Year of Observation	Sun-spots	Promi-nences	Radia-tion	Hori-zontal Mag. Ampl.	Precipi-tation	Temper-ature	Vapor Pressure	Baro-metric Pressure
1900.....	114	128	.....	1589	855	17.81	10.50	749.28
1901.....	33	88	.....	1374	573	17.87	9.88	49.43
1902.....	60	47	.....	1464	716	17.68	10.35	48.59
1903.....	293	118	3.989	1926	775	17.49	10.52	49.22
1904.....	503	291	3.979	1787	914	17.17	10.30	49.12
1905.....	762	302	3.955	2194	719	16.75	10.22	49.24
1906.....	646	261	3.975	2015	613	17.55	10.07	49.33
1907.....	745	427	3.971	2299	648	16.83	9.86	49.52
1908.....	583	346	3.940	2584	615	17.11	9.55	49.97
1909.....	527	365	3.960	2334	566	17.08	9.34	49.90
1910.....	223	259	3.952	2354	651	16.96	9.31	49.59
1911.....	68	163	3.960	1923	751	16.40	9.26	49.75
1912.....	43	126	3.978	1520	811	17.51	10.09	49.72
1913.....	17	135	4.003	1564	704	18.19	10.53	49.44
1914.....	115	198	4.002	1672	928	17.28	10.78	48.77
1915.....	350	250	3.990	2635	773	17.32	10.06	48.92
1916.....	540	400	3.920	2676	486	17.65	8.87	49.00

Sun-spots.—Data from Wolfer's Frequency Numbers, Meteorologische Zeitschrift, May, 1902, May, 1915.

Prominences.—Data from Ricco's Distribuzione delle protuberanze, Mem. della Soc. degli Spettro. Italiani, March, 1914.

Radiation.—Bigelow's reduction of the pyrheliometer data.

Horizontal Magnetic Amplitudes.—Bigelow's compilation from European observatories, and Pilar, Argentina.

Precipitation, Temperature, Vapor Pressure, and Barometric Pressure, from the Argentina stations, Tucuman, Andalgala, Goya, Concordia, Cordoba, Pilar, Buenos Aires, Victorica, Bahia Blanca, Patagones.

second crest occurs near the top of the ascending branch, that a third crest occurs near the top of the descending branch, and a fourth crest near the bottom of the descending branch. Sometimes one or the other of these minor crests fails in the record, or it is distorted by the local conditions, or the sun itself does not always work upon the normal plan, so that the result is complex. Each Argentine station clearly resembles this model under the influence of minor distortions, and they support this proposition.

(5) In the case of the temperature, it is debatable whether the minor crests are not inverse on the long 11-year period, so

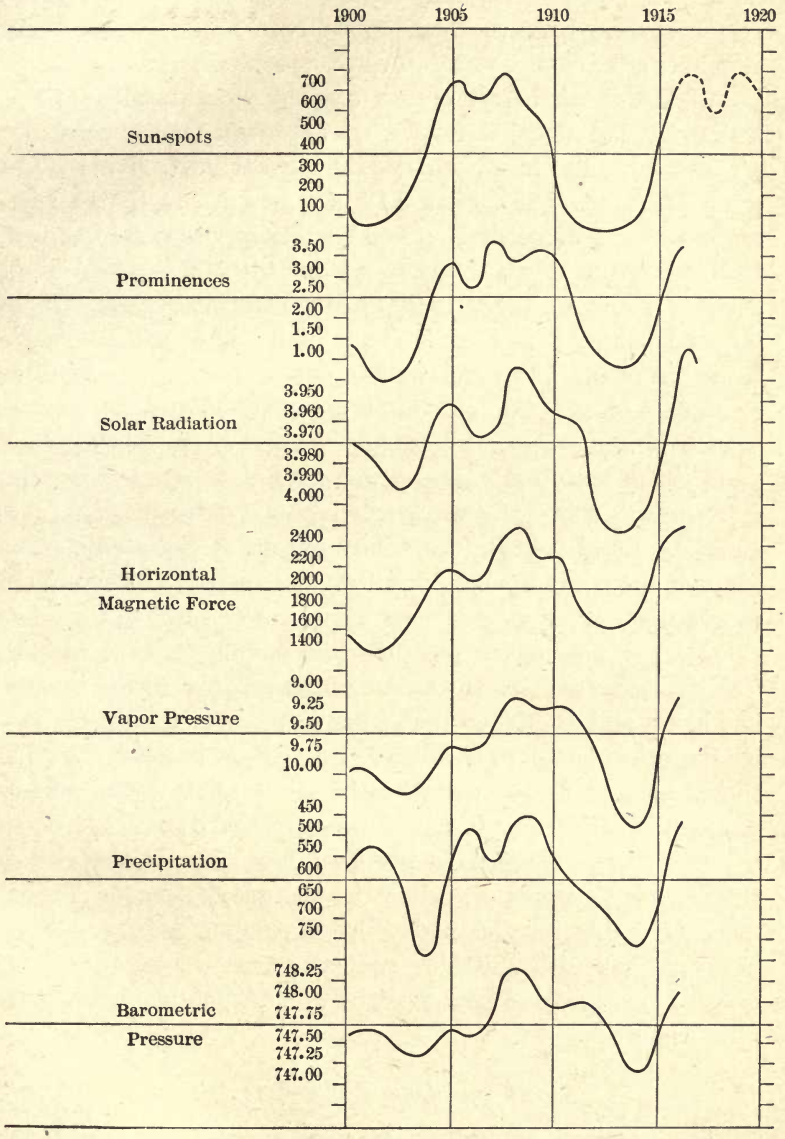


FIG. 27. Synchronism of the Solar and Terrestrial Variations.

as to become practically direct. This must be due to the influence of precipitation and clouds, making the temperature at the surface beneath them opposite to similar temperature data above the cloud sheet, if it could be systematically recorded.

(6) If the rule here proposed conforms to natural solar and terrestrial variations, it follows that one can readily construct a new solar curve in advance of the current year, giving it the mean 11-year and the mean 3.75-year periods, so that a long-range forecast of general annual conditions becomes practical. In this way the 11-year maximum is in 1918 and the four minor maxima occur in 1916, 1918–19, 1920–21, and 1923. These should be the years of minimum rainfall in Argentina, relative temperature maxima, low vapor pressure and high barometric pressure. *If sufficient observations are maintained to keep in touch with any eccentric behavior on the part of the sun, they will enable us to modify this general statement from year to year, and it is generally true that a forecast for a year or two is without doubt possible.* Local peculiarities will require a special study. The effect of the ocean upon coast cities, and the effect of mountain ranges upon cities located near them, make such stations less valuable, as they tend to disturb the smoothness of the result. This is easily seen at Buenos Aires and Bahia Blanca on the one hand, and at Mendoza and San Juan, near the Cordilleras, on the other hand. The inland stations are less liable to disturbances, and these respond to the solar impulses with greater regularity. It is noted that the long period comes out much better in Patagonia than in northern Argentina, in the case of the barometric pressure; while the latitude of Buenos Aires is more favorable for the periods of the vapor pressure. It is believed that with suitable experience such a system of long-range forecasts is easily available for Argentina and many other countries.

Short- and Long-Range Forecasts

The practical application of solar physics in meteorology consists in developing a system of interpretation of solar phenomena, such that the variations of the solar activity may be-

come intelligible in terms of climatic effects. The complexity of the physical mechanism is so great that the progress toward the solution is slow, although there is no reasonable doubt that an effective synchronism exists. The efforts to justify this procedure have been entirely statistical, whereby a succession of maxima and minima has been determined in a general way, coordinating the two parts of the process. Many irregularities are encountered, and in a country of violent cyclonic action, like the United States, the symptoms are much confused, in Europe they are nearly extinct, while in Argentina they are comparatively simple and distinct. Criticism is so mixed up with an incomplete knowledge of the science that the growth of the research is unduly retarded; the enormous accumulation of the statistical data makes its treatment very oppressive; the data are not sufficiently homogeneous to bring out clearly the small true solar residuals; the observations are made in the interests of short-range forecasts to the entire neglect of the long-range forecasts of solar physics; the several branches of the subject are so scattered as to be unmanageable, the solar physics, the bolometric spectra, the magnetic field, the electric ionization, the climatic meteorology, all being separated among different administrative offices; the published reports from these different sources are so retarded that they are useless in any study of current forecasts whatsoever; there exists no systematic machinery for handling this kind of world-wide material.

It is inevitable that several international institutes shall be established—*at least four*, one in the United States, one in Argentina, one in Europe or Asia, and one in Africa or Australia—which shall cooperate upon a fixed general plan, utilize selected data, and treat the subject from the world-wide point of view. The long-range system will begin with annual forecasts, already easy in Argentina, and advance to details in accordance with the progress in solar physics. Such international institutes must be equipped with the highest type of apparatus, continually improved as conditions permit.

The progress in solar physics has been retarded, as far as concerns meteorology, by the insistence upon immediate fore-

cast results, before the fundamental laws have been classified. This demand has already made it unnecessarily difficult to obtain the facilities requisite to work out the problems. It should be remembered how small is the margin of success in short-range forecasts, with all the extensive coordination that the national services actually possess. In the United States the annual amount of fair weather averages about seventy-five per cent and the rain twenty-five per cent. Of this twenty-five per cent of rain, for predictions at fixed 12-hour intervals in advance, 12 to 24, 24 to 36, 36 to 48 hours, the maximum success is about 33 per cent, making 83% the average actual forecast; similarly with the temperature. All the percentages above that amount, *that is, a gain of only eight per cent,* are due to elastic rules of verification. These are changed from time to time, but, in fact, there has been no real improvement in the past thirty years. In Argentina the fair weather percentage is higher, and the corresponding average, also, higher.

The real utility of weather services is in the general climatological records of rainfall and temperature, and in an occasional important forecast of storm conditions, frost conditions, and, especially, in the educational propaganda. The scientific development has been entirely sporadic, depending on some individual initiative. As matters stand, quite aside from the idea of international cooperation by central institutes, there is still a very wide margin for gain in the theory and practice of long-range forecasts. (1) Up to the present time meteorology has had no central unifying analysis. The investigations have been based upon the adiabatic formulas, which are wholly inapplicable in the non-adiabatic atmospheres of the earth and sun. It cannot be doubted that a sound central theory will have the same good influence upon atmospheric thermodynamics, that Newton's gravitation and Kepler's laws of motion had upon the primitive astronomy. (2) In the matter of solar radiation the amount available is not 1.94 calories, as has been claimed, but 5.85 calories, a margin of 200% to the good for future developments. (3) The method of distributing meteorology, radiation, magnetism, and solar physics, among distinct and

independent institutions, is on the same level of efficiency as an army would be to repel invasion with the infantry in New York, the artillery in New Orleans, the cavalry in San Francisco, and the commissariat in Chicago. (4) The value of long-range forecasts is very great economically, and the practical ability to gain 8 or 10% on fair weather forecasts is not so forbidding, as may be supposed. Until the possibilities of science have been exhausted, there is not the least ground for withholding men and money from such an enterprise. Unless the government services approach this subject in an efficient way, there can be no doubt that the demands of commerce and agriculture will seek for other channels of development, which will extend the range of the forecasts far beyond their present limits.

It should be noted that the small range of the solar and terrestrial synchronisms of the radiation, and its various manifestations, culminates in the case of the precipitation in Argentina by registering about 40% of the total as an annual variation. This is enough to distinguish good and bad agricultural years, and fully justifies all the cost of the research. When this is perfected it will be of prime value to the national interests. In the United States the registration approaches 30% of the total in some localities.

CHAPTER VII

Other Solar Phenomena

Restatement of the General Line of Argument

BEFORE entering upon a brief discussion of the other solar phenomena, following from the results of the thermodynamic relations which have been developed on the sun, it is proper to restate the general line of argument that has been explained regarding the solar radiation. It has been shown that all gases of atmospheres open to space on one side, but resting upon a solid globe like the earth, or a viscous core like the sun, possess the same thermodynamic configuration for the elements of the Boyle-Gay Lussac law, $P = \rho R T$. These are illustrated in Figures 2–13. Near the bottom of the isothermal layer, where the temperature gradient suddenly changes from the adiabatic rate, there is a conversion of the chemical elements from their electron-ion-atomic state into their electron-ion-molecular configuration. During this process there occurs a powerful readjustment of the electric charges, accompanied by numerous rapid changes in the velocities of the electrons, the electromagnetic oscillations within the gas, and release of plane electromagnetic waves into space. The thermodynamic elements fix the temperature of the radiation for all elements at about 7655°, and the evidence is clear that such radiation is primarily of black energy, including all the wave lengths in the spectrum. This energy passes through deep layers of superincumbent gas, and the electromagnetic energy suffers its first depletion in passing over from electric oscillations into thermodynamic collisions and free paths, whereby the equivalent value in gram calories per cm.^{2} per minute at the distance of the earth changes from 5.85 calories to 3.98 calories. The latter amount is determined as that which is

received at the earth, since the computation of it within the solar gases involves many physical terms and coefficients that are still difficult of assignment. The loss of brightness between the center and the limb conforms to this 1.87 calories. The determination of the terrestrial intensity, 3.98 calories, depends upon three different lines of research.

I. The observed ordinates of the thermal spectrum by means of the bolometer are best satisfied by several different effective temperatures, such that the wave lengths 1.50 μ – 2.50 μ originate at about 7655°, while the wave lengths of 0.00 μ to 1.50 μ belong to a complex spectrum energy of 6950° with certain depletions.

II. Such a radiant energy in the space through which the earth passes in its orbit is competent to generate certain temperatures in the earth's atmosphere, in equilibrium with the kinetic energy, potential energy, inner energy at the specific heat of constant volume, and the inner energy at the specific heat of constant pressure as required by the force of gravitation. Such conditions of equilibria are built up with the lapse of time; they are independent of the direction of the radiant ray, whether reflected at the earth's surface or direct from space; and they are accompanied by such pressures, densities, thermal coefficients as are determined by observations between the sea level and the vanishing plane of the atmosphere. The several lines of thermodynamic computations all converge upon the value 3.98 calories for the effective intensity of the solar radiation at the distance of the earth, thus agreeing with the bolometer ordinates between 0.00 μ and 1.50 μ.

III. The pyrheliometer measures the kinetic energy of radiation in temperature-degrees at the distance of the sun from the zenith, and by the Bouguer formula its value can be found in the zenith, together with the coefficient of absorption near the station. In the computations the corresponding air-potential energy, the corrections for the specific heat, the depletion by scattering and by absorption, can be supplied, and these again lead to 3.98 calories at stations in the northern and

the southern Americas. The Langley-Abbot method of reducing the pyrheliometer observations involves a number of assumptions and omissions which invalidate it.

1. Extrapolation to the non-mathematical $\sec z = 0$.
2. Assumption that $\sec z = 1$ necessarily is identical with the depth of the earth's atmosphere.
3. Assumption that the line and band depletions used in making up the bolometer factor should be referred to the spectrum of 6000°, instead of 6950°, which is purely arbitrary.
4. Omission of the potential energy term.
5. Omission of the specific heat term.
6. Omission of the ionization energy term. Removing these defects the pyrheliometer data become accordant with the bolometer data, and the thermodynamics of the atmospheres of the earth and the sun.

The terrestrial meteorological data are much better satisfied practicably by reference to a radiation energy of about 4.00 calories than they are by that of 2.00 calories. In fact, it is impracticable to build up the prevailing temperatures, pressures, and densities, together with their climatological changes, by employing Abbot's solar intensity of 1.930 calories.

We shall now proceed to make further use of the solar thermodynamic data in reference to the origin of the sun-spots, faculæ, prominences, sharp disk of the sun, apparent diminution of brightness between the center and the edge of the sun, the corona, the solar magnetic field, the solar electrostatic field, the general circulation in the sun's mass. Each of these subjects requires a prolonged research for developing its details, so that only a short descriptive statement will be practicable in this chapter.

The Effect of the Solar Isothermal Shell upon the Visible Surface Phenomena

The most prominent fact regarding the physical constitution of the sun is that there exists an isothermal shell of the mean temperature of about 7680° located in the midst of the solar gases at a fixed distance from the center of the mass.

This shell is maintained by the thermodynamic processes in this position by the interrelated actions of gravitation and radiation, as has been explained. The outer surface is called the photosphere, and it appears as a sharp circle on the disk, because the heavy gases vanish near those levels, and on account of their large number they produce the observed optical brilliancy of the sun. The lighter gases extend upward through the levels of the flash spectrum, the chromosphere, and the inner corona, the latter being most conspicuous in hydrogen. Beneath the isothermal layer the gases recede at adiabatic rates to great depths, which finally coalesce in the quasi-solid nucleus. Compare Section I, Fig. 28. In the midst of the isothermal layer the gases go through their thermodynamic development in strata having a special depth for each gas, and yet they are entirely independent of one another. The isothermal layer is a complex of numerous gases, each one agreeing with all the others as to temperature, while differing as to the vertical scale of their strata of equal temperatures. We may, therefore, having recognized the very complicated structure of the isothermal shell, proceed with further explanations, substituting the simple idea of a single shell of uniform temperature, at the mean distance $R = 694800800$ meters from the center of the sun. The sun-spots, faculæ, and spectra, all testify to the complex internal structure of the solar gases in the neighborhood of the photosphere, and some progress has been made heretofore in classifying these phenomena, though this was much impeded by the lack of a definite knowledge of the pressures, densities, and gas efficiencies which are associated with them.

The Granulations, Faculæ, Flocculi, and Prominences

The literature regarding the appearance and origin of these solar phenomena is very extensive, and it consists of numerous explanations of the probable causes which produced them, in accordance with some observed physical conditions on the sun. We shall have much advantage in this respect from an accurate

knowledge of the temperatures, pressures, densities, and gas efficiencies that prevail, and it is not too much to say that approximate computations can easily be made, such as would reproduce these appearances in the neighborhood of the photo-

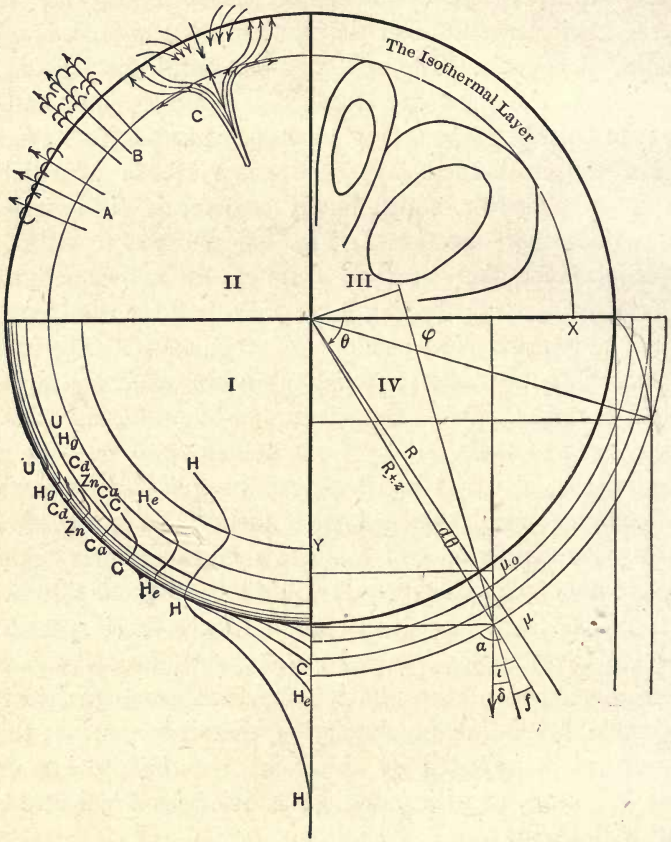


FIG. 28. I. Formation of the Solar Photosphere. II. Formation of the Granulation, Prominences, and Sun-spots. III. Elements of the General Circulation within the Sun. IV. Refraction in the Solar Atmosphere.

sphere. I shall avail myself freely of the results of the Mt. Wilson Observatory, and of the numerous papers from other authors in the *Astrophysical Journal*, in stating the facts pertaining to the several topics.

The granulations, faculæ, flocculi, and prominences are

correlative phenomena, all depending upon thermodynamic processes in several levels. The granulation is a convective result on the top of the photosphere; the faculæ lie in the midst of or just above the photosphere, while the flocculi are in the higher levels of the convective columns; and the prominences may extend by eruption far beyond the normal levels of the hydrogen atmosphere. The forces of segregation depend upon the intrusion of the temperatures, and all the elements associated with them, into levels which are inconsistent with the equilibria of gravitation. If there is an equilibrium, then the equation of condition holds.

$$(176)\quad G\ (z_1 - z_0) = -\frac{P_1 - P_0}{\rho_{10}} - \tfrac{1}{2}\,(q_1^2 - q_0^2) - (W_1 - W_0) - (U_1 - U_0) + \Delta G\,(z_1 - z_0).$$

The gravitation acceleration between two levels balances the hydrostatic pressure, the kinetic energy of circulation, the work of expansion against external forces, and the inner energy. If these are not in equilibrium in the stratum the temperature is no longer normal, the residual $\Delta G\,(z_1 - z_0)$ has value, and the process of readjustment sets in at once. The first step in adjustment is a change in the velocity of circulation between the top velocity q_1 and the bottom velocity q_0, thus resulting in an addition to the gravity, plus or minus, on a rotating globe; and the second step is in a change of inner energy through radiation. The formulas of Chapter I should be kept continually in mind, especially in respect of the fact that no term can change without drawing with it the entire long train of physical terms that are united with it. Hence, to refer any physical solar appearance and effect to a single property, as pressure or density, is very incomplete and even erroneous. This large complex of causes and effects, on the other hand, makes all atmospheric physics difficult, and renders mere description of little value. However, it is not possible to trace out into its proper details all these interrelated terms, but it is proper to insist that they all merely illustrate the several terms in the equation in some of their phases. Thus, the granulation,

faculæ, and flocculi, as convective phenomena, exhibit more the changes in the inner energy, while the prominences are more closely related to kinetic circulation, the direction of the former class being vertical and the latter horizontal. If gases are thrown upward from below by internal convection, through the levels of the isothermal layer and the photosphere, it can be seen that they may pass through very great differences in temperature in a short vertical distance. For example, by Table 6, the changes can be:

H_2 from 7670° to 955°, from the photosphere to 20000 kilometers.
He " 7705° to 740°, " " " " 10000 "
C " 7695° to 0°, " " " " 5000 "
Ca " 7705° to 300°, " " " " 1000 "
Zn " 7715° to 640°, " " " " 600 "
Cd " 7684° to 0,° " " " " 400 "
Hg " 7670° to 0,° " " " " 300 "

Similarly, compare Table 9 for pressure, Table 12 for density, and Table 13 for the gas efficiency. A short vertical movement of a mass of gas is therefore equivalent to an immense amount of kinetic energy of circulation. The same gas at its

TABLE 86

VERTICAL CHANGES

Calcium has the following data

Height	T	A	P	ρ	R
1000 Km	300°	7.654×10^{-10}	7.755×10^{-5}	2.948×10^{-8}	8.7702
700 "	2200	0.003579	362.57	0.000296	556.73
400 "	4700	0.30009	30407	0.0042217	1532.5
100 "	7500	2.8405	287830	0.0162620	2360.1
0 "	7705	5.3060	537620	0.023657	2949.5

Hydrogen has the following data

Height	T	A	P	ρ	R
20000 Km....	955°	3.728×10^{-7}	0.03778	8.051×10^{-8}	4912.8
10000 "	3495	0.0806	8165.6	0.00005006	46669
5000 "	5890	1.0687	108265	0.0003146	58436
1000 "	7680	4.3512	440880	0.0008538	67237
0 "	7670	5.9084	598657	0.0010613	73547

Note the effects of vertical motion on T. A. P. ρ. R.

different levels has such a wide range of temperature for radiation and absorption that it may be able to reverse its spectrum several times.

There are certain bands of calcium, H . K, which are seen to have a double reversal, corresponding to the successive relative emissions and absorptions at different heights. They are classified by Mr. Hale:

H_3 . K_3, narrow line of high level absorption at 700 to 1000 kilometers.

H_2 . K_2, middle lines of emission at 400 to 700 kilometers.

H_1 . K_1, wide base of band of absorption at 100 to 400 kilometers.

H . K, low level continuous spectrum at 0 to 100 kilometers.

It would seem that a change in temperature of about 2000 degrees is sufficient to make the successive steps in alternate emission and absorption from one level to another. In the case of calcium the vertical interval is not far from 300 kilometers; in the case of hydrogen H α, H β, H γ, they are 4000 to 5000 meters, increasing upward. Calcium has the molecular weight 40, and it is located on the hyperbolic curves of Fig. 18 where the rapid rise to the vertical branch begins. Hydrogen extends upward to the maximum height. The elements between these two enter freely into all the visible surface phenomena, while those elements below $m = 40$ enter more difficultly into these appearances. This capacity of vertical extension naturally divides the convectional columns into separate classes, though they merge into one another. Thus, the *flocculi* are the high-level columns of the light gases seen in section by means of the spectroheliograph only; the *faculæ* are low-level sections just above the photosphere, as in the lower parts of the calcium columns; the *granulations* are the heads of the columns on the level of the photosphere, and consist chiefly of the heavy gases. All of the strata of the isothermal layer, and of the non-adiabatic layer, are more or less segregated into columns of different temperature and density in vertical directions, and these extend to different altitudes according to their thermodynamic conditions. *One of the important features of atmospheres is that masses of the same gas having different tem-*

peratures and densities are reluctant to mix thoroughly with one another. Such masses will maintain themselves side by side, standing vertically on a common base, or lying alongside each other on the same horizontal plane, for long intervals without losing their individuality. Such juxta-masses interchange their thermodynamic values slowly at the surfaces, and the shell of mutual equilibrium only penetrates gradually to the middle of large masses. For this reason circulation sets in as minor vortices and mixing currents, which form a sheath of kinetic energy on the surface of the mass, whereby the atomic-molecular equilibria may be transmitted in all their details. This consumes time, and in the case of large masses whose temperature parameters are not very far apart, this may be prolonged; where the temperatures differ widely the kinetic movements integrate into general movements of the entire masses. These movements may be vertical, as where convection currents rise and fall in the immediate neighborhood of one another, or they may be horizontal, as where currents of different temperatures flow along horizontal planes. These have the property, by means of the term $\frac{1}{2}\ (q_1^2 - q_0^2)$, to add to or subtract from the local pressure and density terms, so that, under the force of gravitation, these shall be preserved from abnormal curvatures on the respective levels. Some account of this process is given in the "Meteorological Treatise," but the subject is one of difficulty because of the complexity of the details of the thermodynamics.

These explanations are necessary to be remembered as the causes of the solar segregations of gases. The common descriptive solar physics is a long chapter of these phenomena, but they all illustrate these thermodynamic results. On the other hand, it will probably be possible to proceed from the observed results through the formulas to the primitive energies of circulation and radiation that are involved. This juxtapostion of masses of discontinuous temperatures, in vertical and in horizontal planes, gives rise to anomalies in refraction, and allied phenomena. Some further account of solar atmospheric refraction will be added on a later page.

Granulations. The surface of the photosphere, as seen directly in a telescope, seems to consist of bright and dark patches of irregular forms. They are called "fleecy clouds," "fleecy patches," "rice grains," "granules," and they are in their maximum large, and in their minimum they have been observed down to 2″.6, 1″.4, or even to 0″.3 in diameter. On the surface of the sun, whose diameter = 1389602 kilometers, or 1919″.26, we have 1″ = 724.03 kilometers = 449.89 miles. Hence, we have, 2″.6 = 1882 kilometers = 1170 miles; 1″.4 = 1014 kilometers = 630 miles; and 0″.3 = 217 kilometers = 135 miles. The finest estimated filaments are 0″.03 = 21.7 kilometers = 13.5 miles. We must infer that the filamentary or threadlike structure has no real limit in its tenuity, either in vertical or in horizontal directions. The segregations may appear large, but they are interpenetrated by innumerable vortex lines of complex form, and the individuality in some degree depends upon more or less perfect vortex motions. These interpenetrations may culminate in a snarl of vortex actions, with confused temperatures and densities, favorable to the production of diffuse reflection and scattered radiation in their midst. The tops of the vertical filaments seen in section are evidences of such action, and the bright patches of granulation on the photosphere are thus constructed. There are two causes for their perpetual existence on that level: (1) The heavy metals or gases terminate near that level, so that vertical convection alone will place their heads just above the isothermal levels; (2) Since the gradients of temperature are nearly horizontal for the heavy gases, and these must penetrate through the temperature gradients of the lighter gases, there is a stratum of very pronounced optical confusion just along these levels, due to the mixture of the horizontal and vertical temperature gradients of the heavy and the light gases respectively.

This interpenetration is readily perceived by Figs. 10, 11, 12, and Fig. 28, Section I, when it is remembered that every unit volume of the isothermal and lowest non-adiabatic strata, all over the spherical shell of the sun, is full of such a mixing series of atmospheres. Within the isothermal layer the fila-

ment-structure is generally vertical without serious disturbance, but above this stratum there is a strong tendency to horizontal filaments. The combination may produce curved filaments of every possible shape, such as are seen in sun-spots, prominences, and faculæ. Above this region of special mixture, the filaments tend to become vertical once more throughout the chromosphere and inner corona, which consist of light gases. The irregularities of mixture there take fantastic forms, more or less stationary, and in the prominences the cooling sheath of mixture on the edges of the mass of gas has been repeatedly remarked.

Faculæ. These are the irregular large regions of vertical convection, closely associated with the region of the sun-spots and their production, and they are the evidences of a general convective movement from the interior of the sun on a large scale. They are seen most conveniently in the calcium lines, since it is very abundant, and has a broad base with rapidly diminishing density to 1000 kilometers. The other light gases have similar distributions so far as they can be observed.

Flocculi. These are sectional aspects of the vertical, convectional columns in the light gases at greater altitudes, and in calcium, hydrogen, carbon, they are most readily observed. The great difference in the altitudes possible for calcium and hydrogen makes evident their respective positions. The areas of the sections of the columns increase with the height, as the convectional vortex spreads out horizontally. Every rising column tends to throw off lateral vortex whirls along its sides, and the juxtaposition of such vortices produces mixture, while they develop, also, downward return currents. The crests of the heavy gases become the heads of the granulation, which are bright, but the colder downward currents between them are relatively dark. Likewise, the faculæ and flocculi produce similar curling vortices with crests, mixing sides, and downward interspacial currents. Compare Fig. 28, Section II, A. B.

Prominences. These are the hydrogen flames, seen in a spectroscopic slit on the limb of the sun, and they belong to the so-called eruptive type of convection, whereby the thermo-

dynamic forces of the lower levels develop great spacial expansions when transported upward, on account of the small density of the hydrogen in the high strata. The vertical columns are easily observed, and the curls, more or less pronounced, are recognized. There are large apparent motions, such as 100 miles per second, and this great apparent velocity may be due to the translation of masses, or to the propagation of optical effects by successive local changes in density and scattering through mixtures. Large masses may become visible by surface changes of the thermal conditions, and in many cases this accounts for great apparent velocities of propagation. To distinguish between these two processes is not easy, at the distance of the sun, under the complex conditions which prevail, involving the great force of gravitation in the equilibrium with the other thermal terms of the equation of condition.

The Origin of the Sun-Spots

The literature of solar physics abounds in suggestions and hypotheses regarding the significance and the origin of the sun-spots and their attendant phenomena. These are based upon the different features that are observed, each theory emphasizing some physical characteristic, and doubtless there must be elements of truth in nearly every one of the leading schemes. Yet, it is commonly believed that the subject is still open to research, and that something has been lacking capable of harmonizing the facts derived from direct telescopic and spectroscopic observations. While the analysis of the solar data given in the previous pages may be quite sufficient to open up new paths of research and to reconcile the several outstanding difficulties, it is probable that further experience will enable us to make still further advances, especially in the interpretation of the thermal and the optical spectra. It is impracticable to review carefully the literature regarding sun-spots, but enough may be summarized to bring out the most important facts that must be taken into the account.

Herschel conceived that there are three distinct layers in the solar constitution: a solid nucleus seen as umbra, a quasi-liquid

layer seen as penumbra, and a luminous envelope of condensation forming the cloudy photosphere.

Secchi, *Faye*, and *many others*, make the spots the centers

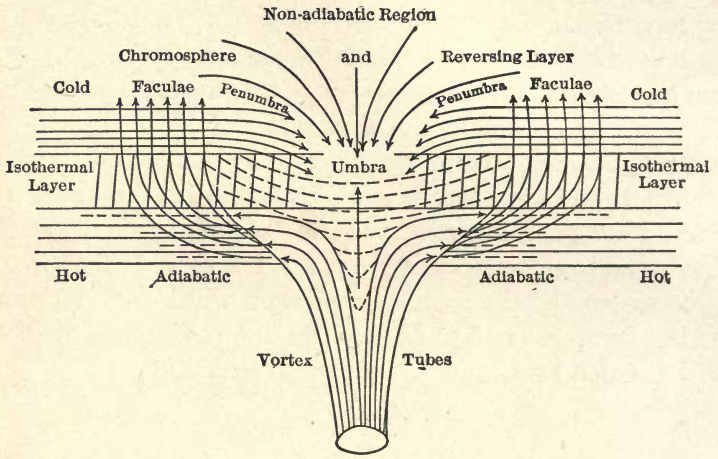


FIG. 29. General Scheme of the Structure of a Sun-spot Vortex, and Its Relations to the Reversing Layer. (*Bigelow.*)

of explosive outbursts of imprisoned gases retained within a quasi-liquid restraining envelope.

Secchi, *Lockyer*, *Oppolzer*, and others, consider the sun-spots the center of downpouring gases, or meteoric material, bringing cooler temperatures from the higher strata. *Young* sees in the spot an area of sinking of the photosphere and elevation of a ring surrounding it. *Fox*, and others, observe that spot-birth is always preceded by an eruption of faculæ in a ring outside of the penumbra, most pronounced on the following side.

Schmidt applies circular refraction to make the spot a mirage effect, while the phenomenon really occurs at considerable depths below the photosphere.

Julius applies the optical effects of anomalous dispersion and complex refraction to the entire series of spot and attendant phenomena.

Schaeberle finds mechanical forces enough to account for the conditions.

Brester applies chemical luminescence and moving flashes, among other phenomena of that kind.

Sidgreaves utilizes the chill of expansion to account for the low temperature in the central areas of spots.

Hale, Adams, and others, find that the spectra indicate an almost universal weakening of the enhanced lines in sun-spots, and consider this as evidence of the low temperature in the reversing layer over the spots themselves.

Abbot determines lower temperatures in the spots than in the surrounding photosphere by bolometer observations.

The observers of the Mt. Wilson Observatory have measured in the displacement of the spectrum lines that there are outward velocities in the lower part of the sun-spot region and inward velocities in the upper part. There are evidences of a slow spiral inward movement from the photosphere at considerable distances into the penumbra, and thence downward to the umbra.

Langley pointed out that the solar surface is striated in vertical columns, like sheaves of wheat, that the quiescent tops of the segregations appear as granules, and that when bent over they form the filaments in the penumbra.

Faye tried to account for a solar vortex as a whirlpool in a stream having relatively different eastward velocities.

Bigelow wrote as follows in respect of the connection between solar vortices and the dumb-bell-shaped vortex, Monthly Weather Review, October, 1908:

"The sun-spots occur on the outer surface of the photosphere and extend inward toward the center of the sun. They consist visibly of a nucleus which is practically structureless, and a penumbra which is striated radially with much regularity. The observed movements of the material composing the penumbra are from the outer edge of the disturbed area in the photosphere toward the umbra, and the radial striæ usually terminate in ends which are bent downward toward the interior of the sun. The motion of a particle starting on the outer edge of the penumbra is primarily inward, and then rather suddenly downward. This corresponds so closely to the motion in the

upper levels of a dumb-bell-shaped vortex where the circulation is downward that it seems proper to suggest this explanation of the origin and structure of the sun-spots. The sun-spots would correspond to the layers between the sections $a\,z = 180°$ and $a\,z = 170°$ (M. W. R., Oct. 1907, p. 475, fig. 3), if the *circulation is downward.* In this limited region there is practically little rotary velocity v, the vertical velocity w becomes important only when approaching the abrupt curvature, which is here assumed to be on the outer edge of the umbra, but in the penumbra the radial velocity is conspicuous. The sun-spot may be caused by layers of matter inside the sun's photosphere operating to draw material downward, warm layers being superposed upon cold layers at the section which corresponds with the lower plane of the sun-spot vortex."

Comparable ideas are quoted by C. E. St. John, from Moore's "Meteorology," in his paper on the radial motion in sun-spots, *Astrophysical Journal*, Vol. XXXVII, 1913. This paper presents certain values of the outward and the inward radial velocities u in different levels, as derived from the displacement spectrum lines. Generally, the sun-spot is now considered to be a complex vortex, and we shall proceed to utilize our solar data in order to examine more closely its probable origin and structure. It will be seen that my views of 1908 have been modified in details in consequence of this new data.

It should be remembered, as explained in my "Meteorological Treatise," that Nature abhors a discontinuity in the curves of pressure and density, and that temporary abnormalities in these curves, caused by imperfect thermodynamic adjustments, are compensated by horizontal currents in general. Since a horizontal current transfers its energy into a vertical force, by the general equations of motion it follows that these currents occur at the boundary of cool and warm strata, whenever the temperature gradient becomes abrupt and abnormal. In the solar atmosphere a small convectional lift of the adiabatic strata, at the lower boundary of the isothermal stratum, produces at once a horizontal outflow from the center or point of greatest discontinuity in T. The lower planes of the several isothermal

strata are therefore the planes of vortical outflow, so long as the internal vertical convection from the interior of the sun persists. Since all gases must pass through the transformation from adiabatic to isothermal temperatures, that is from large negative gradients to small positive gradients, it is evident that the mass of gas containing $-(Q_1 - Q_0)$ can not dispossess itself of this heat content with sufficient rapidity to conform to the change in temperature gradient without other special processes. One of these has been described, namely, the generation of the solar radiation, and the other is the production of the sun-spot vortices. Hence, concentrations of radiation and of sun-spot phenomena happen wherever the internal thermodynamics within the enormous mass of the sun, operating by its own laws, may determine its great currents of general circulation. These concentrations may occur in latitude, as on the maxima of the sun-spot belts, or in longitude, as indicated by the successive maxima and minima along the equator, as heretofore found to be the case by several lines of evidence. The general result is to produce a series of spherical harmonics, and there is little doubt that their characteristics can be worked out from our materials.

Since the same sun-spot endures for months on occasions, it is necessary to determine not only what are the forces which originate them, but what physical process sustains them during such long intervals of time. This, in connection with the minor features enumerated above, is the object of our research. The discovery of the permanent existence of a spherical isothermal shell around the sun, of persistent thermodynamic origin, greatly facilitates our effort. It is seen that all of the gases which by vertical convection, either from below upward or from above downward, attempt to pass through this layer, are necessarily reduced to this isothermal temperature, between 7650° and 7700°, whatever may be their temperature on approaching this shell. For brevity, we shall continue to speak of the parameter T as implying the long train of thermodynamic processes that have already been described, including association and dissociation, radiation and absorption, dynamic move-

ments and thermodynamic transformations. It is especially to be remarked that in the adiabatic strata, below the isothermal layer, a short vertical movement of a mass of gas by convection changes its level of temperature through many degrees, and, similarly, after passing the isothermal layer, there is a further great change of temperature with the height. This is entirely true of the heavy gases, and progressively less true of the light gases.

TABLE 87

VERTICAL DISTANCE REQUIRED FOR A CHANGE OF 1000°

Element	Above the Isothermal Layer		Below the Isothermal Layer
Hydrogen, H_1 1.....	6630 (2)	4360 (3)	2100 Km. (4)
Hydrogen, H_2 2.....	4200 "	1940 "	1450 "
Helium, He 4.....	1700 "	980 "	500 "
Carbon, C 12.....	620 "	340 "	180 "
Calcium, Ca 40.....	145 " $\frac{m}{6330}$	82 " $\frac{m}{3600}$	53 " $\frac{m}{2150}$
Zinc, Zn 65.....	85 "	68 "	33 "
Cadmium, Cd 112.....	56 "	28 "	21 "
Mercury, Hg 198.....	30 "	20 "	11 "

Table 87 collects these vertical temperature gradients into three classes, as from 0° to 4000° marked (2), 5000° to 7000° marked (3) above the isothermal layer, and 8000° to 12000° marked (4) below it in the adiabatic region. The change of gradient is progressive so that the table is merely illustrative. The ratios $\frac{6330}{m}$, $\frac{3600}{m}$, $\frac{2150}{m}$, show how to proceed from one element to another through the atomic weights. Evidently short vertical changes for the heavy gases are matched by long vertical changes for the light gases.

In order to understand the significance of a vertical convective movement, we require the general equation of condition, including the kinetic energy of circulation,

$$G(z_1 - z_0) = -\frac{P_1 - P_0}{\rho_{10}} - \tfrac{1}{2}(q^2_1 - q^2_0) - (Q_1 - Q_0).$$

On a level of equilibrium without circulation, the term $-\frac{1}{2}(q^2_1 - q^2_0)$ disappears, but if by convection the mass containing

$-(Q_1 - Q_0)$ free heat is moved to a higher level, the equilibrium is destroyed temporarily, and in the process of adjustment the term $-\frac{1}{2}(q^2_1 - q^2_0)$ is set in operation, and it will be maintained as long as continuous vertical convection renews the $-(Q_1 - Q_0)$ at the abnormal level. Computing the available velocity, we have for different heights—that is, for different discontinuities—of the thermodynamic levels:

TABLE 88

HORIZONTAL VELOCITY FOR VERTICAL CONVECTION TO DIFFERENT HEIGHTS

Change in Height	H_2	He	C	Ca	Zn	Cd	Hg	Means
50 Kilometers.....	1484	1581	1703	1673	1483	1658	1597	1597
5 "	469	500	539	529	469	524	505	505
1 "	210	223	241	237	210	234	226	226

If the mass containing the free heat, as computed at the top of the adiabatic levels, or in the lower part of the isothermal stratum, is raised 50 kilometers; it can be compensated on the average by a *horizontal* velocity of 1597 meters per second; for 5 kilometers by 505, and for 1 kilometer by 226 meters per second by the formula,

$$(177)\quad \text{Kinetic Energy} = -\tfrac{1}{2}(q^2_1 - q^2_0) = \left[-\frac{P_1 - P_0}{\rho_{10}}\right]_1 - \left[-\frac{P_1 - P_0}{\rho_{10}}\right]_0 = [-(Q_1 - Q_0)]_1 - [(Q_1 - Q_0)]_0$$

The interesting fact appears to be that *each element acquires the same velocity of transportation*, so as to make mixed masses move together at the same speed.

The Circulation in a Solar Vortex or Sun-Spot

With this preliminary explanation, we can proceed to illustrate the sun-spot vortex by diagram and by computations. Fig. 29 contains the general scheme of a solar vortex. Let the normal 1000°—degree temperature levels be drawn for different gases in the adiabatic, isothermal, and the non-adiabatic strata.

Let vertical convection raise a section so that lateral horizontal motion takes place away from a central axis in all directions. This drags after it a simple funnel-shaped vortex. At the extremities of horizontal vortex motion the ends turn up and produce vertical filamentary striæ, which form a ring of faculæ at considerable distances from the axis. The vortex motion lowers the isobars over the center, and into this depression the horizontal strata of the upper non-adiabatic region descend in slow inward spirals, the filaments forming the penumbra. They terminate at certain distances from the axis, according as the cold high-level gases become sufficiently heated to lose their visibility. Over the center of the vortex, called the umbra, the gases may be drawn down from great heights, affecting the reversing layer, the chromosphere, and even the hydrogen of the inner corona. These displacements all respond, in succession, to the *primary vortex under the isothermal layer which is never seen*, where the powerful velocities are produced which generate magnetic field in the core by the rotation of the ions. These are freely disintegrated from the atoms in the processes of radiation, so that the supply is as continuous as the original convection. The upper visible vortex is secondary, and without vortex motion proper, so that students have been misled by this fact in their interpretation of sun-spots. A résumé of the facts given above shows how the several theories are related to the true solar vortex. Professor Hale's preliminary description of the formation of solar vortices is in harmony with the above account, though it lacks the fundamental isothermal action. The spectrum shows a series of line and band changes which can readily be understood by applying these principles. Indeed, a very complete solution of the distribution of $P.\ \rho\ .\ R.\ T.$ at all levels can be drawn out from the Formulas of my Treatise, pages 172, 174, 199–202. We shall adopt these formulas, and compute a few vortex tubes, in order to bring out the dimensions of the vortices on the sun and their attendant phenomena.

Very small spots have a radius of the umbra varying from 400 to 800 kilometers, while in very large spots the umbra has a radius of 40000 to 50000 kilometers, the radius of the pe-

numbra being 150000 kilometers. For a common spot the outside radii may be taken,

for the umbra, 10000 kilometers,
" " penumbra, 25000 "
" " faculæ, 500000 "

We shall compute tubes between 60474 kilometers (1) and 956 kilometers (6), on a level 500 kilometers below the reference plane, and extending downward to 50000 kilometers. Within these adopted data nearly all sun-spots will be readily contained. The horizontal velocities on the 500 kilometer plane will range from 0.212 to 13.39 kilometers per second. To find log ρ, we take, $\frac{1}{5}$ (log 60474 − log 956.25) = $\frac{1}{5}$ (4.78157 − 2.98057) = + 0.36020. By a little trial computing, it results that the current function $\psi = v\varpi = 6\,400\,000$ (6.80618), and the tube constant, $C = \frac{u}{\varpi} =$ 0.0000035 (− 6.54407). Then the successive values of ϖ, C, u, v, w, i, η, are computed on the 500 kilometer level, by the formulas indicated. They are next extended down to the 50000 kilometer level. An examination of the v, or tangential velocities, makes it probable that the actual vortex does not extend much beyond the horizontal lines where the velocity is 2500 kilometers per second. It is likely that, with sufficient study, the velocities indicated by the spectroscope may be made to determine the theoretical size and power of a given vortex.

TABLE 89

TYPICAL SOLAR VORTEX CONFORMS TO HALE'S DATA

Radii in kilometers, $\varpi^2 = \frac{\psi}{C}\frac{1}{z}$. Log ψ = 6.80618

Lines Log C Kilometers	(1) −6.54407 Outside	(2) −5.26447	(3) −5.98487	(4) −4.70527	(5) −3.42567	(6) −2.14607 Center	2 log ρ +0.72040
$z =$ 0.....	∞	∞	∞	∞	∞	∞	
500.....	60474.	26287.	11513.	5023.1	2191.7	956.25	
1000.....	42762.	18658.	8140.7	3551.9	1549.8	676.17	
2000.....	30232.	13193.	5756.3	2511.5	1095.8	478.12	
5000.....	19124.	8344.	3640.6	1588.5	693.06	302.39	
10000.....	13523.	5900.1	2574.3	1123.2	490.08	213.82	
20000.....	9562.	4172.0	1820.3	794.22	346.53	151.20	
30000.....	7807.	3406.5	1486.3	648.49	282.94	123.45	
40000.....	6761.	2950.1	1287.2	561.60	245.04	106.91	
50000.....	6047.	2638.6	1151.2	502.31	219.17	95.63	

TABLE 89—*Continued*

Radial velocity in kilometers/second, $u = C\bar{\omega}$

$z =$							
0.....	∞	∞	∞	∞	∞	∞
500.....	0.2117	0.4851	1.1119	2.5482	5.8404	13.3860
1000.....	0.1497	0.3430	0.7862	1.8019	4.1298	9.4652
2000.....	0.1058	0.2426	0.5559	1.2741	2.9202	6.6929
5000.....	0.0669	0.1534	0.3516	0.8062	1.8469	4.2329
10000.....	0.0473	0.1085	0.2486	0.5698	1.3060	2.9931
20000.....	0.0335	0.0767	0.1758	0.4029	0.9234	2.1164
30000.....	0.0273	0.0626	0.1435	0.3290	0.7540	1.7281
40000.....	0.0237	0.0542	0.1243	0.2849	0.6530	1.4696
50000.....	0.0212	0.0485	0.1112	0.2548	0.5840	1.3386

Tangential velocity in kilometers/second, $v = \frac{\psi}{\bar{\omega}}$

$z =$						
0.....	0	0	0	0	0	0
500.....	105.83	242.56	555.91	1274.1	2920.2	6692.8
1000.....	149.66	343.02	785.02	1801.9	4129.7	9465.0
2000.....	211.66	485.11	1111.8	2548.2	5840.4	13386.
5000.....	334.66	767.02	1758.0	4029.1	9234.4	21164.
10000.....	473.28	1084.7	2486.1	5698.0	13059.	29931.
20000.....	669.33	1534.0	3515.8	8057.8	18468.	42327.
30000.....	819.75	1878.8	4306.1	9869.2	22619.	51842.
40000.....	946.56	2169.4	4972.2	11396.	26119.	59861.
50000.....	1058.30	2425.5	5559.1	12741.	29202.	66928.

Vertical velocity in kilometers/second, $w = -2Cz$

$z =$						
0.....	0	0	0	0	0	0
500.....	0.0035	0.0184	0.0966	0.5073	2.6648	13.998
1000.....	0.0070	0.0368	0.1932	1.0146	5.3211	27.996
2000.....	0.0140	0.0735	0.3863	2.0292	10.659	55.994
5000.....	0.0350	0.1839	0.9658	5.0731	26.648	139.98
10000.....	0.0700	0.3677	1.9312	10.146	53.296	279.96
20000.....	0.1400	0.7354	3.8631	20.292	106.59	559.93
30000.....	0.2100	1.1031	5.7946	30.439	159.89	839.88
40000.....	0.2800	1.4708	7.7262	40.585	213.19	1119.8
50000.....	0.3500	1.8385	9.6576	50.731	266.48	1399.8

Tangential angle, $\tan i = \frac{u}{v}$

$z =$			
0.....	90° 0′ 0″	Constant	90° 0′ 0″
500.....	0 6 53	"	0 6 53
1000.....	0 3 27	"	0 3 27
2000.....	0 1 43	"	0 1 43
5000.....	0 0 41	"	0 0 41
10000.....	0 0 21	"	0 0 21
20000.....	0 0 10	"	0 0 10
30000.....	0 0 7	"	0 0 7
40000.....	0 0 5	"	0 0 5
50000.....	0 0 4	"	0 0 4

Vertical angle, $\tan \eta = -\frac{w}{v}$

$z =$						
0.....	0° 0′ 0″	0° 0′ 0″	0° 0′ 0″	0° 0′ 0″	0° 0′ 0″	0° 0′ 0″
500.....	0 0 7	0 0 16	0 0 36	0 1 22	0 3 8	0 7 11
1000.....	0 0 10	0 0 22	0 0 51	0 1 56	0 4 26	0 10 10
2000.....	0 0 14	0 0 31	0 1 12	0 2 44	0 6 16	0 14 23
5000.....	0 0 22	0 0 49	0 1 53	0 4 20	0 9 55	0 22 44
10000.....	0 0 31	0 1 10	0 2 40	0 6 7	0 14 2	0 32 10
20000.....	0 0 43	0 1 39	0 3 46	0 8 39	0 19 50	0 45 28
30000.....	0 0 53	0 2 1	0 4 37	0 10 36	0 24 18	0 55 42
40000.....	0 1 1	0 2 20	0 5 6	0 12 15	0 28 3	1 4 19
50000.....	0 1 8	0 2 36	0 5 58	0 14 1	0 31 22	1 11 53

Formulas. Meteorological Treatise, p. 172

Inward and Outward Velocities in Solar Vortices

The velocities of inward and outward motions in the solar vortex, made at the Mt. Wilson observatory by C. E. St. John, and summarized in the Annual Report for 1913, are interesting in this connection. Generally the light gases have an inward radial velocity and the heavy gases an outward velocity.

TABLE 90

SUMMARY OF THE INWARD AND OUTWARD RADIAL VELOCITIES

Element	At. Wt.	Line	Height Kilometers	Meters/sec.	Direction
Hydrogen	2	Hα	20000	1500	Inward
		Hγ	13000	1000	"
		Hδ	4000	200	"
Calcium	40	H_3. K_3	25000	2000	"
		H_2 K_2	15000	1300	"
		4277	3800	100	"
Magnesium	24	b_1 b_2	8000	400	"
		(10)	3400	00	"
Sodium	23	D_1 D_2	5000	300	"
Aluminum	26	(20)	3500	—	"
Iron	55	(10)	1500	200	"
		(8)	1300	300	Outward
		(7)	1100	400	"
		(6)	1000	500	"
		(5)	800	600	"
		(4)	600	700	"
		(3)	400	700	"
		(2)	280	800	"
		(1)	250	900	"
		(0)	240	900	"
		(00)	230	1000	"
		(000)	210	1200	"
Lead	207		230	900	"
Lanthanum	139		220	900	"
Barium	137		210	1100	"
Niobium	96		200	1200	"

The same element, Hydrogen, Calcium, Magnesium, has velocities decreasing downward above, but increasing velocities below the plane of velocity inversion. The heights assigned to the inversion velocities, about 3500 kilometers *above* the photosphere, should be modified to variable depths *below* the

plane of reference. An adjustment in this respect is readily interpreted in terms of the displacements of the spectrum lines. The maximum radial velocities observed seem to be about 1500 meters per second. On comparing such very large *radial velocities u* with those computed in the typical vortex tubes, it is seen what enormous velocities, if those are correct as observed, are implied in the interior of the sun near the axis of the vortex. This subject requires a prolonged examination.

The Invisible Deep-Seated Thermodynamic Processes

We have mentioned three processes that take place near the bottom of the isothermal layer during the transition between adiabatic and isothermal conditions. These are (1) the generation of the general radiation, which is afterward depleted by a small amount of true absorption and a large amount of scattering before being reduced to the effective solar radiation toward the earth; (2) the production of abundant free electric charges, afterward apparent in the sun's electrostatic surface charge, in the local magnetic fields, and in their variations; (3) in the generation of true vortices in certain latitudes and longitudes. It should be remarked that not all of these effects can be readily observed from the earth, and that there are many partially developed vortices which have only surface changes in the areas of the faculæ, being the symptoms of deeper seated operations. Consequently, there must be many maxima and minima in the output of the solar radiation, which are only imperfectly registered in the high level phenomena above the photosphere. If the spectroheliograph has such difficulty in fixing accurately the levels and the velocities of the movements of large masses of gas and vapor, it is evident that we need not feel constrained to limit our information to its particular evidences. If there are registered at the earth successive maxima and minima along the solar surface in its synodic rotation, as changes in the intensity of radiation, in the magnetic field, in the temperatures and pressures of the earth's atmosphere, there is no good reason derived from spectrum observations, whether visual or thermal, why such disturbances may not occur be-

neath the isothermal layer, even if there are no spots and no important groups of faculæ above that layer. It is known that magnetic storms originate at the earth from solar impulses, without attendant sun-spots, and there can be no reason why this and similar conditions may not occur under the invisible solar processes.

The General Circulation in the Sun in Latitude

From a study of the effects of the circulation from within the sun, as shown in several surface phenomena, a suggestion may be ventured regarding the thermal movements below the isothermal layer. The statistical studies of the relative frequency of the sun-spots, the faculæ and the prominences, together with the extensions of the solar corona, will serve to indicate the zones of maximum vertical convection in the latitude, that is, along the meridians; also the maxima and minima of the solar convection in longitude, along planes parallel to the equator, are registered in the variations of the terrestrial magnetic field.

The relative frequencies of the solar prominences, faculæ, and sun-spots, as observed by the Italian spectroscopists, Secchi, Tacchini, Ricco, and others; by Wolf, Wolfer, and others, have been studied in 10-degree zones, in a series of papers by Lockeyer, 1902, 1903, 1904, Proc. Roy. Soc.; by Bigelow, Monthly Weather Review, Jan., 1903; by Ricco, Mem. Spett. Ital., 1903, 1914. These papers reach similar conclusions regarding the movements in latitude, and for a summary the data of my paper are quoted covering the mean values, 1872–1900. Three 11-year cycles are superposed, and the mean values are given in Table 91 for each 10-degree zone north and south of the equator during a common period of 11 years. These data were plotted in Fig. 3 of my paper, and there appear to be maxima which have a movement in latitude as well as in intensity. Scale from the diagram the maximum ordinates for successive years, and plot them as on Fig. 30, making at the given latitudes a series of points measured from the limb, which are proportional to these frequencies. The maxima have

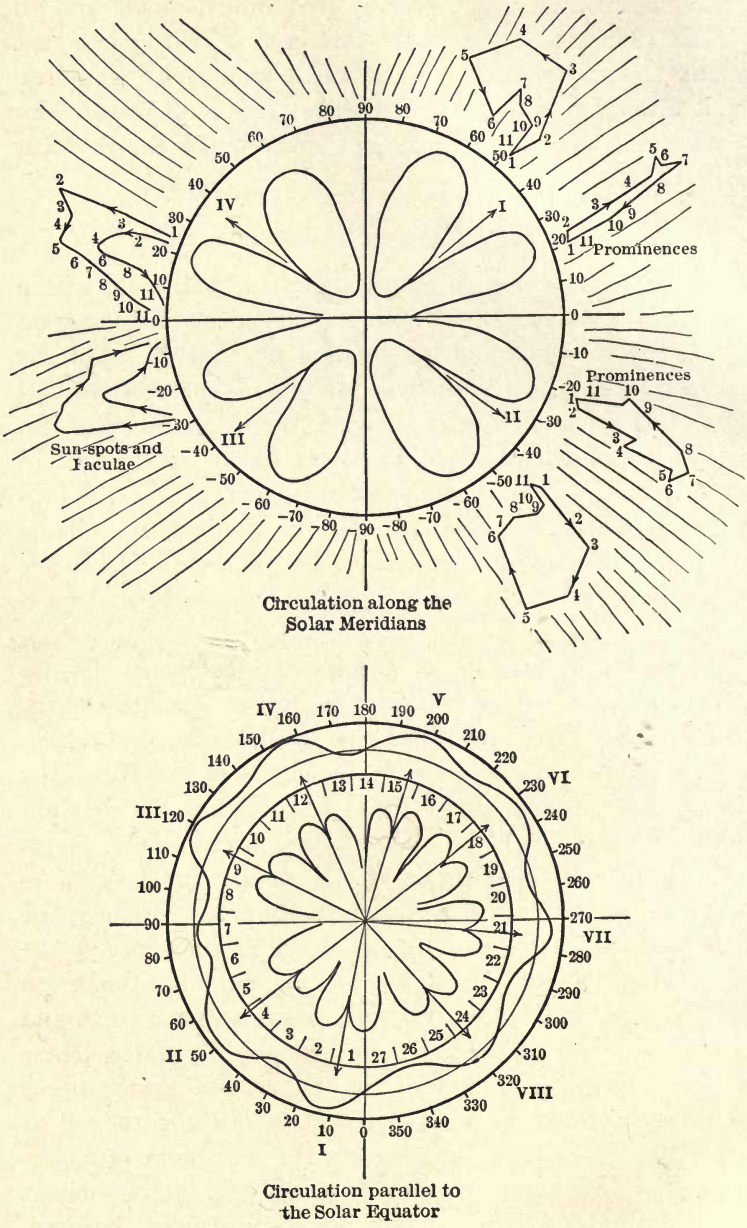


FIG. 30. Circulation In the Interior of the Sun.

two branches for the prominences in each hemisphere, (1) forming a narrow loop near latitudes ± 20°, and (2) forming a broad loop between latitudes ± 50° and ± 70°. The figures show that the maxima progress in two cycles, first, toward the equator, secondly, toward the poles. The solar impulse is apparently outward near latitude ± 40°, in zones surrounding the sun, with quasi-vortical curls on each side of the principal axes or planes. The curl-vortex circulation is common to nearly all free convectional movements, involving a central upward convection, and two lateral downward branches of descending materials. If these ascending and descending branches form parts of a convectional circulation, there will evidently be produced a series of oval-elliptical stream lines, narrow near the interior and broad near the surface of the solar mass. There seem to be two complete circuits in each hemisphere, and the total forms a sort of zonal harmonic with four axes, taking the sun as a whole. There are many details which harmonize with this general view. The maxima of the sun-spot and the faculæ frequencies form circuits from the latitudes ± 30° toward the equator. This is easily seen to be the effect which would follow if a renewal of the general convection, during the sharply ascending branch of the 11-year cycle, should cause the throat of the ascending convection, with its lateral branches, to reach higher up toward the surface, so that during the first years of the period the maximum outbursts are nearer together, ± 50° to ± 30° in latitude; this vortex may then fall back toward the center of the sun, and cause the maximum points to recede toward the equator and the poles, respectively. This general zonal movement in latitude conforms with the observed frequency of these surface phenomena in latitude, and refers them to the general thermodynamic convection within the great interior masses of the sun. They are leisurely and persistent, and they are produced and sustained by the pressure of gravitation, which causes hydrostatic reactions, circulation, and the generation of free heat as one of the terms in the formation of the general solar radiation.

TABLE 91

MEAN OBSERVED DISTRIBUTION IN LATITUDE DURING THE 11-YEAR SOLAR CYCLE, 1872–1900

SOLAR PROMINENCES

11-Year Cycle	90° 80	80° 70	70° 60	60° 50	50° 40	40° 30	30° 20	20° 10	10° 0	0° −10	−10° −20	−20° −30	−30° −40	−40° −50	−50° −60	−60° −70	−70° −80	−80° −90	
Year 1.....	15	17	26	73	113	153	175	166	149	149	185	220	188	109	74	73	57	27	1969
2.....	2	7	28	110	143	172	221	191	186	196	224	224	196	124	68	38	32	21	2183
3.....	6	4	15	90	119	137	170	150	118	135	180	196	147	111	40	8	7	4	1637
4.....	5	9	10	75	91	83	116	113	86	102	115	106	86	84	78	11	5	4	1185
5.....	19	18	15	64	88	82	89	77	65	87	109	115	87	144	59	14	14	12	1158
6.....	13	15	18	33	73	73	59	66	46	61	88	111	109	154	86	15	16	7	1042
7.....	3	6	8	25	42	34	23	24	25	28	31	35	57	100	38	13	10	4	506
8.....	2	3	7	33	80	58	31	26	11	12	17	47	71	143	44	3	2	2	588
9.....	2	5	19	193	172	128	152	85	48	36	81	135	151	184	178	16	5	3	1587
10.....	3	18	195	153	113	152	191	145	96	100	140	225	232	150	227	89	9	2	2235
11.....	11	128	101	43	101	174	201	176	133	132	182	214	192	155	163	192	38	4	2284

SOLAR SPOTS

11-Year Cycle	40° 30	30° 20	20° 10	10° 0	0° −10	−10° −20	−20° −30	−30° −40	Sums
Year 1.....	1	14	64	68	57	118	27	2	350
2.....	–	15	89	73	77	98	23	–	374
3.....	–	3	37	33	52	85	13	1	222
4.....	–	1	19	35	45	34	2	–	135
5.....	–	1	17	28	34	47	—	–	126
6.....	–	—	5	15	30	25	—	–	74
7.....	–	2	2	13	17	9	4	–	46
8.....	–	20	13	1	3	3	21	–	61
9.....	6	41	65	7	4	30	33	5	187
10.....	1	43	93	33	14	84	70	6	344
11.....	1	37	98	47	52	108	60	3	404

SOLAR FACULÆ

Above 40°	40° 30	30° 20	20° 10	10° 0	0° −10	−10° −20	−20° −30	−30° −40	Below −40°	Sums
1	7	66	164	160	136	179	114	39	4	867
—	7	57	148	144	148	168	87	13	1	770
1	5	38	99	98	147	189	98	24	3	698
2	15	35	84	114	139	130	60	15	10	602
4	19	43	72	107	140	120	70	27	8	607
4	8	19	42	70	101	78	32	17	1	371
8	15	36	59	93	121	94	51	30	11	516
—	15	45	31	6	11	16	44	14	1	183
10	59	198	153	41	17	98	115	40	8	731
10	68	177	218	117	60	174	177	82	22	1102
3	40	136	189	128	120	201	147	47	8	1015

The General Circulation of the Sun in Longitude

In Bulletin No. 21, U. S. Weather Bureau, the results were published of a study of the relative intensity of the variations of the terrestrial magnetic field in the adopted equatorial synodic period of 26.68 days. Compare Chapter 5 and Fig. 23; also Meteorological Treatise, pp. 329–335. These statistical studies of the relative maxima and minima were suggestive of effects produced by the solar electro-magnetic radiation upon the terrestrial magnetic field. They were persistent in longitude, the compilations covering the interval 1842–1896; the series has been extended to 1915. There are, of course, many irregularities to consider, but generally it seems as if the solar output is very persistent as to its maxima in certain longitudes. At the time of the original research, 1892–95, before the discovery of the process of the ionization of rarefied gases under the impact of the solar waves of high frequency, it was difficult to assign a physical process competent to produce such magnetic variations. The statistical efforts to connect sun-spots with the magnetic disturbances are only in part successful, but it is easy to see that many maxima impulses may occur in radiation when the isothermal layer has not been broken through in the sun-spot vortical circulation. The radiation maxima may be nearer the equator than that of the sun-spot development in higher latitudes. The great tidal wave of the solar circulation sweeps with its maximum angular velocity along the sun's equator in 26.68 days, but this trails off in higher latitudes to more than 30.00 days near the poles. It is, however, the 26.68-day period which is dominant in the terrestrial magnetic field.

We have reproduced on Fig. 30 the periodic curve of variation, with its eight maxima and minima in Longitude, I, II, . . . VII, VIII. It is seen that they are quite uniformly distributed around the axis of rotation, and we may suppose that there are eight principal vertical axes of vortical convection along and near the solar equator. Admitting the vertical current with lateral descending branches, we have again a circula-

tion in eight branches related as vortical curls. It does not seem to be unnatural that the convectional movements in the sun should sustain two zones in latitude, one in each hemisphere, and that these should concentrate in maxima and minima in longitudes, according to their inherent thermodynamic processes. If this is the natural condition of large bodies of gaseous materials, cooling under gravitation, it then becomes proper to admit that certain longitudes and latitudes accumulate maximum conditions of radiation. It has not seemed to me possible that such magnetic variations can persist so steadily without some corresponding constitution in the solar mass. What the harmonic laws of the thermodynamic process are in themselves we are not in position to explain, but the preceding analysis may provide some valuable suggestions in this connection.

The Chromosphere and the Inner Corona

The gaseous distribution of the solar elements has given rise to three popular names for their visible appearance: (1) The photosphere, or sharp limb; (2) the chromosphere, or thin luminous ring just above the photosphere; (3) the inner corona, or apparent limit of a gaseous atmosphere. These divisions have a general connection with the vanishing heights of the heavy and light gases, as indicated on Table 8 and Fig. 18. For the temperature $T = 0°$ of the several gases, the hyperbolic law for the vanishing height is,

$$m\, z_{T=0} = 45400 \text{ kilometers}$$

Since Calcium, $m = 40$, just floats conspicuously on or near the flash-spectrum stratum, $z = 400 - 500$, this may be taken as the limit of the heavy photospheric layers; Carbon, $m = 12$, may be taken as the limit of the true chromosphere; hydrogen, $m = 2$, is the limit of the inner corona.

Inner Corona, H = 2, Height = 23000 to 3800 kilometers
Chromosphere, C = 12, " = 3800 to 1100 "
Photosphere, Ca = 40, " = 1100 to 200 "

These merge into one another as the gases vanish at heights corresponding with their atomic weights. In violent convections and eruptions there are temporary intrusions of the heavy gases above their normal vanishing levels, but gravitation seeks to restore them to those levels by means of the thermodynamic processes that have been described. The heights that have been assigned by other writers to the gases in the chromosphere were derived generally from a study of the relative length of the arcs as photographed in the several types of spectra. Many of these results are inconsistent with the thermodynamic data. The spectrum lines depend upon so many factors besides density and height that it will be proper to revise this subject in connection with the data that have been obtained in this computation. Compare Abbot's "Sun," pages 137–182, for details and general information.

The Outer Solar Corona

The true gaseous atmosphere of the sun is surrounded by a very extended and excessively tenuous appendage called the outer corona, which is visible during the minutes of total eclipses of the sun. It is closely associated with the underlying gases and their thermodynamic processes. Generally, it has a quadrilateral shape with maximum wings near latitudes $\pm 40°$, just over the axes of the vertical convection, I, II, III, IV, on Fig. 30. At the minimum of the 11-year period the extensions are mainly equatorial; they are not large over the ascending branch of the deep-throated vortex; while the polar regions are covered with short, individual rays, which conform in shape closely to the lines of force that would surround the sun if it were in fact a large spherical magnet. The quadrilateral forms occur during the rise and fall of the relative intensity of the solar convection; at the maximum the sun is surrounded by coronal streamers of fantastic shape and irregular structure. This change of form of the corona is closely connected with the 11-year cyclic changes in the frequency of the sun-spots, faculæ, prominences, the intensity of the solar radiation into space, the

variations in the terrestrial magnetic field, precipitation, temperature, vapor pressure, barometric pressure, and common climatic conditions. As these have been described in the Meteorological Treatise, the reader can be referred to that place for further data. The subject of solar and terrestrial synchronism in the 11-year, 3.75-year, and shorter cycles has been so extensively studied and verified in all parts of the world during long intervals of time that it is accepted as a general phenomena in solar physics. While further researches are required to determine its processes in detail, the consensus of opinion is growing that it is a subject of great economic value, and that, although complex, there is every prospect of complete solutions of the principal phenomena.

The distribution of the intensity of the coronal radiation has been the subject of many researches in eclipse expeditions. There is distinct radial polarization, vanishing at the photosphere and increasing outward, as if due to the scattering of reflected light on isolated particles of matter thrown outward under the pressure of radiation. These minute particles of matter, produced in the general processes of thermodynamics, pervade all the gaseous strata, as well as the corona, in proportion to the local forces in action, more thickly below than above. Hence, polarized reflection above becomes diffused scattering below in conformity with the observations. The bright coronal line in the spectrum 5303 is assigned to an unknown element, "coronium," and there are several other bright lines. The Fraunhofer dark lines of the photospheric spectrum have been observed, and this is evidence of reflected light coming from the lower levels. The incandescence of the coronal matter is apparently a feeble effect, taken by itself, and the luminous appendage is chiefly dependent upon minute particles of matter in the neighborhood of the sun, which reflect and scatter the true radiation passing through them. These small particles are probably charged with electricity, either from local ionization or from electric charges which are transported from the isothermal strata. There is evidence of the structural arrangement of these particles along the magnetic lines of force in the polar

regions, and along the electrostatic lines of repulsion in the equatorial wings, extending to many diameters of the sun in all directions near that plane. Small particles are transported by the pressure of light after they have been detached, just as comet-tails are formed by surface heating from solar radiation, which sets free the matter that is then carried into space on the waves of light. The corona presents many confused phenomena which interact upon one another.

Solar Magnetism

The general view that the sun is a magnetic sphere has been a subject of discussion for a century. There has been unquestioned evidence that the variations of the terrestrial magnetic field follow those of the sun-spot and prominence frequencies in a persistent synchronism. That there is some causal connection between the entire solar and terrestrial fields is beyond debate, although the difficulty of understanding the natural mechanism has not even yet been fully overcome. The theory that the sun is a magnetic sphere, competent to affect the terrestrial magnetic field by direct action to the extent that is observed, has always been opposed by two arguments: (1) The high temperature of the sun precludes its retention of internal magnetism; (2) the great distance between the two bodies would require an excessive magnetization in the interior of the sun. Two equally obstinate facts have stood against these arguments: (1) The earth retains a permanent magnetic field although its interior is at a very high temperature; (2) the synchronous connection can not be denied, and, indeed, its economic importance in connection with long-period forecasts of weather conditions is so great that the subject justifies research till the physical laws become well understood. My own researches in the years 1889–1891 afforded me sufficient reason for proceeding steadily in this general problem without the least hesitation, the difficulties being an incident in the progress of science.

The entire subject has been greatly illuminated by the progress of physics in the direction of atomic ionization, which

showed two facts, first, that the necessary solar impulses can be propagated to the earth along the electromagnetic field, thus relieving us of the necessity of ascribing to the solar magnetization an excessive intensity; secondly, the identification of the Zeeman Effect in the sun-spots, and in other regions of the sun, so that the high-temperature argument against solar magnetism has fallen out of the discussion. The general result is to leave the sun a low-power spherical magnet and the solar radiation as the carrier of the energy which is observed in the earth's atmosphere in many forms of synchronism. Since electrons, or free charges of electricity, in combination with atoms and molecules, or entirely apart from them, exist in the sun's atmosphere, their movements induce magnetic field, under rotation or during their translation, so that general and local magnetic fields are a fundamental part of the sun's constitution. The credit for determining the existence of the Zeeman Effect in the sun is due to the Director of the Mt. Wilson observatory, Dr. George E. Hale, and the allied solar phenomena are receiving constant study with the special equipment that has been acquired for the purpose. The phenomenon of the ionization of gases received its greatest impulse from Sir. J. J. Thomson, of the Cambridge Physical Laboratory, in 1898, and it has now penetrated the remotest branches of Physical Science. Dr. Hale's discovery was published in 1908, and it is now of common astrophysical interest. During the years preceding 1908 the subject of solar magnetism was in the debatable stage, but since that year it has been marked by steady progress. The preceding data from thermodynamic computations, as to the physical conditions under which the solar gases are controlled, afford much additional ground for believing that the solar-terrestrial synchronism will finally become a complete and practical branch of Astrophysical Meteorology. A few details will now be given, as a summary of the most important facts that are known.

The Polar Rays of the Solar Coronas as Evidence of a Magnetized Sphere

During the minimum years of the solar 11-year cycle the polar rays stand apart with much individuality, so that their paths in polar coordinates ($r.\theta$) can be measured at several points along each ray. The coronas of February 29, 1878, January 1, 1889, and December 22, 1889, were studied especially to determine if there is any period of rotation common to them, such that one model rotated in this period, and placed according to the axes S of the sun's rotation, E of the earth's rotation, and K of the ecliptic, would reproduce the successive aspects of the coronal field. The result may be stated in a few paragraphs:

1. Period of sidereal rotation, 27.4117 days.
" " synodic " 29.6358 "

Mean daily motion (sidereal) in longitude 13°.1331. This applies to the pole of the corona C, which does not coincide with the pole of the sun's rotation.

2. Distance between the pole of the sun and the pole of the corona, $S\,C = 4° 30'$ approximately.

3. The south coronal pole is 100° in advance of the north coronal pole. An ephemeris may be constructed from the following data:

Epoch	Period	North Coronal Pole	South Coronal Pole
1878.0	27.4117	Latitude, 85° 32′ Longitude, 201°.2	Latitude, −85° 24′ Longitude, 301°.6

The axis of polarization is therefore at the surface of the sun about $4\frac{1}{2}°$ from the axis of rotation, and the southern end of it precedes by about 100 degrees in longitude.

4. The rays of the solar corona seem to originate in a belt whose mean distance from the poles of the corona is about 32° or 33°. The bases of these polar rays indicate that there is a zone of maximum output of the small particles which, by the arrangement of them in curves under the influence of the solar magnetic lines of force, make up the visible rays.

5. The curvature and the distribution of the group of polar rays conform to the lines of force of a spherical magnet when they are seen projected on a plane which is normal to the line of sight.

(178) Equation of the lines of force, $N = \frac{8\pi}{3} \cdot \frac{\sin^2\theta}{r}$,

where N is a constant along one ray, but varies from one ray to another.

6. It will be noted, on comparing with the diagram of the prominences, Fig. 30, that the coronal belt coincides closely in latitude with the position of the maximum of the prominences during the development of the descending branch of the 11-year curves, that is, from 6 to 11 years, so that they are closely associated with the poleward branch of the general vortex of circulation.

7. The synchronism between the solar and the terrestrial phenomena is based upon the 26.68-day period, along the solar equator so that these are propagated along the electro-magnetic field of radiation.

8. The periods above give us for the

Base of the coronal rays, 29.64 days; 788′ angular velocity.

Equatorial synchronism, 26.68 " 864′ " "

Comparing with Table 81, "Treatise," it seems that at the surface 788′ occurs in latitude 45° as compared with the spectrum displacements. Since the high-level hydrogen has larger angular velocities, as 830′ daily, it is quite likely that the base of the coronal lines is near the top of the chromosphere, at 5000 kilometers.

9. At the time my research was made, 1889–1891, it was commonly accepted that the solar synodic period was from many sources about 26.00. The fact that the terrestrial magnetic field was at that time competent to yield a period so closely conforming to recent spectroscopic results (870′ to 880′) is testimony to the reality of the solar impulses in the earth's atmosphere.

10. There are reasons to think that this transmission is due to a bombardment of solar corpuscles carried along on the light-wave front; other reasons suggest that the terrestrial electric

charges are due to the disintegration of the atoms and molecules in the high, rarefied strata of the earth's atmosphere, under the impact of the solar radiation of short-wave lengths. Since there is clear evidence of a large depletion of short waves in the earth's atmosphere, it would seem that this is the predominant cause of the high-level ionization.

The Zeeman Effect in the Sun's Atmosphere

The Zeeman Effect was discovered in 1896, and it consists in splitting up a simple spectrum line into two circularly polarized or three plane polarized lines, when the ray in a strong magnetic field is viewed along or normal to the lines of magnetic force H.

Let H = the magnetic intensity along the axis z, at right angles to the plane $x \,.\, y$, and let the ion have the mass m, charge e, and let the coefficient of elasticity to the center be k, then the equations of motion of $e\,m$ are:

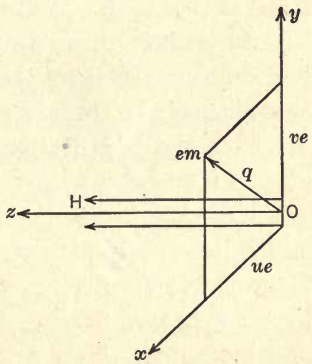


FIG. 31. Elements of the Zeeman Effect.

$$(179)\quad m\frac{d\,u}{d\,t} = -\,k^2\,x + e\,v\,.\,H$$

$$m\frac{d\,v}{d\,t} = -\,k^2\,y - e\,u\,.\,H$$

These equations are solved by the following terms:

(180) $x = x_0\,e^{st}$, provided, $m\,s^2\,x_0 = -\,k^2\,x_0 + e\,H\,.\,s\,y_0$,

(181) $y = y_0\,e^{st}$, provided, $m\,s^2\,y_0 = -\,k^2\,y_0 - e\,H\,.\,s\,x_0$,

(182) where $s = i\dfrac{k}{\sqrt{m}} = i\dfrac{2\,\pi}{T} =$, and $T\dfrac{2\,\pi\,\sqrt{m}}{k}$ is the period.

This occurs when $H = 0$, and there is no magnetic field, so that T represents the undisturbed periodic motion of the line-constituents. If H has a value,

$$(183)\quad s_1 = i\,\frac{k}{\sqrt{m}}\left(1 \mp \frac{e\,H}{2\,k\,\sqrt{m}}\right), \text{ and } T^1 = \frac{2\,\pi\,\sqrt{m}}{k}\left(1 \pm \frac{e\,H}{2\,k\,\sqrt{m}}\right).$$

The sign + signifies a greater periodic motion for the positive rotation, and the sign − for a smaller period.

(1) When viewed *along* the magnetic field the two circular components accelerate or retard the normal rotation, change the wave length and the position of two resultants in the spectrum. The original spectral line becomes *two circularly polarized lines* of equal intensity rotating in opposite directions, and symmetrically displaced in respect of the undisturbed position. (2) When the ion is viewed *across* the magnetic field, at right angles to H, the component along z is unaltered in period and position; the two circular components, seen in the $x\,y$ plane, appear to be plane polarized, the direction of vibration being at right angles to the central z-component. The spectral line is broken up into three distinct plane polarized lines, the central vibrating parallel to the magnetic field and the two outer components vibrating at right angles to it.

Since the polarization may be circular, elliptical, or complex, it follows that there may be developed several component of these two principal types, and numerous lines have been determined by experiment in strong magnetic fields. Such division of a simple spectral line into components is, therefore, proof that a magnetic field exists of sufficient strength to be detected and measured. This method was successfully applied by Dr. G. E. Hale, in 1908, to the sun-spots, wherein the spectrum lines were broadened and subdivided in conformity with these tests. This constitutes proof that free ions or electrons ($e.m$) exist in the sun-spots, and that the vortical motion is sufficiently rapid to produce strong magnetic fields. Similarly, the Zeeman Effect has been generally found in the sun's atmosphere, and its corresponding magnetic field has been determined, so that the subject of solar magnetization of the entire mass, or local magnetic fields accompanied by electric currents, becomes a very important subject of research. The literature is already extensive, so that only a few conclusions can be mentioned. This simple theory of accounting for the Zeeman Effect was first proposed by Lorentz, but it has been extended to comprise very complex motions, including the rotation of

the axes themselves. The ions in high temperature gases may by their motions produce the most varied kinds of optical spectral effects.

The Distribution of the Solar Magnetism as Determined by the Zeeman Effect

If a positive charge rotates anti-clockwise and a negative charge rotates clockwise, a positive magnetic field is generated along the z-axis. Hence, by means of the Zeeman Effect, it is possible to determine the polarity in the sun-spots and in other localities on the sun. The observations on the sun-spots show that there are in each hemisphere about as many of one polarity as the other; that as the sun-spots are commonly generated in pairs they are likely to be of opposite polarities; that the intensity of the magnetic field in the sun-spots increases radially toward the axis and downward from the plane of the photosphere; that the effect is distributed in many anomalous ways among the lines of the spectrum. Dr. Hale has properly associated these phenomena with a true vortex motion, wherein the inner tubes rotate faster than the outer and more rapidly with the depth below the reference plane. It is thought that the introduction of the isothermal shell, the depths of the several gases, the computed P, ρ, R, T, are all in complete harmony with the requirements of the solar spectrum. If the ions are chiefly generated near the bottom of the isothermal layer of each gas, the intrusion of abnormal temperatures from below, as well as from above, will induce segregation of the ions, the negative going to the cooler and the positive to the warmer strata, in a general way. Since the deflecting force of rotation near the equator is small on the sun, it follows that the underflowing hot sheet will rotate indifferently in forming the dependent vortex, so that it is probable that a group of sun-spots consists of independent axes of funnel-shaped tubes, rather than of curved horseshoe vortices terminating on the free surface of a stratum, since no such definite layer exists for the light gases, though by complex interaction between light and heavy gases some such complex layer may

be formed as a photospheric stratum. The structure of this region is so very complicated that the tangled record of the spectrum is to be expected. There are many statements regarding the height of the lines of flow which will need to be modified. The method of double reversal, or that of the long and short arcs, while they give indications of the height, should be interpreted in terms of the general thermodynamic requirements that must prevail on the sun.

The general magnetic field of the sun has been determined with considerable precision, as well as have some of its characteristics. Generally, the magnetic field has opposite signs in the two hemispheres, such that the true positive magnetic pole is on the south side of the plane of the ecliptic on the southern hemisphere of the sun and the negative magnetic pole is on the north side, exactly as is the case with the distribution of the earth's magnetism. It has been suggested that the negative ions as a whole are more distant from the axis of rotation than are the positive ions, and that the rotation of the sun produces its magnetism. It has been thought that the internal magnetization is produced by ampere electric circuital currents rotating about lines which are somewhat parallel to the sun's axis of rotation, as the inner field of a spherical magnet is located. The maximum production of ions is near the bottom of the isothermal layer, and the maximum of disturbed circulation is also near that level. Hence, magnetic field increases from a minimum in the levels above the photosphere to this level. Whether that isothermal value is a true maximum, or only a step toward a higher value in the lower levels, is not now known. The average value of the solar magnetic field is approximately as follows:

Near the coronal poles,	50	gausses
In small sun-spots,	1000	"
In largest sun-spots,	5000	"

The sun's polar magnetism is about 80 times as much as the earth's polar vertical field, which is 0.66000 C. G. S. units. From such data the entire magnetic system of the sun can be readily computed by the well-known formulas.

It has been found that the Zeeman Effect indicates that there is a maximum in each hemisphere in the latitudes $\pm$ 45°, and that a sine curve expresses the distribution, so that there is zero-value at the poles and at the equator. By referring to Fig. 30 of the general solar circulation, it is seen that the maximum vertical circulation has been placed in about latitudes $\pm$ 45° in each hemisphere, with minimum at the poles and at the equator. Hence, it may be inferred that there is a *secondary magnetic solar field due to vertical circulation in latitude*, as if the ions were transported upward and downward in these paths. Very similar convectional circuits in the earth's atmosphere are found to account quite fully for the existing diurnal magnetic variations, and there are other circuits concerned with the general disturbances in latitude.

If circulation in zonal sheets in latitude is sufficient to cause a persistent maxima in certain latitudes, it follows that a similar segregated circulation in longitude is sufficient to account for the maxima of the magnetic field and the maxima of the radiation itself, arranged in longitude, as is suggested by the maxima of Fig. 30. There are persistent and unquestioned impulses from the sun operating on the earth's atmosphere in numerous manifestations, which indicate that there is a general periodic action in 26.68 days, upon which are superposed eight distinct maxima and minima, arranged more or less permanently in longitude. These are marked most clearly in the variations of the terrestrial magnetic field, and less emphatically in the meteorological field. It is stated that these maxima in longitude are not found in the spectrum changes. This can be readily understood from the fact that the spectrum observed belongs generally to the strata lying *above* the level of the isothermal layer, while the maximum impulses themselves originate in convectional operations which are rigorously confined *below* that stratum. The sun-spots penetrate this layer, but they have never been found to synchronize perfectly in their frequency with the system of magnetic pulses that are observed. The minor variations in the spectral lines, due to changes in pressure, density, and motion, are too superficial to take

account of the deep sources of the solar radiant energy and its allied forces. The radiation energy of the sun and the magnetic field of the earth constitute a sensitive mechanism which registers these maximal fluxes of energy, and it would be requiring too much of the spectrum to attempt to see in it a full record of such solar actions. The interpretation of all the symptoms of these fields will doubtless be greatly improved by experience, and it may happen that some telltale signs can be discovered which will serve to indicate the presence of maxima in the radiation. At present the negative results from the spectroheliograph should not be interpreted as conclusive evidence that the problem of maxima has been exhausted as to their synchronisms.

The Solar Spectra

It is not our purpose to study the characteristics of the solar spectra themselves so much as to point out the physical conditions under which they are formed. A good description of them may be found in Abbot's "Sun," and the technical articles are collected in the *Astrophysical Journal* from the Mt. Wilson and other observatories. The following summary of the facts of observation is derived from these sources, and the important matter is to compare them with the data of the preceding tables of solar physical conditions. It will be seen that a very large addition has been made to the knowledge of such conditions, and that while the general harmony between the observed and computed data is excellent, there will be needed many modifications in the inferences that have been made from the behavior of the spectral lines. The present knowledge of the prevailing temperatures, pressures, densities, and gas coefficients, in all strata for gases ranging from hydrogen to mercurial vapor, and the heights at which they occur, will greatly improve the interpretation of the reversals, the shifts, the broadening of the lines, the values of the long and short lines in respect of height, and similar problems. It will be pointed out that we have the material for computing the coefficients of transmission and absorption of the different lines of

the spectrum, and the indices of refraction in all layers of the atmospheres of the earth and the sun. From these data, and those in the tables, studies can be made in atomic and molecular physics, under the conditions of solar temperature and gravitation, and thus escape from many laboratory limitations.

The General Solar Spectrum

The solar spectrum consists of a continuous background, crossed by dark lines which are commonly identified with those of the terrestrial elements. The spectra of all gases become continuous at high temperatures and high pressures. By Table 6 the temperatures in the isothermal layer range from 7650° to 7700°, and in the adiabatic layer they increase rapidly to enormous values; by Table 10 the pressures in terrestrial atmospheres are about 6.08 at the photosphere, about 20 atmospheres at the bottom of the isothermal layer, and they soon become enormous in the adiabatic strata; by Table 12 the density for hydrogen is $\frac{1}{2044}$, that of the earth's normal atmosphere at sea level, and mercury vapor has $\frac{1}{10}$ that density, while at the bottom of the isothermal layer its density is $\frac{1}{5}$ that of air, so that terrestrial sea-level densities occur only in the adiabatic layer, beyond the vision of the spectroscope; the gas efficiencies are indicated in Table 13. These are the conditions for the continuous spectrum. In the reversing layer the temperatures of the light gases, 2 to 60 atomic weight, have been lowered by 2000 to 3000 degrees, while the heavy gases and vapors do not extend to the altitude of 400 to 500 kilometers. These dark lines are formed by interposing a cooler absorbing layer of the same gas between the continuous spectrum and the spectroscope. Most of the dark lines are solar, as iron (55), nickel (58), calcium (40), titanium (48), cobalt (59), chromium (52), magnesium (24), carbon (12), vanadium (51), sodium (23), magnesium (24), hydrogen (2), these being the lines just before reaching the axis of the curves of Fig. 18. The heavy vapors and gases are seen with difficulty, only by intrusion above their normal levels. In the earth's atmosphere there are dark lines

due to absorption at very low temperatures by oxygen, carbonic acid, and aqueous vapor.

Tests are used to determine whether a line is solar or terrestrial: (1) at high and low sun, the terrestrial lines are stronger at low sun; (2) the east and west limbs of the sun directed simultaneously on the slit give the Doppler Effect of shift for solar lines, while the terrestrial lines remain unchanged.

The principal lines of the spectrum are marked, in Ångstrom units, namely, tenth-meter = 0.000 000 000 1 meter.

K	3933.68	calcium,	solar.
H	3968.49	calcium,	"
$H_\gamma = G$	4340.47	hydrogen	"
$H_\beta = F$	4861.35	"	"
b	5183.62	magnesium	"
E	5269.55	iron	"
D_2	5889.98	sodium	"
$H_\alpha = C$	6562.84	hydrogen	"
B	6869.97	oxygen	terrestrial
a	7184.57	aqueous vapor	"
A	7593.83	oxygen	"

The intensities of the lines are classified:

0000 = the most difficult to see.

1 = just clearly visible on Rowland's spectrum map.

1000 = the strong calcium lines, *H*. *K*.

In the solar spectrum the intensities decrease with the increase in the atomic weight, till radium (224) and uranium (236) may exist in the sun without being seen in the lines; there are only a few non-metallic elements; oxygen and helium exist; a very little absorbing gas produces a dark line; lines are dark only by contrast, so that the flash spectrum of the reversing layer consists of bright lines in the places of dark lines, as it is seen in eclipses at the moment the photosphere disappears; short waves are more absorbed than long waves, but the lines from 1.50 μ to 2.50 μ pass through the intervening layers of the solar and terrestrial atmospheres with only selective band absorptions, while the short waves from 0.00 μ to

0.35 μ generally have disappeared; the intermediate lines 0.35 μ to 1.50 μ are depleted in an irregular manner.

Whatever changes the temperature, pressure, and density, relatively to certain normal values on the different levels, also causes variations in the position and shape of the lines. Thus, convection by temporary transportation changes these normal relations in vertical directions, while circulation changes them in horizontal directions, so that on the sun these variations are numerous and very complex.

Increase of pressure shifts the wave lengths toward one end or other of the spectrum, and it broadens certain lines. On the whole, the shifts increase with the wave length, but there are many arbitrary conditions. On the other hand, change in velocity shifts all lines proportional to their wave lengths. Due to the rotation of the sun, all solar lines have a little greater wave length than the corresponding terrestrial lines by a few thousandths of an Ångstrom. The general pressure is from 4 to 6 atmospheres, as determined by the spectrum. There is unsymmetric broadening of some lines, and greater shift on the side of the long waves than on the side of the short waves. There is a common vertical circulation upward in general of 0.1 to 0.3 kilometers per second; there are large vertical movements in the granulations and pores, in the faculæ and sun-spots, and they become at times excessive in the solar prominences. These can all be discussed by the formulas of this Treatise.

Lines are also classified by their temperatures:
Enhanced lines = high temperatures = spark conditions,
Average lines = low temperatures = arc conditions.

The enhanced lines generally indicate vertical convection, bringing high-temperature conditions upward as in the umbra of spots, faculæ, and granulations; low-temperature conditions occur over the center of sun-spots in the reversing layer, in the penumbra and in the pores between the filaments and the granules wherever there is downward circulation; irregular mixtures occur in all levels due to horizontal movements.

The chromosphere, especially in the reversing layer, has a

spectrum opposite to that of the photosphere; there are double reversals in some lines, as 3933.667, wherein K_1 is probably stationary, while K_2, K_2 has a vertical velocity upward of 1.97 kilometers, and K_3 a downward velocity of 1.14 kilometers per second. These represent layers of different levels, K_1 lowest, K_2 higher and rising, K_3 highest and falling. In the sun-spots the vortex circulations bring enhanced lines from below upward into the vortex of the penumbra; the high level gases of the chromosphere descend with their own lower temperatures, as in the penumbra and the reversing layer; there are lines from layers still somewhat normal, so that the sun-spot spectra are very complex and differ from that of the surrounding photosphere as indicated. Seen in superposition, there are many phenomena, such as increase in the shadings and wings; enhanced lines apparently weaker in spots through lowering of temperatures from higher strata; H_α weaker in spots; maximum ordinate of radiation shifts from the penumbra to umbra with the change in the temperature; the short waves are relatively much weaker in the umbra than in the penumbra and the ratio approaches 1.000 in the long waves; temperature over sun-spots less than that of the surrounding photosphere; more scattering and absorption above spots than above the photosphere; many Fraunhofer lines are strengthened and many are weakened in the sun-spots as compared with the photosphere; sun-spot vapors are too cool to produce strong absorption of the enhanced lines; great abundance of flutings in the sun-spots; high temperatures produce complete dissociation in the lower part of the sun-spot vortex, but tendency to association in the central part of the isothermal layer and an abundance of associated compounds above the photosphere itself; reduction of the continuous background in spots is greatest for the short waves.

As between the spectra at the center of the disk and the limb of the sun, the lines are generally displaced toward the red on account of an increase in total pressure under hemispherical curvature. Hydrogen (2), sodium (23), calcium (40), magnesium (24) show no displacement; titanium (48), vana-

dium (51), scandium (44) show moderate displacement; iron (55), nickel (58) show considerable up to 0.007 Ångstroms. That is, high atomic weights are but little displaced; enhanced lines show maximum displacement. The violet edges of the lines do not shift. The limb spectra are weaker than at the center, and the violet lines need much more exposure at the limb; the Fraunhofer lines are much changed, especially in the violet; the strong lines lose their shading or wings; the enhanced, high temperature lines are weakened at the limb; the lines which are strong in spots are strong at the limb; the hydrogen H_a is widened at the limb.

Atmospheric Refraction and Scattering

The general relations between the refraction or change of direction of a ray of light passing through an atmosphere and the loss of the energy by non-selective scattering upon the molecules are expressed by Rayleigh's Formula:

(184) $$k = \frac{32\pi^3 (\mu - 1)^2 l_0 B}{3 \lambda^4 n_0 . B_0} = \frac{32\pi^3 (\mu - 1)^2}{3 \lambda^4 g_0} \cdot \frac{P}{n_0 \rho_0}$$

in the following notation,

k = coefficient of scattering, μ = index of refraction,

λ = the wave length in centimeters, l_0 = the height of the homogeneous atmosphere at the stratum whose pressure is B_0,

B = the barometric pressure at the level under discussion,

n = the number of molecules per cu. cm., System = (C. G. S.)

(185) We have, $\frac{l_0 B.}{B_0} = \frac{\rho_m B_0}{\rho_0} \cdot \frac{B}{B_0} = \frac{\rho_m B}{\rho_0} = \frac{P}{g_0} \cdot \frac{1}{\rho_0}$, Table 3, Treatise.

Hence, by transformations,

(186) $$(\mu - 1)^2 = k . \lambda^4 \cdot \frac{3 g_0}{32\pi^3} \cdot \frac{n \rho}{P} .$$

On the sea level, from which the homogeneous height may be computed, or on the photosphere of the sun, the pressure $P = P_0$. The formula was devised to compute k at any height above the plane of reference, but since we have computed

n, ρ, P, at numerous points in each atmosphere, we can at once take them at the point in any stratum of any gas for which they have been prepared. The conditions are, however, so complex for the different wave lengths that it is possible merely to give some approximate results in the atmospheres of the earth and the sun as examples of a method that can be utilized in discussing these important subjects. It will be necessary to obtain the values of k from the observations with the pyrheliometer and the bolometer, and it will be convenient to summarize the formulas of the theory of refraction.

THE FORMULAS OF REFRACTION, FIG. 28, IV

Terms	Polar Coordinates	Angle Incidence	Angle Refraction	Refraction	Index of Refraction
Upper point.....	$(R+z)$, $(\theta+d\theta)$	$i_1, i_2 \ldots$	$f_1, f_2 \ldots$	$\delta_1, \delta_2 \ldots$	$\mu_1, \mu_2 \ldots$
Lower point.....	R, θ	z			μ_0

φ = the angle which the tangent to the light-curve in the plane $x\,y$ makes with the axis$-x$. ζ = the zenith distance.

α = the angle of the ray with an horizon = $90-f$.

R = the radius to the lower level, $R+z$ to the higher level.

θ = the angle from x to R, $\theta + d\,\theta$ = the angle from x to $R+z$.

(187) $x = r\cos\theta$, generally. $d\,x = d\,s\cos\varphi$. $\dfrac{d\,x}{d\,s} =$

$$-\,r\sin\theta\frac{d\,\theta}{d\,s}+\cos\theta\frac{d\,r}{d\,s}.$$

$y = r\sin\theta$, generally. $d\,y = d\,s\sin\varphi$. $\dfrac{d\,y}{d\,s} =$

$$+\,r\cos\theta\frac{d\,\theta}{d\,s}+r\sin\theta\frac{d\,r}{d\,s}.$$

(188) $f = i-\delta = \varphi-\theta = 90^\circ-\alpha$ = the angle of refraction.
$i = \delta+f = \varphi-\theta+\delta = 90-\alpha+\delta$ = the angle of incidence.

(189) From the laws of optics, $\frac{\mu_0}{\eta} = \frac{\sin i}{\sin f}$,

(190) From the triangle of $(R, R+z)$, $\frac{R+z}{R} = \frac{\sin \zeta}{\sin f}$. Hence,

(191) $\sin f = \frac{\mu}{\mu_0} \sin i = \frac{R}{R+} \sin \zeta$, and generally,

(192) $\mu\,(R+z) \sin i = \mu_0 R \sin \zeta = C = \text{constant.}$

(193) From (189) $\frac{\mu_0 - \mu}{\eta_0 + \mu} = \frac{\sin i - \sin f}{\sin i + \sin f} = \frac{\tan \frac{1}{2}(i-f)}{\tan \frac{1}{2}(i+f)} = \frac{1}{2} \frac{d \,.\, \delta}{\tan i}$.

(194) The differential equation of refraction, $d\,\delta = \frac{d\,\mu}{\mu} \tan i$.

(195) Hence $\mu \cos i \,.\, d\,\delta = d\,\mu \,.\, \sin i$.

More analytically, we have to quote the differential equation (Kummer, Sitz. Berlin Akad, 12 July, 1860),

(196) $$\mu \left(x \frac{d\,y}{d\,s} - y \frac{d\,x}{d\,s} \right) = C.$$

Substituting the auxiliaries, there are two equations,

(197) I. $\mu\,(r \cos \theta \,.\, \sin \varphi - r \sin \theta \,.\, \cos \varphi) = \mu r \sin (\varphi - \theta) = \mu r \sin f = C$

(198) II. $\mu r \cos \theta \left(r \cos \theta \frac{d\,\theta}{d\,s} + \sin \theta \frac{d\,r}{d\,s} \right) - \mu r \sin \theta \left(- r \sin \theta \frac{d\,\theta}{d\,s} + \cos \theta \frac{d\,r}{d\,s} \right) = \mu r^2 \frac{d\,\theta}{d\,s} = C$

(199) $\mu r^2 d\,\theta = C\,d\,s = C\,(d\,r^2 + r^2\,d\,\theta^2)^{\frac{1}{2}}$, $d\,\theta = \frac{C}{r\,(\mu^2 r^2 - C^2)^{\frac{1}{2}}}$.

(200) $\theta = \int_0^z \frac{\mu_0 R \sin \zeta \, d\,z}{(R+z)\,[\mu^2\,(R+z)^2 - \mu^2_0 R^2 \sin^2 \zeta\,]^{\frac{1}{2}}}$.

(201) There are three cases, I. $\mu\,(R+z) > \mu_0 R \sin \zeta$, atmospheric refraction

II. $\mu\,(R+z) = \mu_0 R \sin \zeta$, circular refraction

III. $\mu\,(R+z) < \mu_0 R \sin \zeta$, imaginary refraction.

The index of refraction is connected with the density of the stratum by the general equation,

(202) $\mu^2 - 1 = 4\kappa\rho$, $\mu = 1 + 2\kappa\rho$ and $\frac{d\mu}{\mu} = \frac{2\kappa \cdot d\rho}{1+4\kappa\rho}$, for κ = a constant.

Several hypotheses (Chauvenet) have been employed to determine ρ, the density in the strata above the sea-level value, ρ_0.

(203) $\frac{P}{P_0} = \frac{\rho R T}{\rho_0 R_0 T_0}$, by the Boyle-Gay Lussac Law.

I. It is throughout these discussions assumed that $R = R_0$ = constant, and consequently it is erroneously supposed that the atmosphere is stratified along adiabatic gradients. This being the case, the discussions need complete revision throughout the formulas. This is an exceedingly complicated problem, and it may prove that the empirical formula of Bessel cannot be improved for practical purposes. The first supposition regarding the temperature is that $T = T_0$, and is isothermal. Hence,

(204) $$\frac{P}{P_0} = \frac{\rho}{\rho_0} = e^{-\frac{z}{l_0}}$$

This does not lead to satisfactory results.

II. Assume the following relations, which are easily found:

(205) $$\frac{R}{R+z} = \left(\frac{\mu}{\mu_0}\right)^{n+1} = \left(\frac{1+4\kappa\rho}{1+4\kappa\rho_0}\right)^{\frac{n+1}{2}} = 1 - 2(n+1)\kappa(\rho_0 - \rho) = 1 - \frac{z}{R}.$$

This finally leads to the equation,

(206) $$\frac{P}{P_0} \cdot \frac{\rho_0}{\rho} = \frac{T}{T_0} = 1 - \frac{z}{2 l_0}.$$

It can be easily shown that the temperatures do not conform to this formula in the higher strata.

III. Assume, with Bessel, the following fundamental formula,

(207) $\frac{P}{P_0}\frac{\rho_0}{\rho} = e^{-\frac{z}{h}} = 1 - \frac{z}{h} + \frac{1}{2}\left(\frac{z}{h}\right)^2 - \ldots = \frac{T}{T_0}$. Hence,

(208) $\frac{P}{P_0} = e^{-\frac{h}{l_0}\left(e^{\frac{z}{h}} - 1\right)}$, and $\frac{\rho}{\rho_0} = e^{-\frac{h}{l_0}\left(e^{\frac{z}{h}} - 1\right) + \frac{z}{l_0}} = e^{-\left(\frac{z}{l_0} - \frac{z}{h}\right)}$.

Take $l_0 = 7991.04$, and $h = 227775.7$ meters. In common logarithms,

(209) $$\log \rho = \log \rho_0 - M\left(\frac{z}{l} - \frac{z}{h}\right).$$

TABLE 92

THE DENSITY ρ AS COMPUTED BY THE NON-ADIABATIC, THE ADIABATIC, AND THE BESSEL FORMULAS

Balloon Ascension, Uccle, June 9, 1911

(210) Non-adiabatic. $$\text{Log } \rho_1 = \log \rho_0 + \frac{n}{\kappa - 1}(\log T_1 - \log T_0)$$

(211) Adiabatic. $$\text{Log } \rho_1 = \log \rho_0 + \frac{1}{\kappa - 1}(\log T_1 - \log T_0)$$

(212) Bessel. $$\text{Log } \rho_1 = \log \rho_0 - M\left(\frac{z}{l} - \frac{z}{h}\right)$$

Height in Meters	Non-adiabatic (Bigelow)	Adiabatic	Bessel
$z = 50000$	0.0009	0.0003	0.009
45000	0.0054	0.0022	0.053
40000	0.0160	0.0049	0.097
35000	0.0293	0.0088	0.177
30000	0.0511	0.0188	0.0324
25000	0.0885	0.0407	0.0593
20000	0.1543	0.0901	0.1084
15000	0.2708	0.2012	0.1983
10000	0.4753	0.4179	0.3627
5000	0.7830	0.7229	0.6633
000	1.2132	1.2132	1.2132

Taking the same value of $\rho_0 = 1.2132$ at the sea level, it is seen that the adiabatic is lower in amount than the non-adiabatic value up to 50000 meters; the Bessel value is lower than the non-adiabatic throughout; it is less than the adiabatic from the sea level to 15000 meters, and then remains intermediate between the adiabatic and the non-adiabatic values up to 50000 meters. It will be exceedingly difficult to develop any formula that can supply the place of the non-adiabatic temperature gradient, while depending solely upon the surface conditions. In the case of the Bessel formula for refraction, upon which the working tables are constructed, it is probable

that the exponents A and λ make some compensation for the actual inaccuracy in the density ρ.

(213) $\delta = a\,\beta^{A}\,\gamma^{\lambda}\tan\zeta$ = Bessel's Formula.

The Atmospheric Transmission for Different Wave Lengths p_λ Collected According to the Value of p_w by the Pyrheliometer

Volumes II and III of the Annals of the Astrophysical Observatory of the Smithsonian Institution contain the coefficients of transmission as measured by the pyrheliometer p_w, together with the coefficients for several wave lengths as determined by the bolometer p_λ. These have been collected in groups with p_w at convenient intervals, 0.900 — 0.890, 0.890 — 0.880, 0.880 — 0.870, etc., and the mean values were obtained for the several years. These were united in general means, and they were plotted on large diagrams, with p_λ for ordinates and λ the wave length for abscissas. Each group under the pyrheliometer p_w results in a curve of transmission coefficients, and the series of p_w make a family of curves. These were adjusted to probable values, and from them were scaled the final values which appear in Tables 93, 94, 95, for Washington, Mt. Wilson, and Mt. Whitney, respectively.

Resuming the black spectrum for 6950°, corresponding with 3.98 gr. cal./cm.^{2} min., the spectrum values for the different wave lengths were multiplied by the coefficients of transmission under the several values of p_w, resulting in the groups of p_λ and J_λ. The latter represent the transmitted values of the radiation, and as they are homogeneous throughout they give relative values. Take the sums for the selected wave lengths along the spectrum; the factor 16.7 suffices to reduce these sums to calories, and it is seen that for each value of p_w there are corresponding calories,

such as, 2.79, 2.71, . . . 2.20, 2.13 for Washington,
" 2.95, 2.92, . . . 2.72, 2.63 for Mt. Wilson,
" 3.04, 2.99, . . . 2.78, 2.72 for Mt. Whitney.

The pyrheliometric transmission at different values has its counterpart in the bolometric transmission at different wave

TABLE 93

THE COEFFICIENTS OF TRANSMISSION AND THE ORDINATES IN THE SPECTRUM FOR DIFFERENT VALUES OF THE PYRHELIOMETER p_w. COLLECTED FROM ABBOT'S OBSERVATIONS

Washington, 34 meters

Pyrheliometer	p_w	0.884		0.861		0.840		0.821		0.801		0.780		0.762		0.743		0.697		0.664	
T = 6950°	J_λ	p_λ	J_λ	p_λ	J_λ	p_λ	J_λ	p_λ	J_λ	p_λ	J_λ	p_λ	J_λ	p_λ	J_λ	p_λ	J_λ	p_λ	J_λ	p_λ	J_λ
0.00—0.25μ	7.35	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.30	4.85	.472	2.29	.460	2.23	.435	2.11	.413	2.00	.402	1.95	.384	1.86	.376	1.82	.361	1.75	.350	1.70	.332	1.61
0.35	6.05	.570	3.45	.546	3.30	.517	3.13	.502	3.04	.484	2.93	.469	2.84	.438	2.65	.418	2.53	.392	2.37	.373	2.26
0.40	6.54	.660	4.32	.636	4.16	.613	4.01	.588	3.85	.551	3.60	.537	3.51	.508	3.32	.483	3.16	.458	3.00	.431	2.82
0.45	6.50	.773	5.02	.732	4.76	.690	4.49	.657	4.27	.640	4.16	.627	4.08	.602	3.91	.575	3.74	.550	3.58	.523	3.40
0.50	6.13	.824	5.05	.801	4.91	.767	4.70	.736	4.51	.702	4.30	.683	4.19	.668	4.09	.642	3.94	.616	3.78	.592	3.63
0.55	5.59	.860	4.81	.836	4.67	.810	4.53	.783	4.38	.756	4.23	.732	4.09	.711	3.97	.689	3.85	.663	3.71	.641	3.58
0.60	5.00	.883	4.42	.863	4.32	.844	4.22	.820	4.10	.798	3.99	.772	3.86	.748	3.74	.728	3.64	.703	3.52	.682	3.41
0.70	3.88	.923	3.58	.903	3.50	.883	3.43	.865	3.36	.848	3.29	.822	3.19	.805	3.12	.782	3.03	.753	2.92	.743	2.88
0.80	2.96	.946	2.80	.927	2.74	.910	2.69	.893	2.64	.875	2.60	.856	2.53	.837	2.48	.822	2.43	.803	2.38	.784	2.32
0.90	2.25	.957	2.15	.939	2.11	.924	2.08	.907	2.04	.893	2.01	.874	1.97	.861	1.94	.843	1.90	.829	1.87	.813	1.83
1.00	1.72	.963	1.66	.948	1.63	.933	1.60	.917	1.58	.903	1.55	.887	1.53	.874	1.50	.858	1.48	.843	1.45	.828	1.42
1.10	1.33	.967	1.29	.953	1.27	.940	1.25	.924	1.23	.911	1.21	.900	1.20	.883	1.17	.870	1.16	.854	1.14	.841	1.12
1.20	1.04	.968	1.01	.955	.99	.943	.98	.930	.97	.916	.95	.905	.94	.890	.93	.878	.91	.862	.90	.852	.89
1.30	0.82	.967	.79	.955	.78	.944	.77	.932	.76	.921	.76	.909	.75	.897	.74	.885	.73	.870	.71	.860	.71
1.40	0.66	.966	.64	.955	.63	.945	.62	.933	.62	.924	.61	.912	.60	.902	.59	.890	.59	.877	.58	.867	.57
1.50	0.53	.965	.51	.955	.51	.945	.50	.934	.50	.926	.49	.915	.48	.905	.48	.895	.47	.883	.47	.873	.46
1.60	0.43	.963	.41	.954	.41	.945	.41	.934	.40	.927	.40	.917	.39	.907	.39	.897	.39	.888	.38	.877	.38
1.70	0.36	.962	.35	.954	.34	.944	.34	.936	.34	.928	.33	.918	.33	.909	.33	.900	.32	.890	.32	.880	.32
1.80	0.29	.961	.28	.953	.28	.944	.27	.937	.27	.929	.27	.920	.27	.911	.26	.902	.26	.892	.26	.882	.26
1.90	0.25	.960	.24	.952	.24	.944	.24	.938	.23	.930	.23	.921	.23	.912	.23	.904	.23	.894	.22	.884	.22
2.00	0.21	.960	.20	.952	.20	.944	.20	.938	.20	.931	.20	.922	.19	.913	.19	.905	.19	.896	.19	.886	.19
2.10	0.18	.959	.17	.951	.17	.944	.17	.939	.17	.932	.17	.923	.17	.915	.16	.907	.16	.898	.16	.889	.16
2.20	0.15	.958	.14	.951	.14	.943	.14	.939	.14	.933	.14	.926	.14	.917	.14	.909	.14	.900	.14	.891	.13
2.30	0.13	.957	.12	.951	.12	.943	.12	.939	.12	.934	.12	.927	.12	.918	.12	.911	.12	.902	.12	.892	.12
2.40	0.11	.956	.11	.950	.10	.942	.10	.939	.10	.933	.10	.927	.10	.920	.10	.912	.10	.902	.10	.893	.10
2.50	0.10	.955	.10	.950	.10	.942	.09	.939	.09	.933	.09	.927	.09	.921	.09	.912	.09	.903	.09	.894	.09
2.60	0.09	.954	.09	.950	.09	.942	.08	.938	.08	.932	.08	.927	.08	.921	.08	.912	.08	.904	.08	.895	.08
	65.50		46.00		44.70		43.27		41.99		40.70		39.73		38.65		37.39		36.14		34.96
Factor 16.7			.702		.682		.661		.641		.621		.607		.590		.571		.552		.534
Cal	3.98		2.79		2.71		2.63		2.55		2.47		2.42		2.35		2.27		2.20		2.13

TABLE 94

THE COEFFICIENTS OF TRANSMISSION AND THE ORDINATES IN THE SPECTRUM FOR DIFFERENT VALUES OF THE PYRHELIOMETER p_w

Mt. Wilson, 1720 meters

Pyrheliometer	p_w	0.925		0.915		0.906		0.896		0.886		0.876		0.866		0.856		0.844		0.825	
$T = 6950°$	J_λ	p_λ	J_λ	p_λ	J_λ	p_λ	J_λ	p_λ	J_λ	p_λ	J_λ	p_λ	J_λ	p_λ	J_λ	p_λ	J_λ	p_λ	J_λ	p_λ	J_λ
0.00—0.25μ	7.35	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.30	4.85	.405	1.96	.405	1.96	.405	1.96	.404	1.96	.403	1.95	.403	1.95	.402	1.95	.402	1.95	.401	1.94	.401	1.94
0.35	6.05	.656	3.97	.646	3.91	.636	3.85	.625	3.78	.613	3.71	.601	3.64	.589	3.56	.575	3.48	.562	3.40	.530	3.21
0.40	6.54	.755	4.94	.748	4.89	.740	4.84	.731	4.78	.720	4.71	.708	4.63	.695	4.55	.681	4.45	.666	4.36	.634	4.15
0.45	6.50	.833	5.41	.828	5.38	.822	5.34	.814	5.29	.804	5.23	.792	5.15	.778	5.06	.762	4.95	.745	4.84	.708	4.60
0.50	6.13	.879	5.39	.873	5.35	.867	5.31	.860	5.27	.851	5.22	.841	5.16	.828	5.08	.814	4.99	.800	4.90	.769	4.71
0.55	5.59	.906	5.06	.896	5.01	.887	4.96	.882	4.94	.877	4.90	.868	4.85	.858	4.80	.844	4.72	.833	4.66	.805	4.50
0.60	5.00	.925	4.63	.918	4.59	.911	4.56	.904	4.52	.897	4.49	.889	4.45	.880	4.40	.869	4.35	.858	4.29	.832	4.16
0.70	3.88	.956	3.71	.950	3.69	.944	3.66	.938	3.64	.932	3.62	.926	3.59	.919	3.57	.910	3.53	.899	3.49	.875	3.40
0.80	2.96	.972	2.88	.966	2.86	.960	2.84	.954	2.82	.950	2.81	.945	2.80	.940	2.78	.932	2.76	.924	2.74	.903	2.67
0.90	2.25	.980	2.21	.975	2.19	.969	2.18	.965	2.17	.960	2.16	.955	2.15	.950	2.14	.945	2.13	.937	2.11	.918	2.07
1.00	1.72	.984	1.69	.979	1.68	.974	1.68	.970	1.67	.965	1.66	.960	1.65	.955	1.64	.950	1.63	.944	1.62	.928	1.60
1.10	1.33	.986	1.31	.983	1.31	.978	1.30	.973	1.29	.968	1.29	.963	1.28	.957	1.27	.952	1.27	.946	1.26	.931	1.24
1.20	1.04	.988	1.03	.984	1.02	.980	1.02	.976	1.02	.972	1.01	.966	1.00	.960	1.00	.954	.99	.948	.99	.933	.97
1.30	.82	.988	.81	.984	.81	.980	.80	.976	.80	.972	.80	.966	.79	.960	.79	.955	.78	.950	.78	.935	.77
1.40	.66	.988	.65	.984	.65	.980	.65	.976	.64	.972	.64	.966	.64	.961	.63	.956	.63	.951	.63	.937	.62
1.50	.53	.988	.52	.984	.52	.980	.52	.976	.52	.972	.52	.966	.51	.961	.51	.956	.51	.951	.50	.938	.50
1.60	.43	.988	.42	.984	.42	.980	.42	.976	.42	.972	.42	.967	.42	.962	.41	.957	.41	.952	.41	.939	.41
1.70	.36	.987	.36	.984	.35	.980	.35	.976	.35	.972	.35	.967	.35	.962	.35	.958	.34	.952	.34	.940	.34
1.80	.29	.986	.29	.983	.29	.980	.28	.976	.28	.972	.28	.967	.28	.962	.28	.958	.28	.953	.28	.942	.27
1.90	.25	.985	.25	.983	.25	.980	.25	.976	.24	.972	.24	.968	.24	.963	.24	.959	.24	.953	.24	.943	.24
2.00	.21	.984	.21	.983	.21	.980	.21	.976	.20	.972	.20	.968	.20	.963	.20	.959	.20	.954	.20	.944	.20
2.10	.18	.984	.18	.983	.18	.980	.18	.976	.18	.972	.17	.968	.17	.964	.17	.960	.17	.954	.17	.945	.17
2.20	.15	.983	.15	.982	.15	.979	.15	.976	.15	.973	.15	.968	.15	.964	.14	.960	.14	.955	.14	.946	.14
2.30	.13	.983	.13	.982	.13	.979	.13	.976	.13	.973	.13	.969	.13	.964	.13	.960	.12	.955	.12	.947	.12
2.40	.11	.983	.11	.981	.11	.979	.11	.976	.11	.973	.11	.969	.11	.964	.11	.961	.11	.956	.11	.948	.10
2.50	.10	.982	.10	.980	.10	.978	.10	.976	.10	.973	.10	.969	.10	.964	.10	.961	.10	.957	.10	.949	.09
2.60	.09	.982	.09	.980	.09	.978	.09	.976	.09	.973	.09	.969	.09	.965	.09	.962	.09	.958	.09	.950	.09
	65.50		48.46		48.10		47.74		47.36		46.96		46.48		45.97		45.25		44.71		43.28
Factor 16.7			.740		.734		.729		.723		.717		.710		.702		.691		.683		.661
	3.98		2.95		2.92		2.97		2.88		2.85		2.83		2.79		2.75		2.72		2.63

lengths. This group of results conforms in their average values to those which have been heretofore employed, Table 79, but they express in detail the effects of the atmospheric conditions upon the spectrum processes of scattering.

TABLE 95

THE COEFFICIENTS OF TRANSMISSION AND THE ORDINATES IN THE SPECTRUM FOR DIFFERENT VALUES OF THE PYRHELIOMETER p_w

Mt. Whitney, 4420 meters

Pyrheliometer	p_w	0.938		0.918		0.898		0.878		0.858		0.838	
$T = 6950°$	J_λ	p_λ	J_λ	p_λ	J_λ	p_λ	J_λ	p_λ	J_λ	p_λ	J_λ	λ	λ
0.00 − 0.25μ	7.35	—	—	—	—	—	—	—	—	—	—	—	—
0.30	4.85	.510	2.47	.508	2.46	.506	2.45	.504	2.44	.502	2.43	.500	2.43
0.35	6.05	.670	4.05	.631	3.82	.595	3.60	.574	3.47	.556	3.36	.545	3.30
0.40	6.54	.810	5.30	.772	5.05	.743	4.86	.700	4.58	.665	4.35	.642	4.20
0.45	6.50	.877	5.70	.854	5.55	.826	5.37	.792	5.15	.772	5.02	.750	4.88
0.50	6.13	.913	5.60	.898	5.50	.878	5.38	.852	5.22	.827	5.07	.804	4.93
0.55	5.59	.930	5.20	.919	5.14	.904	5.05	.883	4.94	.861	4.81	.837	4.68
0.60	5.00	.942	4.71	.932	4.66	.921	4.61	.907	4.54	.884	4.42	.859	4.30
0.70	3.88	.961	3.74	.950	3.70	.941	3.66	.929	3.61	.911	3.54	.890	3.46
0.80	2.96	.971	2.87	.962	2.85	.952	2.82	.941	2.79	.927	2.74	.909	2.69
0.90	2.25	.976	2.20	.967	2.18	.958	2.16	.947	2.13	.934	2.10	.921	2.07
1.00	1.72	.978	1.68	.970	1.67	.962	1.65	.952	1.64	.939	1.62	.926	1.59
1.10	1.33	.979	1.30	.972	1.29	.963	1.28	.953	1.27	.943	1.25	.930	1.24
1.20	1.04	.980	1.03	.972	1.02	.964	1.01	.955	1.00	.946	.99	.931	.98
1.30	0.82	.980	.80	.972	.80	.964	.79	.956	.78	.947	.78	.932	.76
1.40	0.66	.980	.65	.973	.64	.965	.64	.956	.63	.947	.63	.932	.62
1.50	0.53	.980	.52	.973	.52	.965	.51	.956	.51	.947	.50	.933	.49
1.60	0.43	.980	.42	.973	.42	.966	.42	.957	.41	.947	.41	.934	.40
1.70	0.36	.980	.35	.973	.35	.966	.35	.957	.34	.948	.34	.935	.34
1.80	0.29	.980	.28	.973	.28	.966	.28	.957	.28	.948	.27	.936	.27
1.90	0.25	.980	.25	.973	.24	.966	.24	.957	.24	.948	.24	.937	.23
2.00	0.21	.980	.21	.973	.20	.966	.20	.958	.20	.948	.20	.938	.20
2.10	0.18	.980	.18	.973	.18	.966	.17	.958	.17	.948	.17	.938	.17
2.20	0.15	.980	.15	.973	.15	.966	.14	.958	.14	.949	.14	.939	.14
2.30	0.13	.980	.13	.973	.13	.966	.13	.958	.12	.950	.12	.940	.12
2.40	0.11	.980	.11	.973	.11	.967	.11	.958	.11	.950	.10	.941	.10
2.50	0.10	.981	.10	.973	.10	.967	.10	.958	.10	.951	.10	.942	.09
2.60	0.09	.981	.09	.973	.09	.967	.09	.959	.09	.951	.09	.942	.08
	65.50		50.09		49.10		48.07		46.90		45.79		44.76
Factor 16.7			.765		.750		.734		.716		.699		.683
	3.98		3.04		2.99		2.92		2.85		2.78		2.72

The Terrestrial Values of the Index of Refraction $(\mu - 1)$

We shall utilize these coefficients of transmission p_λ first to compute the coefficients of absorption,

(214) $$-k_\lambda = \frac{1}{M} \log p_\lambda,$$

and then proceed by the Rayleigh Formula (184) to compute the values of $(\mu - 1)$ for the index of refraction,

$$(215) \qquad (\mu - 1)^2 = k_\lambda \cdot \lambda^4 \frac{3\, g_0}{32\, \pi^3} \cdot \frac{n\, \rho}{P}.$$

From Tables 93, 94, 95, interpolate for four selected wave lengths 0.323 μ, 0.481 μ, 0.670 μ, 1.225 μ, as examples, the values of p'_λ corresponding to p_w. Compute the equivalent coefficients of scattering. In the Rayleigh Formula we have for 0.323 μ, $\lambda^4 = (0.0000323 \text{ cm.})^4$; g_0 is the acceleration of gravitation 980.6; n is the number of molecules per cubic centimeter, Tables 15, 21, 27, note page 55, Bul. No. 4, O. M. A.; ρ is the density from Tables 9, 17, 23; P is the pressure from Tables 9, 17, 23. The mean values for these three balloon ascensions were used in this formula. The resulting values of $(\mu - 1)$ for Washington, Mt. Wilson, and Mt. Whitney are given for four wave lengths, in order to illustrate the variability in the value of $(\mu - 1)$. It increases from large values of p_w, the pyrheliometric coefficient of transmission, to low values of p_w; it increases always with the increase in the wave length; it diminishes generally with the altitude of the station.

The Solar Values of the Index of Refraction

The data are taken for the computations from Abbot's Table 55, Vol. III., which gives the relative brightness for different lines, from the center to the limb at several radial distances. These distances are $\sigma = r \sin \zeta$, from which the zenith distance ζ and the sec ζ are computed. In the sun's atmosphere we can take the path length of the ray, as in Section IV, Fig. 28, proportional to the sec ζ, so that by the Bouguer Formula,

$$(216) \qquad \log p = \frac{\log I_1 - \log I_0}{\sec \zeta_1 - \sec \zeta_0}.$$

In order to compute I_0, I_1, I_2, . . . the relative intensity, we proceed as follows: Take the relative brightness from Table 55; interpolate in the black spectrum of 7655° for the given wave length, and divide by 20.9, in this distribution of the

spectrum line distances, to reduce to gr. cal./cm.^{2} min., and this is the assumed value of I_0 at the center of the disk; multiply I_0 by the observed brightness at the other radial distances, so that we have the successive values, I_0, I_1, I_2, . . . ; these were plotted in pairs on a diagram whose abscissas are sec ζ and whose ordinates are log I, as in the usual pyrheliometric reductions; the resulting values of p_λ appear in the Section I, Table 97. From these values of p_λ compute k_λ, in Section II. From k_λ compute $(\mu - 1)$ in Section III, for four wave lengths of hydrogen, carbon, calcium, mercury. The gravity acceleration $G = 27484.3$ cm./sec.; $n =$ the number of molecules per cu. cm., Table 32; ρ is the density, Table 12; P is the pressure, Table 9.

It is seen that $(\mu - 1)$ decreases from the center to the limb of the sun; it increases with the wave length; and it increases with the molecular weight. The terrestrial values of $(\mu - 1)$ in the lower strata of the atmosphere are about ten times as large as they are for the solar gases in the neighborhood of the photosphere. The values of $(\mu - 1)$ decrease gradually to vanishing values in the outermost strata of both atmospheres.

General Remarks

There can be no greater hardship than to present the results of an extensive logarithmic computation by means of a few selected numerical values at the end. The entire working of the formulas in their details is lost upon the reader, and the relations are in themselves so complex that it is quite impossible to follow them intelligently apart from the actual processes that are involved. In this case the fundamental formulas of thermodynamics are employed in a new field for their application, and, as a matter of fact, the entire series checks within one or two units in the fifth decimal of logarithms. In the solar data the numbers are unusually large, but even the considerable values here quoted in the tables are not sufficient to reproduce the checks. The assumed value of the temperature at any point implies the succeeding terms as computed.

TABLE 96

COEFFICIENTS OF TRANSMISSION, SCATTERING, AND INDEX OF REFRACTION IN THE TERRESTRIAL ATMOSPHERE

Washington, D. C., 34 Meters

p_w	0.884	0.861	0.840	0.821	0.801	0.780	0.762	0.743	0.697	0.664
	p_λ = coeffi	cients of tra	nsmission fo	r a few wave	lengths					
0.323μ	0.517	0.500	0.473	0.454	0.440	0.423	0.405	0.377	0.369	0.351
0.481μ	.805	.775	.738	.706	.676	.662	.643	.617	.591	.566
0.670μ	.911	.891	.872	.852	.833	.807	.788	.766	.738	.725
1.225μ	.968	.955	.943	.931	.917	.906	.892	.880	.864	.854
	k_λ = coeffi	cients of abs	orption or sc	attering	$k_\lambda = \frac{1}{M} \log p_\lambda$					
0.323μ	0.660	0.693	0.749	0.790	0.821	0.860	0.904	0.949	0.997	1.052
0.481μ	.220	.255	.289	.348	.392	.413	.442	.483	.526	.569
0.670μ	.0932	.115	.137	.160	.183	.214	.238	.267	.304	.322
1.225μ	.0325	.0461	.0587	.0715	.0867	.0987	.114	.128	.146	.158
	(μ=1) =	the index of	refraction							
0.323μ	.0002723	.0002790	.0002901	.0002954	.0003037	.0003108	.0003187	.0003265	.0003347	.0003438
0.481μ	3486	3754	3996	4384	4654	4777	4942	5106	5391	5609
0.670μ	4312	4993	5338	5768	6169	6671	7036	7452	7951	8183
1.225μ	8693	10352	11681	12892	14196	15147	16278	17248	18421	19164

Mt. Wilson, 1750 Meters

p_w	0.925	0.915	0.906	0.896	0.886	0.876	0.866	0.856	0.844	0.825
	p_λ = coeffi	cients of tra	nsmission							
0.323μ	0.520	0.516	0.511	0.506	0.500	0.494	0.488	0.482	0.475	0.460
0.481μ	.862	.856	.850	.843	.833	.822	.809	.794	.779	.746
0.670μ	.947	.940	.934	.928	.922	.915	.907	.898	.887	.862
1.225μ	.988	.984	.980	.976	972	.966	.960	.954	.949	.934

	k_λ = coeffi	cients of abs	orption or sc	attering						
0.323μ	0.654	0.662	0.671	0.681	0.693	0.705	0.718	0.730	0.745	0.777
0.481μ	.149	.156	.163	.171	.183	.196	.211	.231	.250	.293
0.670μ	.0545	.0619	.0683	.0747	.0812	.0889	.0977	.1065	.1199	.1485
1.225μ	.0121	.0161	.0202	.0243	.0284	.0346	.0408	.0471	.0624	.0683
	$(\mu-1)$ =	the index of	refraction							
0.323μ	.0002445	.0002463	.0002479	.0002498	.0002519	.0002541	.0002565	.0002586	.0002612	.0002668
0.481μ	2591	2651	2710	2776	2871	2972	3083	3226	3356	3633
0.670μ	3040	3240	3403	3559	3711	3883	4160	4250	4509	5018
1.225μ	4789	5524	6187	6786	7336	8098	8793	9448	9965	11389

Mt. Whitney, 4420 Meters

p_w	0.938	0.918	0.898	0.878	0.858	0.838				
	p_λ = coeffi	cients of tra	nsmission							
0.323μ	0.584	0.565	0.547	0.536	0.527	0.521				
0.481μ	.899	.881	.858	.829	.806	.783				
0.670μ	.956	.945	.935	.922	.903	.881				
1.225μ	.980	.972	.964	.955	.946	.931				
	k_λ = coeffi	cients of abs	orption and	scattering						
0.323μ	0.538	0.571	0.603	0.624	0.641	0.652				
0.481μ	.106	.127	.153	.188	.216	.244				
0.670μ	.0450	.0566	.0672	.0811	.1021	.1267				
1.225μ	.0202	.0284	.0367	.0461	.0555	.0715				
	$(\mu-1)$ =	the index of	refraction							
0.323μ	.0001967	.0002027	.0002083	.0002118	.0002147	.0002165				
0.481μ	1936	2120	2326	2579	2764	2938				
0.670μ	2448	2745	2991	3286	3687	4107				
1.225μ	5483	6501	7390	8245	9112	10315				

TABLE 97

COEFFICIENTS OF TRANSMISSION, SCATTERING, AND INDEX OF REFRACTION IN THE SOLAR ATMOSPHERES

Radius	0.00	0.20	0.40	0.55	0.65	0.75	0.825	0.875	0.92	0.95
I. Computed coefficients of the solar transmission, p_λ										
0.323μ	—	0.269	0.359	0.440	0.500	0.568	0.620	0.680	0.720	0.766
.386	—	.425	.449	.493	.525	.575	.634	.683	.750	.800
.433	—	.430	.485	.522	.560	.620	.670	.729	.761	.820
.456	—	.500	.534	.570	.607	.650	.690	.730	.770	.825
.481	—	.506	.542	.580	.620	.665	.718	.750	.782	.830
.501	—	.506	.543	.582	.622	.666	.720	.749	.780	.827
.534	—	.510	.546	.590	.630	.675	.730	.760	.795	.840
.604	—	.515	.550	.600	.640	.700	.745	.786	.830	.865
.670	—	.575	.620	.660	.700	.702	.780	.816	.845	.882
.699	—	.617	.658	.693	.722	.760	.800	.820	.843	.883
.866	—	.675	.720	.750	.775	.802	.835	.860	.876	.902
1.031	—	.705	.752	.780	.800	.820	.846	.868	.885	.907
1.225	—	.720	.776	.810	.822	.837	.858	.877	.902	.917
1.655	—	.740	.820	.855	.864	.877	.895	.910	.922	.936
2.097	—	.784	.857	.870	.878	.890	.904	.918	.932	.940
II. Computed coefficients of scattering $k_\lambda = \frac{1}{M} \log p_\lambda$										
0.323μ	—	1.313	0.892	0.821	0.693	0.566	0.478	0.386	0.329	0.267
.386	—	.856	.801	.707	.644	.553	.456	.381	.288	.223
.433	—	.844	.724	.650	.580	.478	.401	.316	.273	.198
.456	—	.693	.628	.562	.494	.431	.371	.315	.261	.193
.481	—	.681	.612	.545	.478	.408	.331	.288	.246	.186
.501	—	.681	.610	.541	.475	.405	.329	.289	.249	.190

.534	—	.673	.606	.528	.462	.393	.315	.274	.242	.174
.604	—	.664	.598	.511	.446	.357	.295	.241	.186	.145
.670	—	.554	.478	.416	.357	.298	.249	.204	.169	.126
.699	—	.483	.419	.375	.332	.274	.223	.199	.171	.147
.866	—	.393	.328	.288	.255	.220	.181	.151	.132	.103
1.031	—	.349	.285	.249	.223	.199	.168	.142	.123	.098
1.225	—	.329	.254	.211	.196	.178	.153	.131	.103	.084
1.655	—	.301	.199	.157	.146	.131	.111	.094	.081	.066
2.097	—	.243	.154	.139	.130	.117	.101	.086	.071	.062

III. Computed coefficient of refraction, $(\mu-1)^2 = k.\lambda^4 \frac{3\,G}{32\,\pi^3}\cdot\frac{n\,\rho}{P}$.

Hydrogen	0.323μ	.00000208	.00000171	.00000164	.00000151	.00000136	.00000126	.00000113	.00000104	.00000094
$m = 2$	0.481μ	332	315	297	278	257	231	216	199	173
	0.670μ	581	540	503	466	426	389	353	321	277
	1.225μ	1497	1315	1198	1156	1101	1020	944	837	756
Carbon	0.323μ	.00000567	.00000467	.00000448	.00000412	.00000372	.00000342	.00000307	.00000284	.00000255
$m = 12$	0.481μ	905	858	809	758	700	631	588	544	473
	0.670μ	1583	1470	1372	1271	1161	1061	961	874	.755
	1.225μ	4078	3583	3266	3148	3000	2781	2573	2282	2061
Calcium	0.323μ	.00000981	.00000809	.00000776	.00000713	.00000644	.00000592	.00000532	.00000491	.00000443
$m = 40$	0.481μ	1567	1486	1402	1313	1213	1088	1018	942	819
	0.670μ	2742	2547	2376	2201	2011	1839	1664	1515	1308
	1.225μ	7065	6208	5658	5453	5269	4818	4458	3953	3570
Mercury	0.323μ	.00002262	.00001864	.00001789	.00001643	.00001502	.00001365	.00001226	.00001132	.00001020
$Hg = 198$	0.481μ	3613	3425	3232	3027	2790	2519	2349	2171	1888
	0.670μ	6322	5872	5478	5075	4637	4238	3836	3492	3015
	1.225μ	16287	14310	13043	12571	11979	11106	10277	9113	8229

No attempt has been made to modify the standard formulas beyond adapting them to non-adiabatic strata. Such further changes as may be required by other physical laws must be a matter for further research. An effort will be made to discover a reliable method of procedure, which can be utilized for the complex molecular systems, as well as for the monatomic gases. It is probable that an expert application of the spectroscopic physics to such thermodynamic data will be required for its successful completion. The practical ways to obtain valuable economic forecasts will depend upon the selection of such sensitive variations in some of the thermal or visible lines of the spectrum as will register the internal solar movements more accurately than the sun-spots, faculæ, and prominences can ever do. It is evident that so large a field of discovery has been opened up as to require much intelligent cooperation.

The Variation of the Intensity of the Sun's Radiation in Longitude as Developed by the 26.68-Day Synodic Period

Besides the question regarding the variability of the intensity of the sun's radiation as a whole from year to year, as in the 11-year period, there remains the important problem of the variability from one meridian of the sun to another, as has been indicated. The variations in the intensity of the terrestrial magnetic field have produced the 26.68-day period, as corresponding with one synodic period at the solar equator, and the ephemeris on page 334 of the Meteorological Treatise. It is obvious that the practise of publishing the meteorological data in calendar months is fatal to these studies in solar physics, because it obliterates all minor periodic effects. This unscientific method should be abandoned.

During the years 1912 to 1916 inclusive, the pyrheliometric observations were carried on regularly at Cordoba and Pilar, and during such intervals at La Quiaca as were practical throughout the same years. The observations have all been recomputed by the method described in Chapter VI, and the results were collected according to the 26.68-day ephemeris. It turns

out that the series at La Quiaca has seven observations on the average for each day of the period, and the Cordoba-Pilar series an average of fifteen observations on each day. The curves in hand represent these results for La Quiaca, 3465 meters above the sea-level; for Cordoba, 438 meters, and for Pilar, 380 meters. Similarly, collections of other data follow: the amplitudes of the horizontal magnetic force at Pilar 1910–1916; the maximum temperatures at Pilar for the same interval; the relative frequency of the wind direction from the south, the complement being from the north; and the precipitation. These data are designed to show that *there is a variation in the intensity of the sun's radiation in longitude which is systematic and persistent.* There are a principal maximum near the 15th day, a secondary maximum near the 2d day, and corresponding minima near the 8th and the 21st days, direct for the meteorological data and inverse for the radiation. These are shown by the mean curves. There are eight minor maxima on each curve, and the position of them is quite harmonious throughout the series. *The mean synodic variation in the solar radiation is about three per cent,* and it is better defined at the high level station at La Quiaca than it is at Cordoba-Pilar. It takes a large amount of data to separate this small impulse from the local conditions. The dust-effect is very troublesome to eliminate from the radiation, the magnetic storms must be removed from the amplitudes, and the general circulation disturbs the direct effect of the solar impulse on the meteorological elements. The primary fact is, however, clear that the sun is so constituted as to maintain two axes of maximum and minimum intensity of radiation at right angles to each other, and two minor maxima near the extremity of each axis. These results are in harmony with those published in 1895 and 1898, so that the 26.68-day period is now proved to have maintained itself *without shifting* from 1840 to 1916. These minor crests of radiation have their effects upon the circulation of the earth's atmosphere and the prevailing weather conditions. It follows that forecasts of such conditions can be made by projecting the fundamental curve according to the ephemeris, and experience shows that it is of

practical value for long-range forecast conditions. There is no evidence that a 25-day period, and an 18-day lag in the effect, has any validity; on the other hand, persistent maximum impulses on the 26.68-day period certainly prevail. There is no doubt, also, that an equipment suitable for following the solar action closely will make it possible to interpret many of the seeming discordances in terms of profitable, scientific forecasts of the weather to be expected.

These data and opinions contradict many of the ideas at present prevailing regarding this matter. Meteorologists have generally failed to obtain good results because they do not employ a sufficient amount of homogeneous data, they do not utilize the 26.68-day period in their compilations, nor do they recognize the phenomena of the double annual inversion in the effects under local conditions. Failing these essentials, their views must be disregarded. Solar spectroscopists note the variable frequency in the sun-spots, the faculæ, the flocculi, and the prominences, and it has been proved that they synchronize annually with the magnetic field and the intensity of the radiation. We have now proved that the intensity of the radiation, the amplitudes of the magnetic and the meteorological fields synchronize in the 26.68-day rotation, and the inference must follow that the visible markings on the sun necessarily have definite relations to the maxima, which have their positions in fixed solar longitudes. It should be remembered that *the spectroheliograph is effective in the superficial layers of the solar gases, generally near the top of the sun's isothermal layer, while the radiation is generated primarily near the bottom of the isothermal layer of each constituent gas.* While the surface phenomena must have definite relations to the low-level impulses, it is evident that the circumstances seen in the lines of the spectra are not entirely comprehensive. At present, it is required to trace out, so far as possible, the relations between the variations in the spectral lines, the surface solar conditions, and their connection with the original radiation processes. Similar remarks apply to the spectrobolometer.

Furthermore, it appears from the computations that some

of the adopted theories regarding the Wien-Planck and the Stefan formulas of radiation need modification, and it would be easy to point out several of the details of such a reconstruction. This work is being prosecuted as rapidly as possible, in view of the complexity of the fundamental facts of the structure of matter. It is mentioned in this place only to emphasize the opinion that the subject has been by no means exhausted, and that meteorologists and astrophysicists should generally enter upon the new methods of discussion which have been opened.

Especially, it will be necessary to set apart a few meteorological stations and astrophysical observatories for this purpose. It is supposed that fifty meteorological stations and six solar physics observatories will be sufficient under an international supervision, to follow the solar-terrestrial synchronism practically, and to make progress in the necessary physical studies. The economic value should be considered from the world-view, rather than from that of any nation. Such long-range forecasts annually are already possible in certain localities, such as Argentina, where conditions are favorable. It would be very rash to attempt to set any limit to the possible development of a function already proved to be world-wide and to persist for more than half a century, especially now that it has become possible to determine the 26.68-day variability of the intensity of the solar radiation by means of simple pyrheliometers when the computations are made in a definitive form.

The method of computation of the intensity of the solar radiation employed in this Treatise, makes the pyrheliometer independent of the complex bolometer, and this simplification of the procedure renders it more practicable to extend the observations to many cooperative stations. Simple working instructions can be readily prepared for general use.

CHAPTER VIII

Three Theories of Radiation

In the following notes a brief account will be given of the three theories of the origin of electromagnetic radiation, which seem to suggest further research and verification.

1. *Planck's Theory of Oscillators.* The atoms are conceived to have polarized charges on their spherical surfaces, which oscillate in periodic frequencies equal to those of light. The form of the Wien-Planck function for the intensities in the spectrum was deduced from the theorems of probability and entropy. It seems to be well verified by experiments, at least as to its form. The conditions imposed upon the radiation were *strictly adiabatic*, the coefficients k, h are constants, but these are not applicable in free atmospheres except in specified regions.

2. *Bigelow's Theory of Collisions.* The Planck formula has been reproduced as to its form from the non-adiabatic thermodynamics, wherein k, h are wide, variable coefficients, the periodic frequencies being derived from the collisions reacting upon the electrons of a nucleus-sphere. The negative charges circulate in orbits, arranged upon spherical surfaces, which undergo vibratory oscillations as the result of the shocks from the collision of such dynamic systems.

3. *Bohr's Theory of Non-Radiating Orbits.* The Rutherford positive nucleus controls a series of coplanar orbits of negative electrons circulating in rings of stability which are non-radiative. The periodic frequency is due to the electrons passing from one orbit to another, as the result of changing the configurations, and the radiation is the loss of energy during the readjustments of the circulating electrons.

It seems proper to present briefly the most important features of these three theories, for the sake of facilitating their further discussion and improvement, because it is not probable that the structure of the atom is at present sufficiently understood.

The Derivation of the Wien-Planck Formula for Black Radiation in a Spectrum

The primary question in this branch of physics is whether thermodynamic conditions in the gases are wholly consistent with the electromagnetic radiation, or whether some unknown dynamical term is required to bridge the gap. On the adiabatic basis of k, h as constants, it seems impractical to solve this problem, but on the basis of non-adiabatic variable coefficients the subject enters upon a new phase of discussion.

The Wien-Planck formula for black radiation in the spectrum is based upon an analytic discussion of the Theories of Probability, Equipartition of Energy, Entropy, and Thermodynamic Energy. It is desirable to make the basis as specific as possible, especially in view of the fact that k and h are not constants. The following statement somewhat modifies that followed by Planck,* Richardson,† and others.

Let $k\,T$ represent the kinetic energy of an electron; let $h\,\nu$ represent the potential energy of an electron, where T is the temperature of the medium and ν is the frequency of oscillation, probably between separate electrons, rather than the oscillation of polar charges. Inner energy of one electron $U = k\,T + h\,\nu$.

(217) Let $\eta = \dfrac{h\,\nu}{U} = \dfrac{h\,\nu}{k\,T + h\,\nu}$ = the probability of radiation occurring.

(218) $$\frac{1}{\eta} = \frac{U}{h\,\nu} = \frac{k\,T + h\,\nu}{h\,\nu} = \frac{k\,T}{h\,\nu} + 1.$$

(219) $$\frac{1-\eta}{\eta} = \frac{1}{\eta} - 1 = \frac{k\,T}{h\,\nu} = p\,.\,u_\nu$$ proportional to the radiation density at the ν frequency.

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* Planck, "Wärmestrahlung," p. 139.

† Richardson, "Electron Theory of Matter," p. 353.

(220) $p = \frac{\lambda^3}{8\pi h} = \frac{1}{8\pi h} \cdot \frac{c^3}{\nu^3}$ (Planck).

(221) $\frac{U}{h\nu} + \frac{1}{2} = \frac{kT}{h\nu} + \frac{3}{2}$. $\quad \frac{U}{h\nu} - \frac{1}{2} = \frac{kT}{h\nu} + \frac{1}{2}$.

(222) From the Boltzmann Entropy Law, $S = k \log w$, Planck deduces the probability formula,

(223) $N S = k \log \mathbf{W} = -N k \sum_0^\infty P_m \log P_m$.

Take $P = \frac{1}{\eta}$, and give m successive values,

(224) $N S = -N k \left[\frac{1}{\eta} \log \eta + \left(\frac{1}{\eta} - 1\right) \log \left(\frac{1}{\eta} - 1\right)\right]$.

Substitute $\frac{1}{\eta} = \frac{U}{h\nu} + \frac{1}{2}$, and differentiate for $d S$, $d U$.

(225) $S = k \left[\left(\frac{U}{h\nu} + \frac{1}{2}\right) \log \left(\frac{U}{h\nu} + \frac{1}{2}\right) - \left(\frac{U}{h\nu} - \frac{1}{2}\right) \log \left(\frac{U}{h\nu} - \frac{1}{2}\right)\right]$.

(226) $\frac{dS}{dU} = \frac{1}{T} = \frac{k}{h\nu} \log \frac{\frac{U}{h\nu} + \frac{1}{2}}{\frac{U}{h\nu} - \frac{1}{2}} = \frac{k}{h\nu} \log \frac{\frac{kT}{h\nu} + \frac{3}{2}}{\frac{kT}{h\nu} + \frac{1}{2}} =$

$\frac{k}{h\nu} \log \left(1 + \frac{h\nu}{kT}\right)$ omitting $-\frac{1}{2}\left(\frac{h\nu}{kT}\right)^2 + \frac{1}{3}\left(\frac{h\nu}{kT}\right)^3 - \ldots$

(227) Hence, $\frac{h\nu}{kT} = \log \left(1 + \frac{h\nu}{kT}\right)$, and $e^{\frac{h\nu}{kT}} = 1 + \frac{h\nu}{kT}$.

(228) The radiation $p u_\nu = \frac{kT}{h\nu} = \frac{1}{e^{\frac{h\nu}{kT}} - 1}$, for the ν frequency.

Integrating for the total spectrum,

(229) $u = \int_0^\infty u_\nu \, d\nu = \frac{8\pi h}{c^3} \int_0^\infty \frac{\nu^3 \, d\nu}{e^{\frac{h\nu}{kT}} - 1} = \frac{48\pi h}{c^3} \left(\frac{kT}{h}\right)^4 \left(1 + \frac{1}{2^4} + \frac{1}{3^4} \ldots\right)$.

(230) $u = \frac{48\pi\beta}{c^3} \frac{k^4}{h^3} T^4 = a\,T^4$, Stefan Law.

These formulas are first approximations to natural conditions of radiation, and since k, h, a, are not universal constants, they are of only formal value, though they can be used in succession wherever a certain set of values (k, h) are adopted. They, therefore, lead to families of spectra, which are superposed on the same level in atmospheres consisting of several independent gases. The formula of integration (227) $\frac{h\nu}{kT} = \log\left(1 + \frac{h\nu}{kT}\right)$ is tautological when $\frac{h\nu}{kT}$ is a small term; generally it is not decisive. The following computations show how far the adiabatic case, upon which Planck's discussion is founded, is from satisfying the natural conditions of atmospheric radiation.

Planck's function is a form of the probability curve, which gives the distribution of the ordinates of intensity according to their relative frequency of occurrence in seeking a maximum parameter, the Temperature. It seems, therefore, that the thermodynamic conditions in a given volume of an atmosphere should be capable of reaching the same result from the collisions of n molecules. We shall proceed to deduce two formulas, one for the non-adiabatic branch and one for the adiabatic branch of the curve for h, into which the Planck formula divides itself at the bottom of the solar isothermal layers, as heretofore explained. Compare Fig. 32 for these three curves of the potential h of the vibrating electrons.

Derivation of Bigelow's functions for the potential h. The first of the formulas is for the non-adiabatic branch, and it is derived from the terms $P.\rho.R.T$, taking them in the bulk, as an integrated effect. The second of the formulas is for the adiabatic branch, and it is deduced directly from the constituents of $P.\rho.R.T$, by the derivatives in terms of $n.\ N.\ T$, which are much more fundamental. Indeed, it will be proved that ($n.\ N.\ T$) is a better system for thermodynamics than the usual ($P.\ \rho.\ R.\ T$).

The General Equations of Condition

$$(231)\quad \frac{P}{P_0} = \left(\frac{T}{T_0}\right)^{\frac{n\kappa}{\kappa-1}}. \qquad \log\left(\frac{P}{P_0}\right) = \frac{n\kappa}{\kappa-1}\log\left(\frac{T}{T_0}\right).$$

$$\frac{dP}{P} = \frac{n\kappa}{\kappa-1}\cdot\frac{dT}{T} + \frac{\kappa}{\kappa-1}\log\left(\frac{T}{T_0}\right)dn.$$

$$(232)\quad \frac{\rho}{\rho_0} = \left(\frac{T}{T_0}\right)^{\frac{n}{\kappa-1}}. \qquad \log\left(\frac{\rho}{\rho_0}\right) = \frac{n}{\kappa-1}\log\left(\frac{T}{T_0}\right).$$

$$\frac{d\rho}{\rho} = \frac{n}{\kappa-1}\frac{dT}{T} + \frac{1}{\kappa-1}\log\left(\frac{T}{T_0}\right)dn.$$

$$(233)\quad \frac{R}{R_0} = \left(\frac{T}{T_0}\right)^{n-1}. \qquad \log\left(\frac{R}{R_0}\right) = (n-1)\log\left(\frac{T}{T_0}\right).$$

$$\frac{dR}{R} = (n-1)\frac{dT}{T} + \log\left(\frac{T}{T_0}\right)dn.$$

The Boyle-Gay Lussac Law, with all terms variable.

(234) $Pv = RT$. Differentiate and make the substitutions.

(235) $P\,dv + v\,dP = R\,dT + T\,dR.$

$$(236)\quad P\,dv = \frac{P}{\rho}\left[-\frac{n}{\kappa-1}\frac{dT}{T} - \frac{1}{\kappa-1}\log\left(\frac{T}{T_0}\right)dn\right] =$$

$$-R\frac{n}{\kappa-1}dT - RT\frac{1}{\kappa-1}\log\left(\frac{T}{T_0}\right)dn.$$

$$(237)\quad v\,dP = dP\cdot\frac{RT}{P} = RT\left[\frac{n\kappa}{\kappa-1}\frac{dT}{T} + \frac{\kappa}{\kappa-1}\log\right.$$

$$\left.\left(\frac{T}{T_0}\right)dn\right] = R\frac{n\kappa}{\kappa-1}dT + RT\frac{\kappa}{\kappa-1}\log\left(\frac{T}{T_0}\right)dn.$$

$$(238)\quad R\,dT \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots = R\,dT.$$

$$(239)\quad T\,dR = T\left[R(n-1)\frac{dT}{T} + R\log\left(\frac{T}{T_0}\right)dn\right] =$$

$$(n-1)\,R\,dT + RT\log\left(\frac{T}{T_0}\right)dn.$$

$$(240)\quad P\,dv + v\,dP = R\,dT + T\,dR = nR\,dT + RT\log\left(\frac{T}{T_0}\right)dn.$$

This is the non-adiabatic form. For $n = 1$, we have,

(241) $P\,dv + v\,dP = R_a\,dT$, the adiabatic form.

Since $n = \frac{d T_a}{d T}$, and $g\, d z = - C p_a\, d T_a$, we have by substitution and addition,

(242) $P\, d v + v\, d P + g\, d z = -(C p_a - R)\, d T_a + R T \log \left(\frac{T}{T_0}\right) d n.$

This is the equation of condition in non-adiabatic atmospheres. Hence, by integration, and with $P\, d v = d W$,

(243) $g_{10}\,(z_1 - z_0) = - \frac{P_1 - P_0}{\rho_{10}} - (C p_a - R_{10})\,(T_a - T_0) - (W_1 - W_0) + R_{10}\, T_{10} \log \left(\frac{T_1}{T_0}\right)\,(n_1 - n_0).$

(244) $g_{10}\,(z_1 - z_0) = - \frac{P_1 - P_0}{\rho_{10}} - (U_1 - U_0) - (W_1 - W_0) + R_{10}\, T_{10} \log \left(\frac{T_1}{T_0}\right)\,(n_1 - n_0).$

In my computations the last term in $(n_1 - n_0)$ has been omitted, but more exactly it must be included. My equations for radiation depend upon $\frac{d U}{d v} = K$, while Planck's depend upon $\frac{d U}{d (h \nu)} = J$. It is interesting to note that they have such close relations as have been already pointed out, besides others, as in the examples on the following pages.

It is evident that the Wien displacement law $\lambda_m T = 0.2891$ constant applies only to the adiabatic case, while $\lambda_m T$ is a variable in all non-adiabatic atmospheres. It is thought that these values are too small, according to certain bolometric data, and that the $\lambda_m T$ values for λ_m in centimeters may have to be decreased in the logarithm by about 0.30000 to make the λ_m somewhat larger. This amounts to moving the Planck plotted curve a little to the left of its position. This discrepancy may be due to imperfections in the computation, such as the omission of the term $R T \log \left(\frac{T}{T_0}\right) d n$ in the equation of condition. Until further progress has been made in these details the subject must be left open for study.

Additional Formulas

It is seen that the derivatives are only for $P.\rho.R.T$, which treat of the gas from the integrated or the bulk conditions, rather than from their individual molecular constituents.

The full significance of the change of the adiabatic constants to the non-adiabatic variable coefficients in atmospheres cannot be completely anticipated, especially in molecular-atomic-electronic physics. However, there emerge from the preceding computations certain general formulas, of which a few may be mentioned.

Kinetic energy in monatomic gases, $\kappa = \frac{Cp}{Cv} = \frac{5}{3}$.

$$(245)\quad \frac{1}{3} m q^2 = P V = m R T = K T = \rho R T V = n k T V = N k T = \frac{n}{N} K T V.$$

Kinetic energy in gases whose $\kappa = \frac{Cp}{Cv}$ is not $\frac{5}{3}$

$$(246)\quad \frac{1}{\kappa - 1} \cdot \frac{1}{3} m q^2 = \frac{1}{\kappa - 1} P V = \frac{1}{\kappa - 1} n k T V = \frac{1}{\kappa - 1} N k T = \frac{1}{\kappa - 1} \frac{n}{N} k T V = H V = Cv \rho T V = Cv m T.$$

Specific heat at constant volume Cv,

$$(247)\quad Cv = \frac{1}{\kappa - 1} \cdot \frac{P V}{m T} = \frac{1}{\kappa - 1} \frac{N k T}{m T} = \frac{H V}{m T} = \frac{1}{\kappa - 1} \frac{1}{3} q^2 \frac{1}{T}.$$

The specific heat in terms of k, h, ν (Wien-Planck),

$$(248)\quad Cv = N \frac{\partial}{\partial T} \left[\frac{h \nu}{e^{\frac{h\nu}{kT}} - 1} + \frac{h \nu}{2} \right] = N \frac{h^2 \nu^2}{k T^2} \frac{e^{\frac{h\nu}{kT}}}{\left(e^{\frac{h\nu}{kT}} - 1\right)^2}, \text{ (Einstein).}$$

The inner energy in the kinetic theory of gases is related to the pressure as follows:

$$(249)\quad (U_1 - U_0)_\kappa = \frac{1}{\kappa - 1} (P_1 - P_0) = \left(\frac{Cv}{R}\right)_{ad} (P_1 - P_0)$$

by the kinetic theory.

$$(250)\quad \text{If } (U_1 - U_0)_{Th} = Cv_{10} (T_a - T_0)_{Th} = (Cp_a - R_{10}) (T_a - T_0)$$

is the inner energy as derived from the non-adiabatic thermodynamics; the two are connected by the following formula:

$$(251)\quad (U_1 - U_0)_\kappa = \rho_{10}\left[\frac{\kappa}{\kappa - 1}\frac{P_1 - P_0}{\rho_{10}} + g\,(z_1 - z_0) + (U_1 - U_0)_T + (W_1 - W_0)_T\right].$$

Pressure, number of molecules, and the kinetic energy.

$$(252)\qquad P = n\,k\,T = \frac{1}{3}\rho\,q^2. \qquad P\,V = N\,k\,T = \frac{1}{3}m\,q^2.$$

$$(253)\quad \frac{3}{2}k\,T = \frac{1}{n}\cdot\frac{1}{2}\rho\,q^2 = \frac{1}{N}\cdot\frac{1}{2}m\,q^2, \text{ for three degrees of freedom.}$$

$$(254)\quad \int\frac{\partial P}{P} = \frac{P_0 - P_1}{P_{10}} = \frac{1}{M}\log\frac{P_0}{P_1} = \int\frac{\partial n}{n} = \frac{n_0 - n_1}{n_{10}} = \frac{1}{M}\log\frac{n_0}{n_1}.$$

$$(255)\quad P_0 - P_1 = n_{10}\,k\,T\,\frac{1}{M}\log\frac{n_0}{n_1} = c\,(z_1 - z_0)\,\frac{m}{2}\,P_{10}\,T_{10}\,h.$$

It follows that Planck's Wirkungsquantum h can be expressed in thermodynamic terms.

$$(256)\quad h = \frac{1}{c\,(z_1 - z_0)}\cdot\frac{1}{\frac{m}{2}}\cdot\frac{1}{T_{10}}\cdot\frac{P_0 - P_1}{P_{10}}.$$

c is the velocity of light, $(z_1 - z_0)$ the depth of the column of gas where P_0 is the pressure at z_0, and P_1 that at z_1; T_{10} is the mean temperature of the column in which the gradient has the same value through the stratum; $\frac{m}{2}$ is half the atomic weight in monatomic gases, and it, therefore, is the number of the electrons in each molecule of this kind. Hence, h represents the potential energy which exists between pairs of electrons per degree in the simplest form of the gaseous structure. This seems to be fundamentally true in solar gases, but to need an additional term in terrestrial gases.

It was proved by trial computations that $\frac{m}{2}$ must be used

and not m, as if about one-half of the number of electrons forming the atom were concerned in this potential action. Since the adiabatic branch has a different function it seems to be implied that the electrons may be in another form of configuration, perhaps wholly dissociated under the very high prevailing temperature, more than 8000°. It should be noted that in the Planck formula, wherein $c \,.\, h \,.\, k$ are assumed to be universal constants in the exponent, $\frac{h\,c}{k\,\lambda\,T}$, such that for $c_2 = \frac{h\,c}{k}$ = constant. For the *adiabatic case*, the distribution is different in the *non-adiabatic case.* Here $\frac{h\,c}{k\,\lambda\,T}$ becomes $\frac{h\,\nu}{k\,T}$ and $k\,T$ *is a constant for each atmosphere under its gravitation, while* h *and* ν *are both variables.* It is entirely probable that the potential h between electrons of the same gas should vary in value from one level to the other while the kinetic energy is a constant, $\frac{3}{2}\,k\,T = \frac{1}{2}\,m\,u^2$ = constant for each atom.

The Variable k in All Atmospheres

It has been verified throughout the computations that $(k\,T)_E$ has one constant value in the earth's atmosphere, and that $(k\,T)_S$ has a constant value in the sun's atmosphere, such that,

(257) $(k\,T)_E \times \gamma^2 = (k\,T)_S$. $\log \gamma^2 = 2.89514$.

		C. G. S.	M. K. S.
(258)	Earth, $(k\,T)_E = 3.7145 \times 10^{-14}$	−14.56990	−21.56990
(259)	Sun, $(k\,T)_S = 2.9179 \times 10^{-11}$	−11.46508	−18.46508

The kinetic energy of each atom is one constant in the earth's atmosphere and another constant in the sun's atmosphere. They are so related that

(260) $\frac{(k\,T)_S}{(k\,T)_E} = \gamma^2$, where $\gamma = \frac{G}{g} = 28.028$,

which is the ratio of the surface gravity accelerations.

This is correct since we assumed $k\,T = \frac{2}{3}E_0$ = constant.

The Atmospheric Pressure

Since $P = n\,k\,T$, it follows that the pressure in gases depends upon the number of molecules present in the unit volume. Furthermore,

(261) $P = n\,k\,T$ . . . . . pressure,

(262) $\rho = \dfrac{n\,m}{N}$ . . . . . density,

(263) $R = \dfrac{N}{m}\,k$ . . . . . the gas efficiency, and

(264) Boyle-Gay Lussac Law, $P = \rho\,R\,T = n\,k\,T = \dfrac{n\,m}{N} \cdot \dfrac{N}{m} \cdot k\,T.$

These propositions greatly simplify the theory of gases, especially in view of the probable numerical number of the electrons $= \dfrac{m}{2}$.

The Variable Potential Coefficient h

It remains to illustrate the relation of the variable potential coefficient to the computed volume density of the radiation by the two formulas:

(265) Thermodynamic, $h_B = \dfrac{1}{c\,(z_1 - z_0)} \cdot \dfrac{1}{\frac{m}{2}} \cdot \dfrac{1}{T_{10}} \cdot \dfrac{P_0 - P_1}{P_{10}}$, . . .

Bigelow.

(266) Radiation, $h_P = \left(\dfrac{48\,\pi\,\beta}{a}\right)^{1/3} \dfrac{k^{4/3}}{c}$, ... Planck.

In tables 98–102 is collected the summary of the computed values of u, using these two values of h for the sun and the earth, respectively. They are collected by temperatures, instead of by heights, since the several gases give nearly the same values of k, h, at the same temperatures, as computed. This applies to the monatomic elements and hydrogen in the Bigelow formula, but hydrogen is given separately under the Planck formula.

In the Bigelow computation the exponent α is taken 4.00, as in the Stefan formula, while in the Planck formula both h and α vary in such a way as to produce very nearly the same value of the volume density of the radiation u, as can be seen

by comparing Tables 98 and 99. *This is an additional proof that the solar radiation is black,* and it has been hereby computed directly from the thermodynamic non-adiabatic data.

In the case of the earth's atmosphere, there is brought out a large discrepancy in the Bigelow formula, as in Tables 100 and 101. The h_B (Bigelow) is much smaller than h_P (Planck), and the resulting u is impossible. On computing the correction Δh, which is required to change h into $h_1 = h + \Delta h$, such that $u_1 = u$ in Tables 100 and 101, the values of Δh are seen in Table 101.

In order to exhibit these relations, they are plotted in Fig. 32. The h_B formula extends from -24.60000 to -20.00000 in a smooth curve; the h_P extends from -26.60000 to -26.00000, thence by a sudden saltum to -23.10000, whence it follows nearly along the h_B curve. The saltum occurs, as already indicated, near the bottom of the sun's isothermal layer, and it is apparently related to the change of configuration from free electrons into atoms or monatomic molecules. The diatomic H_2 is intermediate.

TABLE 98

THE SOLAR VOLUME DENSITY OF RADIATION

$$u = \frac{48\,\pi\,\beta}{c^3} \cdot \frac{k^4}{h^3} \cdot T^a = a\,T^a.$$

Using $h_B = \dfrac{1}{c\,(z_1 - z_0)} \cdot \dfrac{1}{\frac{m}{2}} \cdot \dfrac{1}{T_{10}} \cdot \dfrac{P_0 - P_1}{P_{10}}$ (Bigelow)

T	k	h	a	u
	Log.	Log.		Log.
679	−14.65022	−22.94069	4.000	−11.88767
1484	−14.34924	−22.46783	4.000	− 7.46057
2485	−14.10323	−22.07325	4.000	− 6.55587
3719	−15.94234	−23.79901	4.000	− 6.43543
4564	−15.82945	−23.60079	4.000	− 5.93421
5526	−15.74361	−23.46848	4.000	− 5.32002
6604	−15.66212	−23.25572	4.000	− 4.94194
7629	−15.58445	−23.13332	4.000	− 3.24510
7687	−15.59363	−23.12281	4.000	− 3.33051
8369	−15.56757	−23.01921	4.000	− 3.68471
9443	−15.51222	−24.94727	4.000	− 3.88839
10511	−15.46630	−24.85549	4.000	− 2.29867
11552	−15.42037	−24.78105	4.000	− 2.37027
12572	−15.38579	−24.69859	4.000	− 2.62641
13405	−15.34803	−24.63058	4.000	− 2.79080

TABLE 99

Using $h_P = \left(\frac{48\pi\beta}{a}\right)^{1/3} \frac{k}{c}^{4/3}$ (Planck)

For the monatomic elements

T	k	h	α	u
679.....		−22.83036	3.7556	−9.52655
1484.....		−22.23233	3.9103	−7.88278
2485.....		−23.83910	3.9701	−5.15670
3719.....		−23.60358	3.9860	−5.97200
4564.....		−23.45863	3.9923	−4.33229
5526.....		−23.34590	3.9975	−4.67842
6604.....		−23.22775	3.9836	−4.97341
7629.....	Same values of k	−23.09578	4.0179	−3.42749
7687.....		−26.92865	2.4100	−3.73345
8369.....		−26.88846	2.4185	−3.87328
9443.....		−26.81290	2.4182	−3.96063
10511.....		−26.75073	2.4222	−2.26759
11552.....		−26.66880	2.4189	−2.27328
12572.....		−26.63955	2.4217	−2.33339
13405.....		−26.59442	2.4191	−2.36464

For the diatomic hydrogen H_2

T	k	h	α	u
7687.....		−24.06865	3.3316	−3.89554
8369.....	Same values of k	−24.00975	3.3468	−2.15091
9443.....		−25.94549	3.3483	−2.04349
10511.....		−25.88354	3.3490	−2.59640
11552.....		−25.82674	3.3487	−2.59076

TABLE 100

THE TERRESTRIAL VOLUME DENSITY OF RADIATION

$$u = \frac{48\pi\beta}{c^3} \cdot \frac{k^4}{h^3} \cdot T^\alpha = a\,T^\alpha.$$

Using $h_B = \frac{1}{c\,(z_1 - z_0)} \cdot \frac{1}{\frac{m}{2}} \cdot \frac{1}{T_{10}} \cdot \frac{P_0 - P_1}{P_{10}}$ (Bigelow)

z	T	k	h	α	u
Meters		Log.	Log.		Log.
66000...	52.0	−15.03573	−20.99536	4.000	−26.80222
56000...	86.7	−16.71084	−19.14377	4.000	−27.94551
46000...	109.3	−16.55127	−20.83233	4.000	−26.64395
36000...	216.3	−16.23881	−20.22718	4.000	−24.37532
26000...	222.5	−16.22207	−20.19276	4.000	−24.48070
16000...	217.3	−16.23216	−20.22384	4.000	−24.38674
6000...	257.0	−16.16597	−20.06449	4.000	−24.89151
2000...	279.8	−16.12525	−21.99349	4.000	−23.08931
500...	285.5	−16.11579	−21.98890	4.000	−23.10028

TABLE 101

Using $h_P = \left(\frac{48\pi\beta}{a}\right)^{1/3} \frac{k}{c}^{4/3}$ (Planck).

z	T	k	h	a	u
66000...	52.0		−25.47579	2.737	−11.19360
56000...	86.7		−26.62427	3.399	− 8.33910
46000...	109.3		−26.26206	3.601	− 7.54148
36000...	216.3	Same	−27.86323	3.577	− 6.49953
26000...	222.5	values	−27.77386	3.668	− 6.95808
16000...	217.3	of k	−27.71347	3.768	− 5.36961
6000...	257.0		−27.58766	3.819	− 5.88588
2000...	279.8		−27.52770	3.826	− 4.06088
500...	285.5		−27.51076	3.829	− 4.11486

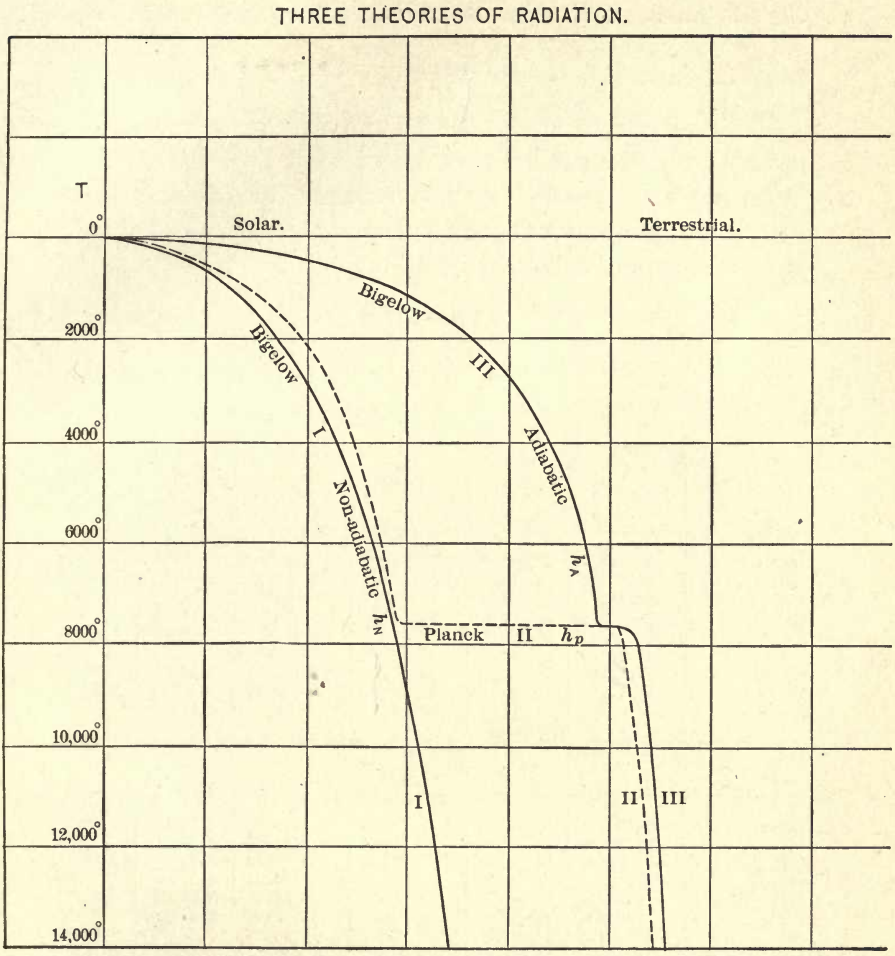
TABLE 102

Correcting Bigelow's Formula, $h_1 = h + \Delta k$

z	T	Δh	h_1	a	u_1
66000...	52.0	−6.20287	−24.19823	4.000	−11.19361
56000...	86.7	−7.86880	−25.01257	4.000	− 8.33911
46000...	109.3	−7.70082	−26.53315	4.000	− 7.54149
36000...	216.3	−7.96526	−26.19244	4.000	− 6.49954
26000...	222.5	−7.84087	−26.03363	4.000	− 6.95809
16000...	217.3	−7.67271	−27.89655	4.000	− 5.36861
6000...	257.0	−7.66854	−27.73303	4.000	− 5.88589
2000...	279.8	−7.67614	−27.66963	4.000	− 4.06089
500...	285.5	−7.66175	−27.65071	4.000	− 4.11485

Extend the branch of the h_P curve to −24.30000. Since the transformation of energy is accompanied by an emission of radiation, and an accession to the potential energy, we may regard the area between h_B and h_P as a measure of this energy of stored potential in the structure of the atoms.

In the earth's atmosphere, the h_P curve appears at the top of the diagram, −27.50000 to −25.00000, while the other curve h_B is near the vanishing point of the solar curves at −21.00000. The differences between h_B and h_P are given in Table 102. It will be desirable to investigate further the relations between these formulas.



The variable potential energy of radiation h.

I, Non-adiabatic, $h_N = \frac{1}{(z_1 - z_0)} \cdot \frac{1}{\frac{m}{2}} \cdot \frac{1}{c\,T_{10}} \cdot \frac{P_0 - P_1}{P_{10}}$.

II, Planck, $h_P = \left(\frac{48\,\pi\,\beta}{a}\right)^{1/3} \frac{k^{4/3}}{c}$.

III, Adiabatic, $h_A = -\frac{g\,(z_1 - z_0)}{(v_1 - v_0)_{10}} \cdot \frac{1}{n} \cdot \frac{1}{\nu_m}$.

FIG. 32. The Non-Adiabatic and the Adiabatic Branches of the Curve of the Functions of Solar Radiation h.

Certain Relations Between the Adiabatic and the Non-Adiabatic Values of $\frac{h\nu}{kT}$.

It is proper to study the factors which are required for reducing the non-adiabatic values of $h\nu/kT$ in the solar and terrestrial atmospheres to those required to conform to the Planck formula, as developed for the variables $(k \cdot a)$. Table 103 gives the factor of reduction for h_B, which is $\Delta c \cdot \gamma^3$.

TABLE 103

EVALUATION OF $\left(\frac{h\nu}{kT}\right)_B$ AND $\left(\frac{h\nu}{kT}\right)_P$.

I. Terrestrial Conditions.

T	$\log \frac{hB}{\Delta c \cdot \gamma^3}$	$\log \nu_m$	$\log \frac{hB\,\nu_m}{\Delta c \cdot \gamma^3}$	$\log h_P \nu_m$	$\log\left(\frac{h\nu}{kT}\right)_B$	$\log\left(\frac{h\nu}{kT}\right)_P$
52.....	−26.51601	12.73207	−13.24808	−13.69795	0.67818	1.12805
86.7.....	−26.66442	12.95409	−13.61851	−13.45265	1.04861	0.88275
109.3.....	−26.35298	13.05469	−13.40767	−13.34496	0.83777	0.77506
216.3.....	−27.74783	13.35113	−13.09896	−13.20430	0.52906	0.63440
222.5.....	−27.71341	13.36340	−13.07681	−13.17228	0.50691	0.60238
217.3.....	−27.74449	13.35313	−13.09762	−13.09123	0.52772	0.52133
257.0.....	−27.58514	13.42600	−13.01114	−14.99801	0.44124	0.42811
279.8.....	−27.51414	13.46292	−14.97706	−14.99233	0.40716	0.42243
285.5.....	−27.50955	13.47168	−14.98123	−14.98743	0.41133	0.41753
II. Solar Conditions.						
273.....	−21.50777	13.45223	−8.96000	−8.75200	3.49492	3.28629
679.....	−22.94069	13.84794	−8.78863	−8.59493	3.32355	3.12985
1484.....	−22.46783	14.18750	−8.63533	−8.33040	3.17025	2.86532
2485.....	−22.07325	14.41140	−8.48465	−8.20021	3.01957	2.73513
3719.....	−23.79901	14.58650	−8.38551	−8.12100	2.92043	2.65592
4564.....	−23.60079	14.67542	−8.27621	−8.09364	2.81113	2.62856
5526.....	−23.46848	14.75848	−8.22696	−8.05470	2.76188	2.58962
6604.....	−23.25572	14.83588	−8.09160	−8.04626	2.63652	2.58118
7629.....	−23.13332	14.89854	−8.03186	−8.01561	2.56678	2.55053
Below the Isothermal Layer.						
7687.....	−23.12281	14.90183	−11.88806	−11.80980	0.42298	0.34472
8369.....	−23.01921	14.93874	−11.82137	−11.78979	0.35629	0.31471
9443.....	−24.94727	14.99118	−11.80187	−11.77086	0.33679	0.30578
10511.....	−24.85529	15.03771	−11.75662	−11.75561	0.29154	0.29053
11552.....	−24.78105	15.07848	−11.72295	−11.73234	0.25687	0.26726
12572.....	−24.69859	15.11548	−11.67749	−11.72891	0.21241	0.26383
13405.....	−24.63058	15.14335	−11.63735	−11.71841	0.17227	0.25333

The entire system is reduced to the equivalent of the exponent 4.00, as in the Stefan Law.

In the terrestrial section I the divisor $\Delta c \,.\, \gamma^3$ consists of the mean factor $\Delta c = 2.13658$ required to reduce from the coefficients corresponding with the exponent 2.42 to that for 4.00 and γ^3. Since $\Delta c.\gamma^3$ is equal to a little more than $\gamma^4 = \gamma^{4/3 \times 3}$ for h^3 in the equation, it is possible that γ^4 is the fundamental reduction for h from the solar adiabatic layer to the terrestrial non-adiabatic layer. The values of $\frac{h\nu}{kT}$ are thus nearly the same in the Bigelow $\left(\frac{h\nu}{kT}\right)_B$, and the Planck $\left(\frac{h\nu}{kT}\right)_P$, as derived from their respective formulas for each. The Planck formula is somewhat indeterminate, because it contains the unknown coefficient a $(J = a\,T^4)$ in $h = \left(\frac{48\,\pi\,\alpha}{a} \cdot \frac{k^4}{c^3}\right)^{1/3}$; ν_m refers to the frequencies for the maximum wave lengths, $\lambda_m = \frac{2891}{T}$.

The Variations in the Wien Displacement Law

The Wien Displacement Law is reduced to the form $\lambda_m\,T = \frac{c}{\beta} \,.\, \frac{h}{k}$, and since (h, k) are variables it follows that $\lambda_m\,T$ must vary with them. For the selected temperatures T, compute $\frac{h}{k}$ and $\lambda_m\,T$ in succession, and finally the corresponding maximum wave length λ_m in terms of (μ). On multiplying $\lambda_m\,T$, in $\lambda\,(c\,m)$, the displacement constant is about 0.1465 in the sun's non-adiabatic strata; about 0.1185 in the sun's adiabatic strata, about 0.2891 in the higher levels of the earth's atmosphere, but gradually diminishing to about 0.1500 near the sea level. This suggests that the Wien Displacement Law for $(h \,.\, k)$ variables is only relatively a constant under special thermodynamic conditions, and that these differ from one body to another. The law was deduced as a function of $(\lambda.T)$, but it must be made a more complex function of $(h \,.\, k \,.\, \lambda \,.\, T)$. This difficult subject will require further investigations.

TABLE 104

THE WIEN DISPLACEMENT LAW, $\lambda_m T = \frac{c}{\beta} \cdot \frac{h}{k}$.

T	$\frac{h}{k}$	$\lambda_m T$	λ_m (μ)	T	$\frac{h}{k}$	$\lambda_m T$	λ_m (μ)
Sun's Atmosphere				Earth's Atmosphere			
220...	−11.40081	3.18137	6.912	52.0	−10.44006	4.22125	320.00
283...	−11.39881	3.18000	5.348	86.7	−11.91343	3.69462	45.35
679...	−11.38978	3.17097	2.183	109.3	−11.71079	3.49198	28.40
1484...	−11.39576	3.17695	1.013	216.3	−11,62442	3.40561	11.76
2485...	−11.37677	3.16796	0.592	222.5	−11.55179	3.33298	9.68
3719...	−11.38766	3.16885	0.388	217.3	−11.48131	3.26250	8.42
4564...	−11.39555	3.17674	0.329	257.0	−11.42169	3.20288	6.21
5526...	−11.38639	3.16758	0.266	279.8	−11.40245	3.18364	5.46
6604...	−11.36788	3.14907	0.213	285.5	−11.39497	3.17616	5.26
7629...	−11.34555	3.12674	0.176				
7687...	−11.33502	3.11621	0.170				
8369...	−11.32089	3.10208	0.151				
9443...	−11.30074	3.08193	0.128				
10511...	−11.28443	3.06562	0.111				
11552...	−11.25189	3.03308	0.094				
12572...	−11.25376	3.03495	0.086				
13405...	−11.24639	3.02758	0.080				

$c = 3 \times 10^{10}$ 10.47712; $\beta = 4.9651$ 0.69593; $\frac{c}{\beta}$, 13.78119 (μ)

Another Formula for the Quantity h_B, Computed from Thermodynamic Data

We will proceed to develop a series of formulas which will connect together definitely the radiation function that has been used, $\frac{U_1 - U_0}{v_1 - v_0}$, and the thermodynamic potential and kinetic energies, together with the radiation terms which enter the Wien-Planck formula. Furthermore, it will be possible to derive the terms of the ionization which make the free electricity of atmospheres, and its relation to the variable potential energy h, and the wave length or the wave frequency related thereto. It may be stated that there is a complete confirmation of the results of the discussion of the radiation as developed in the preceding chapters, so that the data are thoroughly self-consistent, and can be derived from at least three independent methods of computation. They throw much light upon the problems of atomic physics, which will be briefly mentioned.

The Planck formula for the Wirkungsquantum h_P was derived from a statistical analysis of the probable distribution of the elementary oscillators, coupled with the thermodynamic data. We have attempted to produce similar results by referring to the pressure distribution of the *electrons* in the atoms of different weights. There has evidently been a partial success, since one branch of the solar curve has been obtained. There was, however, a difference in the extension to the adiabatic strata in the sun, and there was a wide discrepancy in applying the formula to the earth's atmosphere.

Another formula has been derived from the fundamental equations of thermodynamics, in which it is the *molecules* that are considered as the units, rather than the *atoms* or *electrons*. The purpose has been to compute the amount of the potential energy which is expended in one oscillation of a molecule during the passage of one wave. Hence, the rate of the change of the potential energy per unit variation in the volume, the number of molecules in the unit volume, and the wave frequency are the significant terms. A large number of primary formulas are developed in terms of n, the number of molecules per unit volume, and N, the number of molecules per unit mass. It is possible to develop the Boyle-Gay Lussac Law and the First Law of Thermodynamics in terms of n and N, so that these two fundamental quantities are the proper bases for all thermodynamic discussions of the problems of radiation.

The Kinetic and the Potential Energies in Radiation, Determined from Thermodynamics

It is evident that the complete relations between the radiation and the thermodynamics of atmospheres must be expressed in the terms of the kinetic and the potential energies. Specifically, it is necessary to compute the values of k, h, in the Wien-Planck formulas directly from the primary atmospheric quantities. The following development shows how the Boyle-Gay Lussac Law and the First Law of Thermodynamics can be expressed in terms of n, the number of molecules in the unit volume, and N, the number of molecules in the unit mass, in place of

the usual P, ρ, R, T. We have the following thermodynamic definitions:

(267) Boyle-Gay Lussac Law. $P = \rho R T$.

(268) Pressure, $P = n k T$. $dP = k T . dn$, for kT = constant.

(269) Density, $\rho = m \frac{n}{N}$. $d\rho = m \left(\frac{dn}{N} - \frac{n}{N} \cdot \frac{dN}{N} \right)$.

(270) Volume, $v = \frac{1}{m} \frac{N}{n}$. $dv = \frac{1}{m} \left(\frac{dN}{n} - \frac{N}{n} \frac{dn}{n} \right)$.

(271) Efficiency, $R = k \frac{N}{m}$. $dR = \frac{1}{m} \left(k\,dN + N\,dk \right)$.

(272) Kinetic Energy, $PV = mRT = N . kT = KT$.

(273) Specific Kinetic Energy, $Pv = RT = N . \frac{kT}{m} = \frac{K}{m} T$.

Differentiate and substitute in succession:

(274) $P\,dv + v\,dP = R\,dT + T\,dR = R\,dT_a$.

(275) $P\,dv = n\,k\,T \frac{1}{m} \frac{dN}{n} - n\,k\,T \frac{1}{m} \frac{N}{n} \frac{dn}{n} = \frac{kT}{m} \left(dN - N \frac{dn}{n} \right)$.

(276) $v\,dP = \frac{kT}{m} N \frac{dn}{n}$.

(277) $R\,dT = \frac{k}{m} N\,dT = \frac{\kappa T}{m} N \frac{dT}{T}$.

(278) $T\,dR = \frac{T}{m} (k\,dN + N\,dk) = \frac{kT}{m} \left(dN + N \frac{dk}{k} \right)$.

(279) $P\,dv + v\,dP = \frac{kT}{m} \left(dN - N \frac{dn}{n} + N \frac{dn}{n} \right) = \frac{kT}{m} dN$.

(280) $R\,dT + T\,dR = \frac{kT}{m} \left(N \frac{dT}{T} + dN + N \frac{dk}{k} \right) = \frac{kT}{m} dN$.

The last step comes from kT = Constant, so that,

(281) $\frac{dT}{T} + \frac{dk}{k} = 0$.

This checks the differentiated equation.

Integrate each term for the First Law of Thermodynamics:

(282) Work, $(W_1 - W_0) = P_{10} (v_1 - v_0) = \frac{kT}{m} \left[(N_1 - N_0) - N_{10} \frac{n_1 - n_0}{n_{10}} \right]$.

(283) Hydrostatic pressure, $\left.\right\} \dfrac{P_1 - P_0}{\rho_{10}} = v_{10}\,(P_1 - P_0) = \dfrac{k\,T}{m}\,N_{10}\,\dfrac{n_1 - n_0}{n_{10}}.$

(284) Efficiency I, $R_{10}\,(T_1 - T_0) = \dfrac{k\,T}{m}\,N_{10}\,\dfrac{T_1 - T_0}{T_{10}}.$

(285) Efficiency II, $T_{10}\,(R_1 - R_0) = \dfrac{k\,T}{m}\left[(N_1 - N_0) + N_{10}\,\dfrac{k_1 - k_0}{k_{10}}\right].$

(286) Efficiency III, $R_{10}\,(T_1 - T_0) + T_{10}\,(R_1 - R_0) = R_{10}\,(T_a - T_0) = \dfrac{k\,T}{m}\,(N_1 - N_0).$

(287) Free Heat, $Q_1 - Q_0 = Cp_a\,(T_a - T_0) - Cp_{10}\,(T_a - T_0).$

$$= -\,g\,(z_1 - z_0) - \frac{\kappa}{\kappa - 1}\cdot\frac{k\,T}{m}\,(N_1 - N_0).$$

$$= \frac{\kappa}{\kappa - 1}\cdot\frac{k\,T}{m}\cdot N_a\cdot\frac{T_a - T_0}{T_{a0}} - \frac{\kappa}{\kappa - 1}\cdot\frac{n_{10}\,k\,T}{m}\cdot\frac{N_1 - N_0}{n_{10}}$$

$$= -\,g\,(z_1 - z_0) + \kappa\,n_{10}\,k\,T\,(v_1 - v_0).$$

It is easily proved that,

(288) $\dfrac{d\,n}{n} = \dfrac{\kappa}{\kappa - 1}\,\dfrac{d\,N}{N}.\qquad d\,v = -\,\dfrac{1}{\kappa - 1}\,\dfrac{1}{m}\,\dfrac{d\,N}{n}.$

(289) $\dfrac{n_1 - n_0}{n_{10}} = \dfrac{\kappa}{\kappa - 1}\,\dfrac{N_1 - N_0}{N_{10}}\cdot v_1 - v_0 = -\,\dfrac{1}{\kappa - 1}\cdot\dfrac{1}{m}\,\dfrac{N_1 - N_0}{n_{10}},$

which were used in the last transformations.

(290) Work, $W_1 - W_0 = R_{10}\,(T_a - T_0) - Cp_{10}\,(T_a - T_0).$

$$= \frac{k\,T}{m}\,(N_1 - N_0) - \frac{\kappa}{\kappa - 1}\,\frac{k\,T}{m}\,(N_1 - N_0).$$

$$= -\frac{1}{\kappa - 1}\cdot\frac{n_{10}\,k\,T}{m}\cdot\frac{N_1 - N_0}{n_{10}}.$$

$$= n_{10}\,k\,T\,(v_1 - v_0).$$

Total Inner energy

(291) $U_1 - U_0 = Cp_a\,(T_a - T_0) - R_{10}\,(T_a - T_0).$

$$= -\,g\,(z_1 - z_0) - \frac{k\,T}{m}\,(N_1 - N_0).$$

$$= -\,g\,(z_1 - z_0) - \frac{n_{10}\,k\,T}{m}\,\frac{N_1 - N_0}{n_{10}}.$$

$$= -\,g\,(z_1 - z_0) + n_{10}\,k\,T\,(\kappa - 1)\,(v_1 - v_0).$$

From the last equation the radiation function is found:
Radiation function

$$(292)\quad \frac{U_1 - U_0}{v_1 - v_0} = -\,g\,\frac{(z_1 - z_0)}{v_1 - v_0} + (\kappa - 1)\,n_{10}\,k\,T$$

$$= c\,T^b = g\,\frac{(z_1 - z_0)\,(\kappa - 1)\,m\,n_{10}}{(N_1 - N_0)} + (\kappa - 1)\,n_{10}\,k\,T$$

$$= K_{10} = g\,(z_1 - z_0)\,\kappa\,.\,m\,\frac{n_{10}}{N_{10}}\,.\,\frac{n_{10}}{n_1 - n_0} + (\kappa - 1)\,n_{10}\,k\,T$$

$$= g\,\rho_{10}\,(z_1 - z_0)\,\kappa\,\frac{n_{10}}{n_1 - n_0} + (\kappa - 1)\,n_{10}\,k\,T.$$

The density of the radiation is the potential energy together with the kinetic energy for the mass in the column $(z_1 - z_0)$, $g\,\rho_{10}\,(z_1 - z_0) = M_{10}$, which contains n_{10} molecules. We may assume that this potential energy is also expressed by $n_{10}\,h\,\nu$, where ν = the frequency, $\nu = \frac{c}{\lambda}$, and h is the potential energy per oscillation in the path of the ray. Hence, we have, using the first form,

$$(293)\quad n_{10}\,h\,\nu = +\,g\,\frac{(z_1 - z_0)}{(v_1 - v_0)}, \text{ so that,}$$

$$(294)\quad h = +\,g\,\frac{(z_1 - z_0)}{(v_1 - v_0)}\,.\,\frac{1}{n_{10}}\,.\,\frac{1}{\nu}.$$

If we assume the Wien displacement law,

$$(295)\quad \nu_m\,T_m = 0.2891 \text{ (Planck), we have,}$$

$$(296)\quad \nu_m = \frac{c}{\lambda_m} = \frac{c\,T_m}{0.2891}. \text{ Hence,}$$

$$(297)\quad h = +\,g\,\frac{(z_1 - z_0)}{(v_1 - v_0)}\,\frac{0.2891}{c\,T_m}\,\frac{1}{n_{10}}$$

where the temperature is that of $T_m = \frac{0.2891}{\nu_m}$.

$$(298)\quad h_P = \left(\frac{48\,\pi\,\alpha}{a}\right)^{1/3}\,.\,\frac{k^{1/3}}{c} \quad \text{.....Planck's formula.}$$

$$(299)\quad h_B = g\,\frac{(z_1 - z_0)}{(v_1 - v_0)_{10}}\,.\,\frac{1}{n}\,.\,\frac{1}{\nu_m} \quad \text{.....Bigelow's formula.}$$

h_B is the change in the potential energy per unit volume, per molecule, per wave vibration, and it therefore is the mean amplitude of the potential energy of each molecule in one oscillation.

Planck's Wirkungsquantum should be interpreted in this manner. The values of h increase with decreasing temperatures, and h is a wide variable. ν_m can be computed for each T_m.

The diagram of h_B and k_P, Fig. 32, shows that the curves are similar except that the amplitude of h_P is considerably greater than that of h_B at the isothermal layer.

TABLE 105

THE MOLECULAR POTENTIAL ENERGY IN THE EARTH'S ATMOSPHERE

Balloon Ascension, Uccle, September 13, 1911.

$$C.\ G.\ S.\ h_B = \frac{g\,(z_1 - z_0)}{(v_1 - v_0)_{10}} \cdot \frac{1}{n} \cdot \frac{1}{\nu_m} \quad \text{for} \quad \nu_m = \frac{c}{\lambda_m} = \frac{c \cdot T_m}{0.2891}$$

ATMOSPHERIC AIR, $m = 28.736$			ATMOSPHERIC AIR, $m = 28.736$		
T	Log h_B	Log h_P	T	Log h_B	Log h_P
°			°		
6.....	−23.26522	−24.87100	215.0.....	−27.89619	−27.86071
11.....	−24.54772	−24.38936	216.7.....	−27.87693	−27.85048
16.....	−24.36712	−24.06853	218.5.....	−27.85846	−27.84036
21.....	−24.18255	−25.83527	220.0.....	−27.83852	−27.83007
25.....	−24.02646	−25.67431	221.0.....	−27.81995	−27.82077
29.....	−25.88035	−25.53936	222.0.....	−27.80147	−27.81150
33.....	−25.75124	−25.42188	223.0.....	−27.78081	−27.80257
37.....	−25.63559	−25.31896	223.5.....	−27.75996	−27.79344
41.....	−25.53110	−25.22653	224.0.....	−27.74117	−27.78630
45.....	−25.43576	−25.14329	223.6.....	−27.72689	−27.77903
49.....	−25.34825	−25.06905	223.1.....	−27.70765	−27.77231
53.....	−25.26741	−25.00024	222.8.....	−27.68779	−27.76609
57.....	−25.19222	−26.93678	221.7.....	−27.66777	−27.76096
61.....	−25.12184	−26.87784	221.2.....	−27.64765	−27.75494
65.....	−25.10459	−26.82108	220.4.....	−27.62869	−27.74904
68.....	−26.99386	−26.77527	220.1.....	−27.61086	−27.74384
71.....	−26.93365	−26.73127	219.8.....	−27.59003	−27.73645
74.....	−26.87620	−26.68964	219.0.....	−27.57062	−27.73111
77.....	−26.82071	−26.64962	218.0.....	−27.55197	−27.72509
80.....	−26.76907	−26.61150	217.1.....	−27.53594	−27.72083
83.....	−26.71771	−26.57385	216.0.....	−27.51127	−27.71711
86.....	−26.66646	−26.53964	216.5.....	−27.49137	−27.70810
89.....	−26.61849	−26.50511	217.1.....	−27.47081	−27.70350
92.....	−26.57217	−26.47190	218.3.....	−27.45316	−27.69099
95.....	−26.52723	−26.44065	220.5.....	−27.43505	−27.67917
98.....	−26.48460	−26.40964	227.0.....	−27.41406	−27.66168
101.....	−26.44156	−26.38119	233.7.....	−27.39562	−27.64418
104.....	−26.40049	−26.35157	240.3.....	−27.37797	−27.62738
107.....	−26.36057	−26.32412	248.1.....	−27.36947	−27.60787
110.....	−26.32076	−26.29646	255.1.....	−27.34308	−27.59043
112.....	−26.28411	−26.27335	261.7.....	−27.32628	−27.57564
114.....	−26.24687	−26.25144	267.4.....	−27.30934	−27.56082
116.....	−26.20997	−26.22934	270.9.....	−27.29594	−27.55130
118.....	−26.17416	−26.20745	278.4.....	−27.28524	−27.53482
120.....	−26.14021	−26.18839	281.4.....	−27.27666	−27.52762
124.....	−26.10516	−26.16340	284.5.....	−27.26710	−27.52060
130.....	−26.07145	−26.13304	287.2.....	−27.25455	−27.51321
138.....	−26.04097	−26.09912	289.0.....	−27.24332	−27.50851
150.....	−26.01089	−26.05750	290.4.....	−27.19248	−27.50438
166.....	−27.98406	−26.00915	292.5.....	−27.17990	−27.49951
187.....	−27.95397	−27.94784			
202.....	−27.93361	−27.90550			
211.....	−27.91444	−27.87792			

TABLE 106

THE MOLECULAR POTENTIAL ENERGY IN THE SUN.

$$C.\,G.\,S.\ h_B = \frac{g\,(z_1 - z_0)}{(v_1 - v_0)_{10}} \cdot \frac{1}{n} \cdot \frac{1}{\nu_m} \quad \text{for} \quad \nu_m = \frac{c}{\lambda_m} = \frac{c \cdot T_m}{0.2891}.$$

	Hydrogen 1.00		Hydrogen 2.00		Helium 4.00		Carbon 12.00	
	T	Log h_B	T	Log h_B	T	Log h_B	T	Log h_B
Non-Adiabatic Strata.			160	−20.10922				
	250	−21.94535	245	−21.86380	240	−21.99583		
			330	−21.52664				
			415	−21.21621			400	−21.90457
	500	−21.39246	500	−22.94440	490	−21.29088		
			585	−22.70645			610	−21.18744
			675	−22.49559				
	750	−22.94654	765	−22.35991	740	−22.86200		
			860	−22.14042			850	−22.72569
			955	−23.99073				
	1025	−22.54442	1055	−23.85296	1000	−22.46554		
			1155	−23.73776			1120	−22.25270
	1300	−22.21134	1260	−23.65236	1270	−22.14394		
			1365	−23.50624				
			1475	−23.40681			1430	−23.89265
	1575	−23.90338	1585	−23.31500	1550	−23.87751		
			1700	−23.15858				
	1850	−23.63666	1815	−23.15270	1830	−23.64833	1790	−23.62562
			1935	−23.07209				
			2055	−23.00079				
	2150	−23.51698	2180	−24.93299	2110	−23.45087	2190	−23.38746
			2305	−24.86936				
	2475	−23.34117	2435	−24.80839	2400	−23.27727		
			2565	−24.72080			2630	−23.19183
			2695	−24.69632	2710	−23.12253		
	2800	−23.18871	2835	−24.64426				
			2985	−24.59380	3040	−24.98431		
	3150	−23.05115	3145	−24.52125			3110	−23.02073
			3315	−24.50106				
	3500	−24.92993	3495	−24.45832	3390	−24.86087	3620	−24.97693
			3685	−24.41730	3780	−24.73894		
	3900	−24.81793	3885	−24.37848				
			4095	−24.34152	4200	−24.64921	4160	−24.75180
	4300	−24.71832	4315	−24.30639				
			4545	−24.27298				
	4700	−24.62845	4785	−24.24133	4680	−24.55788	4720	−24.64203
			5045	−24.21100				
	5150	−24.54440	5325	−24.18455	5180	−24.47627	5300	−24.54450
	5600	−24.46466	5610	−24.15442				
			5890	−24.12830	5700	−24.40198	5900	−24.45604
	6100	−24.39754	6160	−24.10343	6230	−24.33423		
			6420	−24.07941			6450	−24.37770
	6600	−24.33859	6660	−24.04641	6750	−24.27258		
			6880	−24.02639			7000	−24.31002
	7100	−24.27664	7218	−24.00748	7300	−24.21116		
Isothermal Layer.	7450	−24.21528	7500	−25.98655			7500	−24.23340
	7650	−24.16071	7685	−25.96720	7600	−24.15831		
	7690	−24.10892	7680	−25.95259	7715	−24.10382	7715	−24.16797
	7701	−24.05411	7675	−25.93409	7705	−24.05359	7705	−24.10742
	7691	−24.00129	7670	−25.90632	7695	−24.00160	7695	−24.04491
	7681	−25.94928	7665	−25.88558	7685	−25.94759	7685	−25.97958
	7671	−25.89508	7660	−25.82240	7675	−25.89577	7675	−25.91712
	7661	−25.84133	7652	−25.90050	7665	−25.83907	7665	−25.84929
	7652	−25.72315	7662	−25.99194	7652	−25.78725	7652	−25.78587
	7665	−25.75970	7676	−25.78850				
			7640	−25.74790			7640	−25.70234
			7652	−25.71170				
			7665	−25.67275			7750	−25.66468
	7690	−25.73580	7690	−25.63693	7680	−25.73808		
			8000	−25.59774				
Adiabatic Strata.	8476	−25.67732	8500	−25.56087	8500	−25.68037	8550	−25.59687
	8951	−25.65733	9187	−25.52482	9451	−25.63315	9691	−25.53836
	9427	−25.63427	9874	−25.49587				
	9902	−25.62351	10561	−25.46506	10402	−25.59186	10833	−25.49080
	10378	−25.59313	11248	−25.43779	11353	−25.55426		
	10853	−25.57395	11935	−25.42004	12304	−25.51977	11974	−25.44933
					13255	−25.48769	13115	−25.40844
					14207	−25.45774	14257	−25.41550
					15158	−25.42963	15398	−25.39486

TABLE 106—*Continued*

THE MOLECULAR POTENTIAL ENERGY IN THE SUN.

$$C.\ G.\ S.\ h_B = \frac{g\ (z_1 - z_0)}{(v_1 - v_0)_{10}} \cdot \frac{1}{n} \cdot \frac{1}{\nu_m} \text{ for } \nu_m = \frac{c}{\lambda_m} = \frac{c\ .\ T_m}{0.2891}.$$

	Calcium 40.00		Zinc 64.85		Cadmium 111.51		Mercury 198.41	
	T	Log h_B	T	Log h_B	T	Log h_B	T	Log h_B
Adiabatic Strata.			230	−21.55292				
	300	−21.40258			300	−21.51639		
			420	−21.14156			550	−22.72159
	600	−22.76097			600	−22.86002		
			640	−22.68030				
	900	−22.37932	890	−22.30522	900	−22.44028	850	−22.35401
	1200	−22.03800	1180	−22.00821	1200	−22.06172	1150	−22.01314
	1500	−22.75701	1480	−23.77339	1500	−23.74720	1450	−23.72689
	1850	−23.51871	1790	−23.57788	1800	−23.48390	1800	−23.48438
	2200	−23.32328	2110	−23.41177	2150	−23.25608	2150	−23.28636
			2440	−23.26827				
	2550	−23.15603			2500	−23.07288	2550	−23.11613
			2780	−23.14226				
	2950	−23.00686			2900	−24.97003	3000	−24.97029
			3120	−23.03067				
	3350	−24.87851						
			3460	−24.93021	3400	−24.76029	3450	−24.84759
	3750	−24.76517						
			3800	−24.83917	3900	−24.68863	3950	−24.73723
	4200	−24.66204	4140	−24.75554				
			4480	−24.67869				
					4400	−24.52947	4450	−24.64143
	4700	−24.56977	4830	−24.60720				
					5000	−24.43197	4950	−24.55546
	5225	−24.48668	5190	−24.54078				
			5560	−24.47676			5450	−24.48681
	5750	−24.39883	5940	−24.42049	5700	−24.34787		
							5900	−24.40644
	6300	−24.33804	6320	−24.36625			6350	−24.33924
			6700	−24.31533	6600	−24.26731		
	6800	−24.29567					6750	−24.27724
Isothermal Layer.	7200	−24.23752	7100	−24.26657	7300	−24.20161	7100	−24.21855
	7500	−24.14207	7500	−24.22694			7400	−24.16189
	7715	−24.12398	7745	−24.17468	7650	−24.14205	7600	−24.11135
	7705	−24.07731	7735	−24.13613			7670	−24.06122
	7695	−24.02532	7725	−24.09650	7690	−24.08396	7667	−24.00043
	7685	−25.96788	7715	−24.04370	7684	−24.02540	7663	−25.94067
	7675	−25.90878	7705	−24.00437	7677	−25.96701		
	7665	−25.81193	7695	−25.96132	7669	−25.90505	7659	−25.88993
	7652	−25.94886	7685	−25.91737	7660	−25.84675	7656	−25.83591
			7675	−25.87538	7652	−25.79100	7652	−25.78992
			7665	−25.82680				
			7652	−25.78635				
			7640	−25.74322				
Non-Adiabatic Strata.	7700	−25.69642	7700	−25.70688	7720	−25.73446		
							7800	−25.74041
			8400	−25.65650	8500	−25.67005	8550	−25.68319
			9171	−25.61771				
	9600	−25.57183	9942	−25.58258	9561	−25.61561	9494	−25.63361
			10713	−25.55075	10621	−25.57053	10437	−26.59501
	11504	−25.49726	11484	−25.52071	11682	−25.52968	11381	−25.55770
			12255	−25.49272	12742	−25.49247	12324	−25.52798
	13406	−25.43299	13026	−25.46650	13803	−25.45793	13268	−25.49152
	15308	−25.37649			14864	−25.42503	14211	−25.46186
	17210	−25.32660			15924	−25.40965	15155	−25.43731
	19112	−25.28173						
	21014	−25.19050						

It should be remembered that in computing,

$$h_P = \left(\frac{48\,\pi\,\alpha}{a}\right)^{1/3} \frac{k^{1/3}}{c} \tag{300}$$

the value of a in the denominator is the same as the coefficient c in the quasi-Stefan Law

$$K_{10} = c\,T^b \tag{301}$$

and that the coefficient c and the exponent b were obtained by the method of trials. Since these vary together, and b is about 4.00 in the non-adiabatic layers, and about 2.42 in the adiabatic layers, it follows that the mutual adjustment of c, b, will produce the same values for h by the Planck and the Bigelow formulas, for the same value of $\frac{U_1 - U_0}{v_1 - v_0}$. It will be necessary to study more fully the relations between them before reaching any opinion as to their meaning. But it is clear that they have the same horizontal branch in the isothermal layer, and that they can be reduced to the same values in the adiabatic and non-adiabatic layers.

It is seen, furthermore, that if the line radiation in the spectrum of any element is due to a definite value of h, which represents a given pressure and temperature, then the series of spectral lines will be spaced as commonly observed, by giving h certain fixed increments in steps. In this case the series of lines of an element is due to its independent radiations at such levels as produce the given $P.\ T.$, and if the scale can be fixed these lines can be located in the levels z_0, z_1, z .....

In the earth's atmosphere the agreement between h_P and h_B is very close, and these small variations can be adjusted through the c coefficient and b—exponent.

Summary

It should be noted that the h_B equation in its expanded form,

$$h_B = \frac{\kappa}{\kappa - 1} \cdot \frac{k\,T}{m} N_a \frac{T_a - T_0}{T_{a0}} \cdot (\kappa - 1)\, m \frac{n_{10}}{N_1 - N_0} \cdot \frac{1}{n_{10}} \cdot \frac{0.2891}{c\,T_m}, \tag{302}$$

$$h_B = \kappa \,.\, k\,T \frac{T_a - T_0}{T_{a0}} \cdot \frac{N_a}{N_1 - N_0} \cdot \frac{0.2891}{c\,T_m}, \tag{303}$$

reduces finally to terms in (T, N), and includes the Wien Displacement Law. A new system of physics can be founded upon (T, n, N) since these include the potential energy h, and the kinetic energy k for each oscillation. Whether the molecule and the atom are constructed as electric oscillators (Planck), or as a large volume nucleus of positive charges (Thomson), or as a small condensed positive nucleus (Rutherford), *the h, k, are not coefficients of the internal structure, but of the outer forces of collision between independent molecules.* The secondary effects of such collisions in producing electromagnetic radiation, and vibratory adjustments of structures between the positive and the negative charges, lead to the fundamental problems in physics, chemistry, and the constitution of matter. It is hoped that some further progress can be made in this direction by the study of the spectral series, the atomic numbers and allied problems, from the point of view that h, k, n, N, are all primary variable quantities in the standard equations of thermodynamics and electromagnetics.

Derivation of Certain Formulas for Radiation, Ionization, and Atmospheric Electricity

It seems that the fundamental formula for the radiation,

(304) $$\frac{U_1 - U_0}{v_1 - v_0} = + g(z_1 - z_0)(\kappa - 1) m \frac{n_{10}}{N_1 - N_0} + (\kappa - 1)\, n_{10}\, k\, T,$$

is competent to produce the Wien-Planck formula for radiation, though the terms are differently interpreted; the fundamental relation between ionization, potential energy, and frequency of the vibration; and the quantity of the electric charge in the unit volume. Generally, we have for the single H-atom, of which there are n_{10} in the unit volume, the potential energy $= h\nu = g(z_1 - z_0)\ (\kappa - 1)\ m \frac{1}{N_1 - N_0}$, and the kinetic energy $= (\kappa - 1)\, k\, T$, so that, for single H-atoms,

(305) $$\frac{U_1 - U_0}{v_1 - v_0} = h\nu + (\kappa - 1)\, k\, T,$$

as the primitive form, whereas Planck assumed,

(306) $$u = \frac{U_1 - U_0}{v_1 - v_0} = h\nu + k\, T.$$

This will lead to the Planck form of the function with the change of 1 to $\kappa - 1$, that is, 0.666 for the monatomic gases and approximately 0.410 for the diatomic gases. Thus,

$$(307) \qquad u = \int_0^\infty u_\nu \, d\nu = \frac{8\pi h}{c^3} \int_0^\infty \frac{\nu^3 \, d\nu}{e^{\frac{h\nu}{kT}} - (\kappa - 1)} = a T^4.$$

In this formula it is $k T$ which is the variable constant in any given atmosphere, while $h \nu$ is the variable potential energy. It has become evident that this potential energy is *external* to the atoms and molecules, such as occurs in the thermal collisions between atoms or molecules consisting of m unit charges e. *Hence the theory of the generation of radiation is transferred from the saltum changes within the circulating orbits of an atom into the mutual repulsions of the atoms and molecules in the collisions which integrate into the existing pressure under the force of gravitation.* Evidently the potential energy changes from level to level.

Ionization, Potential Energy, and Frequency

It will be convenient to make the following transformations: From the equation for the specific volume, $v = \frac{1}{m} \frac{N}{n}$, we have,

$$(308) \qquad d v = \frac{1}{m} \left(\frac{d N}{n} - \frac{N}{n} \frac{d n}{n} \right). \quad \text{Integrate,}$$

$$(309) \quad (v_1 - v_0) = \frac{1}{m} \left(\frac{N_1 - N_0}{n_{10}} - \frac{N_{10}}{n_{10}} \cdot \frac{n_1 - n_0}{n_{10}} \right),$$

$$= \frac{1}{m} \left(\frac{N_1 - N_0}{n_{10}} - \frac{N_{10}}{n_{10}} \cdot \frac{\kappa}{\kappa - 1} \frac{N_1 - N_0}{N_{10}} \right),$$

$$= \frac{1}{m} \left(\frac{N_1 - N_0}{n_{10}} - \frac{\kappa}{\kappa - 1} \frac{N_1 - N_0}{n_{10}} \right),$$

$$= - \frac{1}{\kappa - 1} \cdot \frac{1}{m} \cdot \frac{N_1 - N_0}{n_{10}}.$$

Substitute in the general equation of radiation (304),

$$(310) \qquad \frac{U_1 - U_0}{v_1 - v_0} = - g \frac{(z_1 - z_0)}{(v_1 - v_0)_{10}} + n_{10} \, k \, T, \text{ for } n_{10} \text{ charges of } e.$$

Hence we shall assume that the potential energy of one H-atom in thermal collisions is,

(311) $$h\nu = -g(z_1 - z_0) \cdot \frac{1}{(v_1 - v_0)_{10}} \cdot \frac{1}{n_{10}}, \text{ or,}$$

(312) $$h\nu = +g(z_1 - z_0)(\kappa - 1)\, m \frac{1}{N_1 - N_0},$$

that is, in terms of the number of H-charges per unit volume n, or per unit mass N.

We can separate this equation into the terms for the ionization, the potential energy, and the frequency changes, as follows:

Take the second differences of $(N_1 - N_0)$,

(313) $$d(N_1 - N_0) = -g(z_1 - z_0)(\kappa - 1)\, m \left(\frac{h\, d\nu + \nu\, dh}{h^2 \nu^2}\right).$$

Integrate,

(314) $$\Delta(N_1 - N_0) = -g(z_1 - z_0)(\kappa - 1)\, m \frac{1}{h_{10}\nu_{10}} \left(\frac{h_1 - h_0}{h_{10}} + \frac{\nu_1 - \nu_0}{\nu_{10}}\right).$$

Since for one H-charged atom,

(315) $$(N_1 - N_0) = +g(z_1 - z_0)(\kappa - 1)\, m \frac{1}{h_{10}\nu_{10}} \text{ on the average,}$$

(316) $$-\frac{\Delta(N_1 - N_0)}{N_1 - N_0} = \frac{h_1 - h_0}{h_{10}} + \frac{\nu_1 - \nu_0}{\nu_{10}}.$$

This equation is valid in all atmospheres. Compare the Tables 107, 108, and 109.

Atmospheric Electricity

The potential equation is immediately applicable to the problem of the origin of atmospheric electricity and the quantity of electric charges in the unit volume by adopting the Einstein formula,

(317) $$h\nu = Ve.$$

The fundamental electrostatic equations are,

(318) Potential $= V = \frac{e}{r}$.

(319) Electric force $= E = -\frac{dV}{dr} = \frac{e}{r^2}$.

(320) Work of potential $= W = Ve = \frac{e^2}{r} = h\nu$.

The preceding development assumes that the fundamental charges $(e \,.\, e)$ are alike, being in repulsion during the collisions and along the free path, so that we have only to transform the expressions for $h\nu$ into volts per unit distance in order to compute the accumulated charge in the stratum $(z_2 - z_1)$.

$$Q = \frac{W}{V} = (V_2 - V_1)\ (z_2 - z_1). \tag{321}$$

We have 1 E. S. U. = 300 volts.

$$1 \text{ Volt} = 0.00333 \text{ E. S. U.}$$

$$\frac{1 \text{ Volt}}{\text{cm.}} = 0.00333 \frac{\text{E. S. U.}}{\text{cm.}^3}. \quad \text{Hence,}$$

$$\frac{\text{Volts}}{\text{meter}} = \frac{\text{Volts}}{100 \text{ cm.}} = \frac{300}{e} \,.\, h\nu \,.\, 100\, m. \tag{322}$$

Preliminary trials on the data of atmospheric electricity indicate that m must be identified with the molecular weight in the earth's atmosphere, $m = 28.74$, approximately. Thence the formula for Q in successive strata gives the quantities which when summed together produce the electric charges on each level above the plane of reference, as the earth's surface or the top of the sun's adiabatic strata. The results are indicated in Tables 110 and 111.

Tables 107, 108, and 109 give the results for the terms,

$\frac{h_1 - h_0}{h_{10}}$, the rate of change of the potential in the stratum,

$\frac{\nu_1 - \nu_0}{\nu_{10}}$, the rate of change of the frequency in the stratum,

$-\Delta \frac{(N_1 - N_0)}{N_1 - N_0}$, the rate of change of the ionization in the stratum,

for the balloon ascension, Uccle, September 13, 1911, in the terrestrial atmosphere, and for helium and zinc in the sun's atmosphere, the other elements being similar in their development. Figs. 33 and 34 show the relative values for the earth's atmosphere, $m = 28$, and for the solar atmosphere of helium,

$m = 4.00$. The potential is the value which occurs in each wave length and its oscillation so that it can be analyzed according to the formulas of Chapter I. The ionization term marks the disintegration of the mass according to the changes in the density per unit mass. These data, therefore, become of primary importance in studies of the atomic physics.

In the earth's atmosphere the convectional, isothermal, and non-adiabatic regions are well defined. At the top of the isothermal strata there is a rapid change in the frequency and ionization, while the potential increases quite steadily in value.

In the solar atmosphere there is no ionization in the adiabatic strata; there are sudden large changes in the frequency and ionization at the isothermal layer, including the photosphere, where the radiation is especially generated; in the non-adiabatic region they increase together to very large values. Figs. 33, 34.

It should be remarked that Abbot's value of the solar constant of radiation, corresponding with the temperature 5810°, finds no support in these results. They are in harmony with the view that the solar radiation originates at the temperature of 7655°.

The Terrestrial Atmospheric Electricity

There are many hypotheses in the literature of atmospheric electricity regarding the origin and the distribution of the powerful electric charges which are a characteristic and permanent constituent of all atmospheres, but they are generally deficient and unsatisfactory. We shall employ the Einstein relation, $Ve = h\nu$, as the electrostatic form of the prevailing potential energy of radiation, reducing it to the form,

$$\frac{\text{Volts}}{\text{meter}} = \frac{300}{e} h\nu. \quad 100 \text{ m, for voltage, and}$$

$$(Q_1 - Q_0) = (z_1 - z_0)(V_1 - V_0), \text{ for quantity.}$$

Table 110 contains the results for $(V_1 - V_0)$, $(Q_1 - Q_0)$ arranged by the arguments z and T. The values of $(V_1 - V_0)$ have three distinct stages:

$(V_1 - V_0)$ varies between 1.00 and 2.00 in the convection strata; it increases in the isothermal region to about 10.00 at

TABLE 107

UCCLE, SEPTEMBER 13, 1911

$$\frac{-\Delta(N_1 - N_0)}{N_1 - N_0} = \frac{h_1 - h_0}{h_{10}} + \frac{\nu_1 - \nu_0}{\nu_{10}}$$

z Meters	T	$\frac{h_1 - h_0}{h_{10}}$	$\frac{\nu_1 - \nu_0}{\nu_{10}}$	Sum	$\frac{-\Delta(N_1 - N_0)}{N_1 - N_0}$
80000.....	0°.0	—	—	—	—
78000.....	11.0	0.40997	−.37017	.02980	.34468
76000.....	21.0	.35561	−.17392	.18169	.25819
74000.....	29.0	.30862	−.12903	.17959	.18751
72000.....	37.0	.23946	−.10255	.13691	.15143
70000.....	45.0	.20080	−.08517	.11563	.12360
68000.....	53.0	.17266	−.07271	.09995	.10518
66000.....	61.0	.15453	−.06348	.09105	.10040
64000.....	68.0	.14302	−.04318	.09984	.09899
62000.....	74.0	.12761	−.03972	.08789	.09122
60000.....	80.0	.11813	−.03682	.08191	.08354
58000.....	86.0	.11035	−.03428	.07607	.07876
56000.....	92.0	.10339	−.03210	.07129	.07437
54000.....	98.0	.09032	−.03012	.06020	.06923
52000.....	104.0	.09166	−.02841	.06325	.06514
50000.....	110.0	.08437	−.01806	.06631	.07018
48000.....	114.0	.08491	−.01735	.00756	.06761
46000.....	118.0	.07813	−.01701	.06712	.05769
44000.....	124.0	.07763	−.04727	.03036	.02031
42000.....	138.0	.06920	−.08318	−.01398	−.02941
40000.....	166.0	.06925	−.11897	−.04972	−.04545
38000.....	202.0	.04413	−.04359	.00054	.00177
36000.....	215.0	.04433	−.01017	.03416	.03823
34000.....	218.5	.04590	−.00681	.03909	.03830
32000.....	221.0	.04254	−.00452	.03802	.04068
30000.....	223.0	.04800	−.00458	.04392	.03799
28000.....	224.0	.03289	.00181	.03470	.03050
26000.....	223.1	.04572	.00149	.04721	.04747
24000.....	221.7	.04633	.00296	.04929	.03775
22000.....	220.4	.04103	.00140	.04243	.03978
20000.....	219.8	.04467	.00360	.04827	.04097
18000.....	218.0	.04230	.00416	.04646	.05027
16000.....	216.0	.04586	−.00192	.04394	.03674
14000.....	217.1	.04065	−.00553	.03502	.04092
12000.....	220.5	.04829	−.02907	.01922	.01405
10000.....	233.7	.04062	−.02781	.01281	.01156
8000.....	248.1	.03772	−.02788	.00984	.01234
6000.....	261.7	.03900	−.02156	.01744	.01901
4000.....	270.9	.02463	−.02730	−.00267	−.00497
2000.....	284.5	.02869	.00937	.01932	.01986
1000.....	289.0	.02690	−.00486	.02204	.01975
500.....	290.4	—	—	—	—
100.....	292.5	—	—	—	—

TABLE 108

HELIUM, $m = 4.00$

$$\frac{-\Delta (N_1 - N_0)}{N_{1_\bullet} - N_0} = \frac{h_1 - h_0}{h_{10}} + \frac{\nu_1 - \nu_0}{\nu_{10}}$$

z Kilometers	T	$\frac{h_1 - h_0}{h_{10}}$	$\frac{\nu_1 - \nu_0}{\nu_{10}}$	Sum	$\frac{-\Delta (N_1 - N_0)}{N_1 - N_0}$
11000.....	240°	1.34090	−.94892	0.39198	1.46400
10500.....	490	.91438	−.40648	.50790	2.75790
10000.....	740	.85434	−.29885	.55549	.91804
9500.....	1000	.70840	−.23779	.47061	.66130
9000.....	1270	.59495	−.19963	.39532	.57256
8500.....	1550	.51579	−.16564	.35015	.42196
8000.....	1830	.44697	−.14211	.30486	.35483
7500.....	2110	.39452	−.12863	.26589	.29792
7000.....	2400	.35252	−.12130	.23122	.25232
6500.....	2710	.31562	−.11484	.20078	.21132
6000.....	3040	.28231	−.10883	.17348	.18633
5500.....	3390	.27827	−.10878	.16949	.15458
5000.....	3780	.20587	−.10526	.10061	.11610
4500.....	4200	.20952	−.10886	.10066	.10376
4000.....	4680	.19040	−.10141	.08899	.08683
3500.....	5180	.17294	−.09560	.07734	.07686
3000.....	5700	.15337	−.08885	.06452	.06935
2500.....	6230	.14159	−.08009	.06150	.06373
2000.....	6750	.14122	−.07850	.06272	.07444
1500.....	7300	.12146	−.04025	.08021	.10524
1000.....	7600	.12532	−.01503	.11028	.13702
500.....	7715	.11500	.00215	.11715	.11000
0....	7705	.11953	.00045	.11998	.11217
−500....	7695	.12139	.00133	.12272	.11282
−1000....	7685	.11918	.00129	.12047	.15327
−1500....	7675	.13037	.00132	.13169	.12795
−2000....	7665	.11918	.00174	.12092	.13338
−2500....	7652	.11311	−.00371	.10948	.06938
−3000....	7680	.13272	−.10136	.03136	.00805
−3500....	8500	.10862	−.10595	.00267	—
−4000....	9452	.09498	−.09579	−.00081	—
−4500....	10402	.08654	−.08744	−.00090	—
−5000....	11353	.07939	−.08041	−.00102	—
−5500....	12304	.07382	−.07437	−.00056	—
−6000....	13255	.06892	−.06927	−.00035	—
−6500....	14206	.06473	−.06478	−.00005	—
−7000....	15158	—	—	—	—

The curves for the other solar elements are similar.

TABLE 109

ZINC, $m = 64.85$

$$\frac{-\Delta(N_1 - N_0)}{N_1 - N_0} = \frac{h_1 - h_0}{h_{10}} + \frac{\nu_1 - \nu_0}{\nu_{10}}.$$

z	T	$\frac{h_1 - h_0}{h_{10}}$	$\frac{\nu_1 - \nu_0}{\nu_{10}}$	Sum	$\frac{-\Delta(N_1 - N_0)}{N_1 - N_0}$
650.....	230°	0.88216	−0.58461	0.29755	0.35806
625.....	420	.97240	− .41512	.55728	1.28730
600.....	640	.81372	− .32679	.48693	.74920
575.....	890	.65837	− .28017	.37820	.49726
550.....	1180	.52790	− .22564	.30236	.37027
525.....	1480	.44272	− .18960	.25312	.29049
500.....	1790	.37788	− .16408	.21380	.23516
475.....	2110	.32768	− .14508	.18260	.19575
450.....	2440	.28813	− .13022	.15791	.16789
425.....	2780	.25552	− .11528	.14024	.14861
400.....	3120	.23009	− .10338	.12671	.13217
375.....	3460	.20909	− .09363	.11546	.11993
350.....	3800	.19197	− .08566	.10631	.10972
325.....	4140	.17649	− .07887	.09762	.09979
300.....	4480	.16424	− .07519	.08905	.09004
275.....	4830	.15272	− .07369	.07903	.08076
250.....	5190	.14706	− .06698	.08108	.07448
225.....	5560	.12940	− .06612	.06328	.06818
200.....	5940	.12470	− .06199	.06271	.06330
175.....	6320	.11710	− .05835	.05875	.05903
150.....	6700	.11246	− .05799	.05447	.05350
125.....	7100	.09117	− .03176	.05941	.06189
100.....	7500	.12020	− .05517	.06503	.08657
75.....	7745	.08871	.00130	.09001	.10455
50.....	7735	.09117	.00130	.09247	.10450
25.....	7725	.12147	.00130	.12277	.12488
0.....	7715	.08956	.00130	.09086	.08666
− 25.....	7705	.09907	.00128	.10035	.12484
− 50.....	7695	.10111	.00133	.10244	.08619
− 75.....	7685	.09661	.00129	.09790	.12522
−100.....	7675	.11174	.00129	.11303	.10237
−125.....	7665	.09308	.00176	.09484	.10645
−150.....	7652	.09922	.00151	.10073	.10274
−175.....	7640	.08363	− .00782	.07581	.05130
−200.....	7700	.11588	− .08697	.02891	.00429
−225.....	8400	.08925	− .08775	.00150	—
−250.....	9171	.08084	− .08067	.00017	—
−275.....	9942	.07324	− .07465	− .00141	—
−300.....	10713	.06916	− .06946	− .00030	—
−325.....	11484	.06442	− .05893	+ .00549	—
−350.....	12255	.06036	− .06047	− .00011	—
−375.....	13026	—	—	—	—

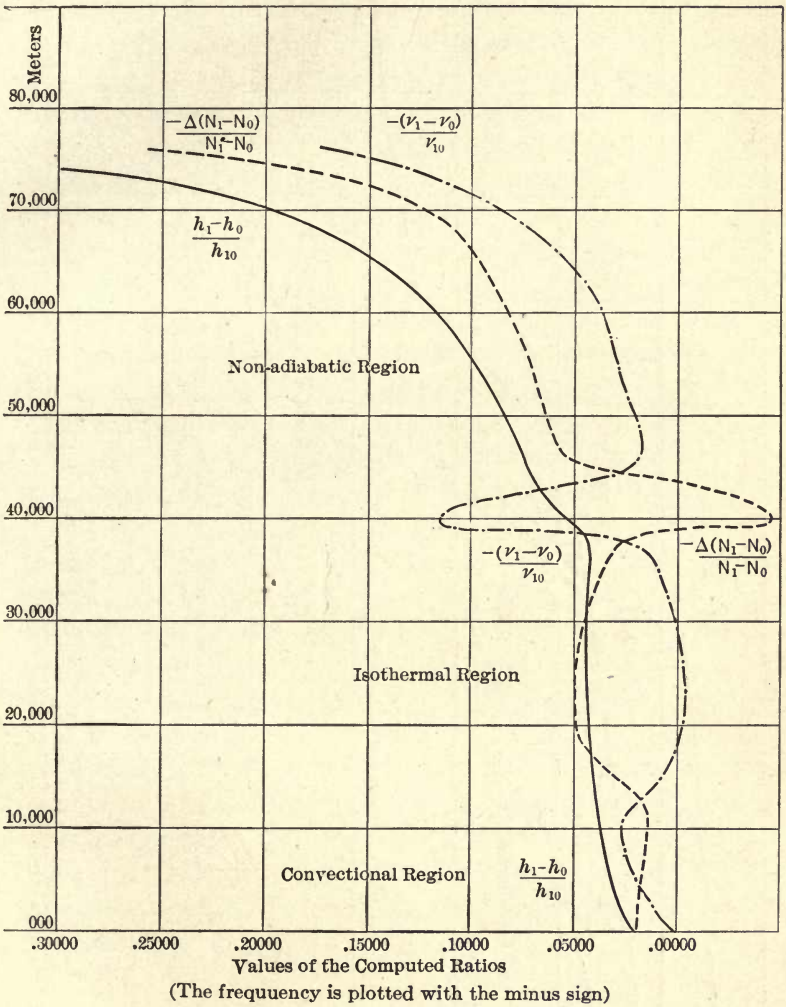


FIG. 33. Ionization, Potential Energy and Frequency in the Terrestrial Atmosphere.

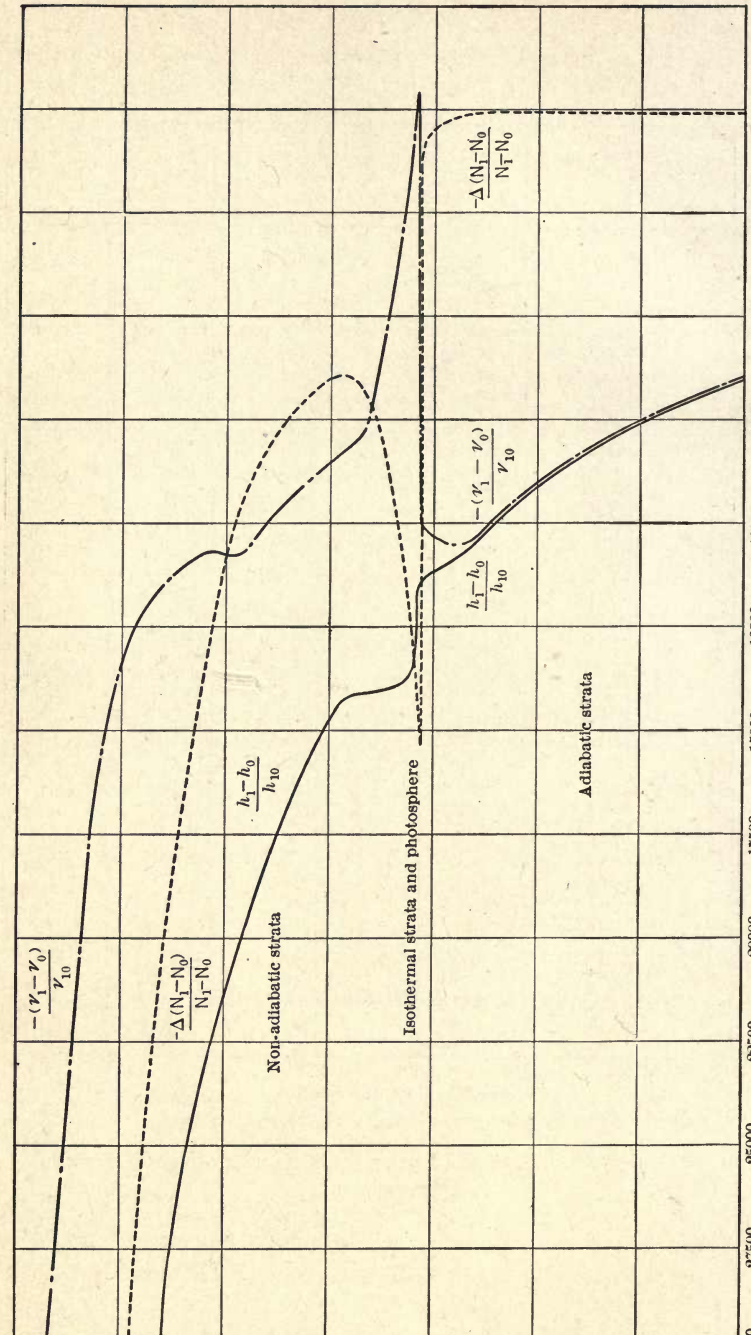


FIG. 34. Ionization, Potential Energy and Frequency in Solar Atmospheres. Helium, $m = 4.00 - \frac{\Delta(N_1-N_0)}{N_1-N_0} = \frac{h_1-h_0}{h_{10}} + \frac{\nu_1-\nu_0}{\nu_{10}}$.

36,000 meters, but falls to −15.00 at 40,000 meters, then rises steadily to very large values as 1000.00 on the vanishing plane. The values of the computed quantity by summation range from 80 volts on the surface upwards to about 200,000 volts at the top of the isothermal layer; this value is nearly the same through the stratum 34,000 − 45,000 meters; above that level $(Q_1 - Q_0)$ increases to very large values. Comparing with the average charges observed in balloon ascensions, Drexel, Nebraska, November and December, 1915, the increases in vertical value are in close accord up to 4,000 meters.

This distribution is in agreement with the known conditions, generally, with increasing values in the convectional and isothermal region, a layer of steady values at about 200,000 volts, and a supercharged non-adiabatic region to the vanishing plane. The curve of voltage follows closely that of the free heat $(Q_1 - Q_0)$. When the charge overpasses the insulating capacity of the gaseous medium there is a discharge as in accumulations in the lower strata during temporary thunderstorms, or permanently in the strata above the isothermal layers. This latter is, then, the fundamental cause of the free electric charges which go over into the polar auroras and the disturbances of the normal magnetic field as was explained in the Meteorological Treatise, Chapter VI.

The cause of atmospheric electricity appears to be a thermodynamic-radiation effect which is due to the deficiencies of the non-adiabatic physical conditions relative to the adiabatic conditions. Adiabatic strata have no free electricity, but, on diverging into the non-adiabatic temperature gradients, there is set up a process of transformation of energy which is of the electrostatic type, though it is really an integrated electromagnetic form of potential energy.

The Solar Atmospheric Electricity

On Table 111 are collected the values of $(V_1 - V_0)$ computed for helium and zinc by the same formula. It is seen that the potential $(V_1 - V_0)$ vanishes in the adiabatic strata; in the

TABLE 110

Terrestrial Atmospheric Electricity

$$\frac{V}{meter} = \frac{300}{e} h\nu. 100 \text{ m.} \qquad (Q_1 - Q_0) = (z_1 - z_0)(V_1 - V_0)$$

z	T	$(V_1 - V_0)$	z	T	$(V_1 - V_0)$	$\Sigma (Q_1 - Q_0)$	Observed at Drexel, Neb. Nov., 1915	Dec., 1915
							Elevation 396 Meters	
79000...	6°.0	—	36000	215°.0	10.66	222307		
78000...	11 .0	293.3	35000	216 .7	11.01	211647		
77000...	16 .0	990.9	34000	218 .5	11.32	200637		
76000...	21 .0	1012.1	33000	220 .0	10.66	189317		
75000...	25 .0	723.4	32000	221 .0	10.21	178657		
74000...	29 .0	638.3	31000	222 .0	11.11	168447		
73000...	33 .0	491.4	30000	223 .0	11.28	157337		
72000...	37 .0	386.0	29000	223 .5	8.69	146057		
71000...	41 .0	310.0	28000	224 .6	8.38	137367		
70000...	45 .0	252.6	27000	223 .6	10.64	128987		
69000...	49 .0	209.0	26000	223 .1	9.82	117347		
68000...	53 .0	175.5	25000	222 .8	10.13	107527		
67000...	57 .0	149.6	24000	221 .7	9.30	97397		
66000...	61 .0	132.0	23000	221 .2	8.37	88097		
65000...	65 .0	124.9	22000	220 .4	7.30	79727		
64000...	68 .0	114.4	21000	220 .1	8.09	72427		
63000...	71 .0	99.2	20000	219 .8	7.57	64337		
62000...	74 .0	87.84	19000	219 .0	7.09	56767		
61000...	77 .0	74.02	18000	218 .0	5.84	49677		
60000...	80 .0	68.89	17000	217 .1	8.40	43837		
59000...	83 .0	64.30	16000	216 .0	5.65	35437		
58000...	86 .0	54.83	15000	216 .5	5.40	29787		
57000...	89 .0	49.09	14000	217 .1	4.18	24387		
56000...	92 .0	44.37	13000	218 .3	3.62	20207		
55000...	95 .0	38.87	12000	220 .5	2.15	16587		
54000...	98 .0	37.35	11000	227 .0	1.46	14437		
53000...	101 .0	33.00	10000	233 .7	1.39	12977		
52000...	104 .0	30.27	9000	240 .3	1.14	11587		
51000...	107 .0	28.37	8000	248 .1	1.05	10447		
50000...	110 .0	28.72	7000	255 .1	1.37	9397		
49000...	112 .0	25.55	6000	261 .7	1.80	8027		
48000...	114 .0	24.72	5000	267 .4	0.82	6227		
47000...	116 .0	22.31	4000	270 .9	0.73	5407	6052	7232
46000...	118 .0	19.65	3000	278 .4	0.91	4677	4451	5226
45000...	120 .0	14.53	2500	281 .4	1.09	4227	4149	4815
44000...	124 .0	8.84	2000	284 .5	1.69	3682	3519	4219
43000...	130 .0	2.99	1500	287 .2	2.09	2837	2071	2970
42000...	138 .0	− 4.13	1000	289 .0	1.96	1792	946	1422
41000...	150 .0	−11.64	500	290 .4	1.83	812	127	240
40000...	166 .0	−15.32	100	292 .5	—	80	—	—
39000...	187 .0	− 9.69						
38000...	202 .0	0.18						
37000...	211 .0	7.45						

These are the approximate mean electric potentials as observed in balloon ascensions. The range is wide among the original data.

TABLE 111

SOLAR ATMOSPHERIC ELECTRICITY

$$\frac{V}{meter} = \frac{300}{e} h\nu. \text{ 100 m.}$$

HELIUM, $m = 4$				ZINC, $m = 65$			
z	T	$(V_1 - V_0)$		z	T	$(V_1 - V_0)$	
11000.....	240°	3.703×10^{7}	Non-adiabatic.	650	230°	1.014×10^{8}	Non-adiabatic.
10500.....	490	1.093×10^{7}		625	420	1.1632×10^{8}	
10000.....	740	6.428×10^{6}		600	640	5.371×10^{7}	
9500.....	1000	3.005×10^{6}		575	890	2.507×10^{7}	
9000.....	1270	1.565×10^{6}		550	1180	1.371×10^{7}	
8500.....	1550	9.254×10^{5}		525	1480	8.438×10^{6}	
8000.....	1830	5.697×10^{5}		500	1790	5.676×10^{6}	
7500.....	2110	3.689×10^{5}		475	2110	3.892×10^{6}	
7000.....	2400	2.481×10^{5}		450	2440	2.824×10^{6}	
6500.....	2710	1.635×10^{5}		425	2780	2.154×10^{6}	
6000.....	3040	1.229×10^{5}		400	3120	1.696×10^{6}	
5500.....	3390	1.014×10^{5}		375	3460	1.367×10^{6}	
5000.....	3780	5.204×10^{4}		350	3800	1.125×10^{6}	
4500.....	4200	4.746×10^{4}		325	4140	9.311×10^{5}	
4000.....	4680	3.668×10^{4}		300	4480	7.730×10^{5}	
3500.....	5180	2.922×10^{4}		275	4830	6.436×10^{5}	
3000.....	5700	2.433×10^{4}		250	5190	5.761×10^{5}	
2500.....	6230	2.103×10^{4}		225	5560	4.331×10^{5}	
2000.....	6750	2.016×10^{4}		200	5940	4.032×10^{5}	
1500.....	7300	2.421×10^{4}		175	6320	3.551×10^{5}	
1000.....	7600	2.985×10^{4}		150	6700	3.082×10^{5}	
500.....	7715	2.840×10^{4}		125	7100	3.215×10^{5}	
				100	7500	3.229×10^{5}	
0.....	7705	2.619×10^{4}	Isothermal.	75	7745	4.285×10^{5}	
− 500.....	7695	2.333×10^{4}		50	7735	3.956×10^{5}	
−1000.....	7685	2.019×10^{4}		25	7725	4.717×10^{5}	
−1500.....	7675	1.945×10^{4}					
−2000.....	7665	1.594×10^{4}		0	7715	3.167×10^{5}	
−2500.....	7652	1.248×10^{4}		− 25	7705	3.140×10^{5}	Isothermal.
−3000.....	7680	−		− 50	7695	2.900×10^{5}	
				− 75	7685	2.308×10^{5}	
−3500.....	8500	−		−100	7675	2.607×10^{5}	
−4000.....	9452	−		−125	7665	3.161×10^{5}	
−4500.....	10402	−	Adiabatic.	−150	7652	0.706×10^{5}	
−5000.....	11353	−		−175	7640	1.306×10^{5}	
−5500.....	12304	−					
−6000.....	13255	−		−200	7700	0.473×10^{5}	
−6500.....	14206	−		−225	8400	−	Adiabatic.
−7000.....	15158	−		−250	9171	−	
				−275	9942	−	
				−300	10713	−	
				−325	11484	−	
				−350	12255	−	
				−375	13026	−	

isothermal region helium has a nearly constant value of about 20,000 volts per meter, and zinc 350,000 volts; in the non-adiabatic strata they increase to very large values. This is in harmony with the distribution of the terrestrial atmospheric electric charges.

It is noted that in the adiabatic layers where the umbra of sun spots is developed, that is, in the low-level primary vortex, there are no free charges. The magnetic effects noted must be due to the charged atoms and molecules, rather than to the free electricity; *in this region the Stark electrical effect is lacking. In the penumbra and the upper secondary vortex there is free electricity, but without rapid rotations and no Stark effect.* The upper levels are highly charged, and this constitutes the source of the solar surface electrical density. Violent readjustments are continuously necessary in the high levels of all atmospheres.

Atmospheric Electricity and the Diurnal Convection

Similar computations were applied to the data of the diurnal distribution of the atmospheric electricity, using the same elements that were described on pages 98 to 112 of the Meteorological Treatise, from the surface up to 3,000 meters. Table 112, and Figs. 35 and 36, briefly indicate these results. The figures are somewhat smoothed, because a single computation is not sufficient to produce accurate normal values, but the general distribution is clearly indicated. The ionization is shown on Fig. 35. It is seen that we have,

1. A maximum at the 8 A.M. and 8 P.M. hours at the surface, becoming 6 A.M. and 10 P.M. on the 3,000-meter level.

2. There is a minimum from 2 P.M. at the surface to 4 P.M. at 3,000 meters, and there is a morning minimum at about 2 A.M.

3. At the 1,000-meter level, generally near the cumulus cloud base, there is a minimum, with negative ionization (absorption). This leaves positive ionization above and below, so that this level is favorable for great electrical disturbances,

as in the production of thunderstorms, and lightning discharges either horizontally or to the earth.

4. The lowest curve represents the surface electric potential, with maxima at 8 A.M., 8 P.M., and minima at 4 A.M., 2 P.M. These must be closely associated with the thermodynamic ionization as computed. The system of arrows for the horizontal and vertical electric currents, H, V, which produce the observed diurnal components of the magnetic field in Argentina (Meteorological Treatise, pages 317–329), evidently has about the same distribution with abrupt changes at 8 A.M. and 8 P.M. Similarly, the 8 A.M., 8 P.M. maxima of the diurnal vapor pressure, coefficient of electrical dissipation, evaporation, and other surface phenomena, must depend upon this thermodynamic process. Further researches will be continued.

TABLE 112

THE DIURNAL VARIATIONS OF ATMOSPHERIC ELECTRICITY

Cordoba-Pilar Data—Volts per Meter

z	2 A.M.	6 A.M.	10 A.M.	2 P.M.	6 P.M.	10 P.M.
3000	98.07	98.11	97.99	97.71	97.77	97.86
2500	97.54	97.43	97.32	97.43	97.66	97.43
2000	96.71	96.91	96.90	97.15	97.52	96.91
1500	96.80	96.29	96.15	96.92	97.21	96.29
1000	97.09	95.26	95.86	95.88	96.39	95.46
800	96.55	95.78	95.17	96.27	96.99	95.65
600	95.97	95.57	94.87	96.69	97.09	95.57
400	95.09	95.01	94.96	96.83	96.89	95.34
200	94.23	93.82	95.11	96.36	96.42	94.85
000	93.38	92.44	94.94	95.49	95.42	94.06
Total Voltage = $\Sigma\,(z_1-z_0)\,(V_1-V_0)$						
3000	1325	2081	1341	1090	977	1574
2500	1060	1742	1006	950	922	1359
2000	645	1481	796	810	852	1099
1500	690	1171	421	695	697	789
1000	835	656	276	175	287	374
800	727	760	138	251	407	412
600	611	718	78	335	427	396
400	435	606	96	363	389	350
200	263	368	126	269	295	252
000	93	92	92	95	95	94

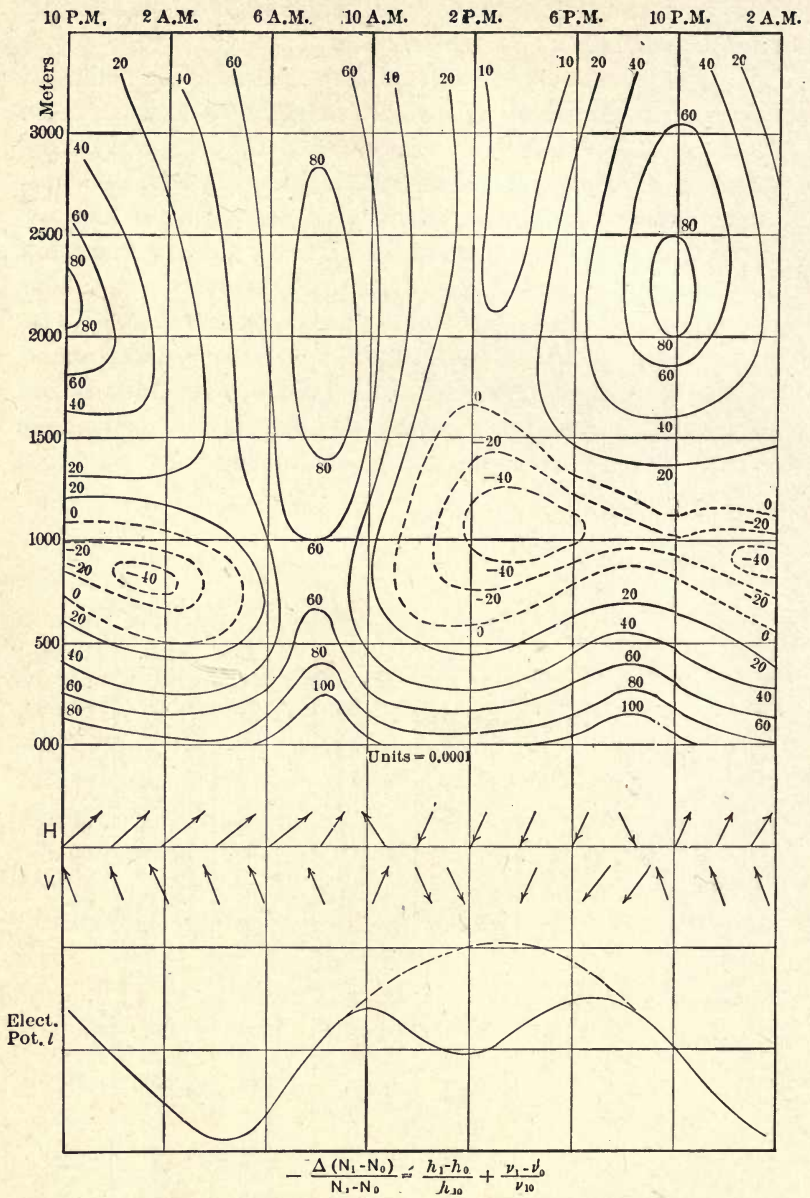


FIG. 35. Ionization, Potential Energy and Frequency.

Table 112 and Fig. 36 contain the results of the computations of the total voltage in the distribution caused by the diurnal period, that is, the solar radiation during the day and the obscuration during the night. It is seen that there exists on a given level z, generally from the surface to 3,000 meters, a minimum value at 2 P.M., with maxima averaging 8 A.M. and 8 P.M. The computed and the observed systems are practically in accord throughout the twenty-four hours, and in all the levels of record. There is an isolated maximum extending along near the 1,000-meter level, from 10 P.M. to 6 A.M., at the top of the level of vapor convection. The midday minimum extends upward toward the right on the diagram, and it gradually spreads out into a semi-diurnal curve in the high levels. This entire series of phenomena is, therefore, closely connected with the temperature convection, and they are all the by-products of it.

There have been many speculative conjectures regarding the origin of these semi-diurnal meteorological periods, but they have been usually of a secondary character. *The primary cause is clearly to be ascribed to the many complex processes which are due to the thermodynamics of radiation.* It is thought that with sufficient experience the formulas that have been deduced here, and illustrated, can be made to yield other valuable data regarding the atomic and subatomic activities which are concerned in the variations of the fundamental terms expressed by $T_1\, n_1\, N_1\, h_1\, \nu_1\, \lambda$ and their very numerous derivatives.

In Figs. 35 and 36 it is seen that, by interpolating the values of V/meter along any level, the well-known distribution is obtained, with maxima at 8 A.M. and 8 P.M., and with minima at 4 A.M. and 2 P.M., especially above 1,000 meters.

The Fundamental Quantities in Meteorology and in Astrophysics

It has become evident that the quantities which are to be most useful in the higher problems of Meteorology and Astrophysics are those which develop the fundamental terms of

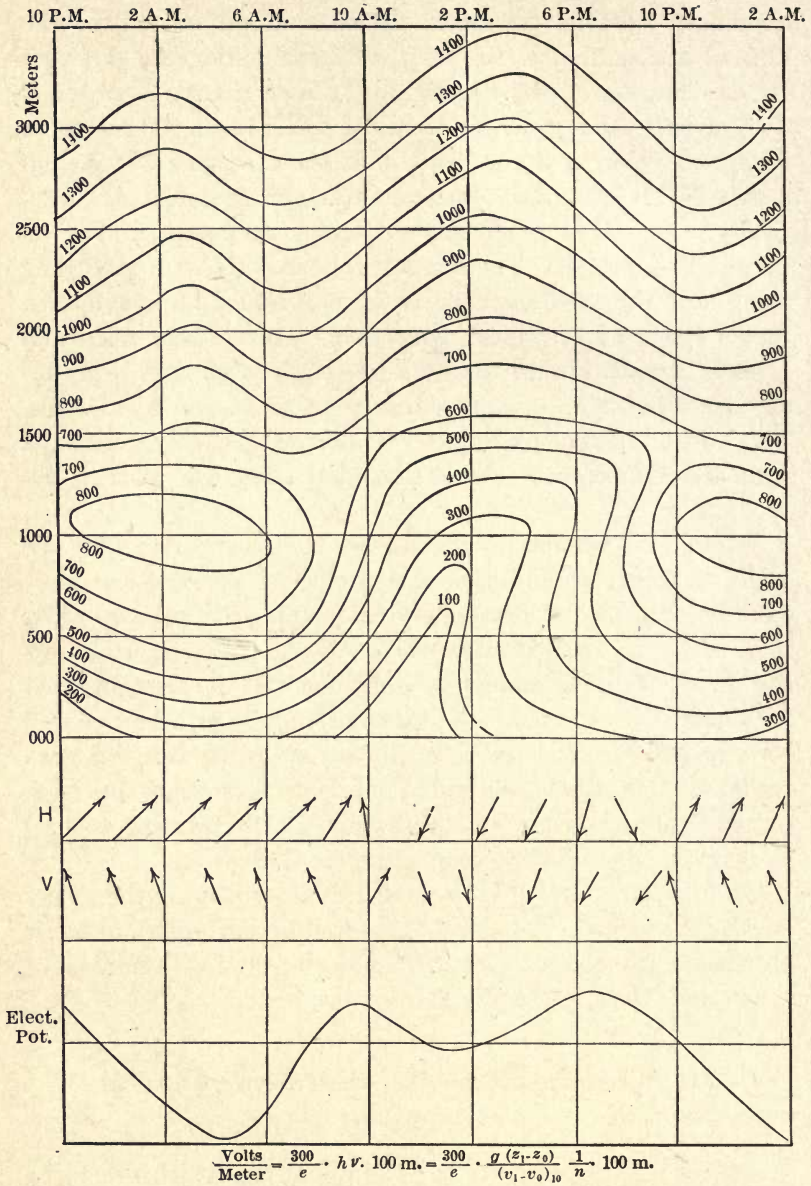


FIG. 36. Atmospheric Electricity.

our equations, namely, $(T,\ n,\ N,\ h,\ k)$. The usual forms of the Boyle-Gay Lussac Law,

$$P = \rho R T, \qquad P v = R T, \tag{323}$$

$$P = \frac{\rho}{m} . m R . T, \qquad P V = K T$$

may be written,

$$(n k T)\, m v = m (v n k)\, T. \tag{324}$$

This involves the following definitions:

Pressure. Since $k T$ is constant $= 3.7145 \times 10^{-14}$ (C. G. S.) earth, and " " " $= 2.9179 \times 10^{-11}$. " sun, it represents the mean kinetic energy of one H-atom, and the pressure per unit volume is proportional to the number of H-atoms in the specific volume, $P = n k T$.

For any other gas whose molecular weight is m the pressure is $(n k T)\, m$ per unit volume, and in the volume v the pressure or kinetic energy is,

$$P V = (n k T)\, m v. \tag{325}$$

Gas Efficiency. Grouping the same quantities in the second form $(v n k)$, k is now a variable, and we have the gas efficiency of one H-atom for one degree of temperature,

$$R = (v n k) \tag{326}$$

so that the total kinetic energy for m atoms and the temperature T, is

$$K T = m (n v k) T. \tag{327}$$

The corresponding dimensions are:

$$P V = (n k T)\, m v = \left[\frac{1}{L^3 (m)} \cdot \frac{(m) L^2}{T^2 \text{ deg.}} \cdot \text{deg.} \frac{L^3 (m)}{M} = (m^2) \frac{L^2}{T^2}\right] = K T. \tag{328}$$

Hence, the primitive equations become:

$$P = n k T \text{ for the kinetic energy of } H\text{-atoms per unit } v. \tag{329}$$

$$P V = N k T \text{ for the kinetic energy of } m \text{ and } H\text{-atoms in } v. \tag{330}$$

The entire thermodynamic system, therefore, depends upon the variations in $(n.\ N)$ and these are more fundamental than $(P.\ \rho.\ R.\ T)$, since the latter are summation or integral results.

Practical Series of Thermodynamic Terms

Since $k\,T$ = constant, n is immediately found from

(331) $$n = \frac{P}{k\,T} = \frac{g_0\,\rho_m\,B}{k\,T}, \text{ from barometric data.}$$

N must be computed from the density by the non-adiabatic formulas. This can be shortened somewhat, as follows:

(332) $$\frac{B}{B_0} = \left(\frac{T}{T_0}\right)^{\frac{n\,\kappa}{\kappa-1}}, \quad \frac{\rho}{\rho_0} = \left(\frac{T}{T_0}\right)^{\frac{n}{\kappa-1}}. \text{ Hence,}$$

(333) $$\log \frac{B}{B_0}\,\frac{1}{\kappa} = \log \frac{\rho}{\rho_0} = \log \frac{B}{B_0} \times 0.711.$$

The value of $\frac{1}{\kappa} = \frac{1}{1.406} = 0.711$ [9.85194]. Assuming an initial pressure by the barometer B_0, and an initial air density ρ_0, it follows that the difference of the barometric logarithms multiplied by 0.711 is the difference of the density logarithms.

For an example, Uccle, September 13, 1911,

Height z in meters	4000	5000	6000 meters
B (*M.K.S.*)	0.4695	0.4136	0.3636
log B	−1.67164	−1.61658	−1.56058
(log B_1 − log B_0)		−0.05506	−0.05600
Factor		0.711	0.711
(log ρ_1 − log ρ_0)		−0.03915	−0.03982
log ρ_0	−1.93158	—	—
log ρ_1	—	−1.89243	−1.85261
ρ density	0.8542	0.7806	0.7122
log v	0.06842	0.10757	0.14739
v volume	1.1706	1.2811	1.4041

$N = n\,m\,v = n\,m/\rho$ is now readily computed.

(334) $R = v\,n\,k$ can be found at once from $R = \frac{P}{\rho\,T} = \frac{n\,k\,T}{\rho\,T}$.

We, therefore, have obtained,

P, ρ, v, R, n, N, at the temperature T.

Then $k = \frac{k\,T}{T} = \frac{\text{constant}}{T}$, the kinetic coefficient.

(335) Then $h = g\,(z_1 - z_0)\,.\,\frac{1}{v_1 - v_0}\,.\,\frac{1}{n_{10}}$ the potential coefficient.

This system is compact and simple, and it at once provides the data for physical studies in atmospheres. The entire future

of meteorology and astrophysics depends upon obtaining the correct non-adiabatic density ρ and volume v. It is evident that but little progress has been made heretofore for lack of this fundamental quantity.

The Relations Between the Kinetic and the Potential Energies in Orbital Oscillations

Assume the following data:

v_0 = the velocity of the mass P in an orbit of amplitude a, with the angular velocity $\omega = \frac{2\pi}{\theta} = 20\nu$ in the periodic time θ.

t = the time elapsed from the epoch θ on the axis $O\,B$.

t_1 = the time elapsed from the epoch α on the axis $O\,C$.

$\nu = \frac{1}{\theta} = \frac{\omega}{2\pi}$ = the frequency.

φ = the phase angle $= \omega\,(t - \beta) = \omega\,(t_1 - \alpha)$.

m = an integer number of rotations.

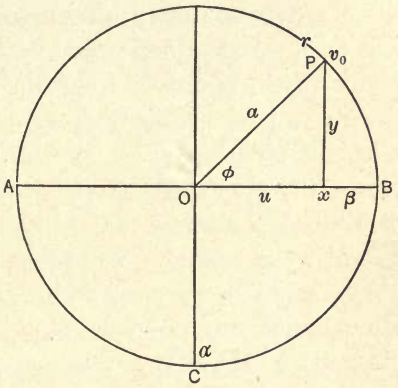


FIG. 37. Potential and Kinetic Energies.

(336) Distance. $x = a \cos \omega\,(t - \beta)$, for the epoch β and time t.

(337) Velocity. $u_x = -a\,\omega \sin \omega\,(t - \beta)$, along the axis x.

(338) Acceleration. $f_x = \frac{v_0^2}{a} \cos \varphi = -a\,\omega^2 \cos \omega\,(t - \beta)$, for $\varphi = (t - \beta)$.

(339) *Kinetic Energy.* $E = \frac{1}{2} m v_0^2$ in the orbit.

$$E_x = \tfrac{1}{2} m \omega^2 a^2 \cos^2 \omega\,(t - \beta) = \tfrac{1}{4} m \omega^2 a^2 [1 + \cos 2\,\omega\,(t - \beta)].$$

E_x ranges from $+1$ to -1, and its integral value in one period vanishes.

(340) Average. $E_x = \frac{1}{4} m \omega^2 a^2 = \frac{1}{4} m v_0^2 = \frac{1}{2}$ maximum value.

Potential Energy. Kinetic Energy + Potential Energy = a constant along the axis x of the free path $= \frac{1}{2} m v_0^2$.

(341) $P_x + \frac{1}{2} m u_x^2 = \frac{1}{2} m v_0^2 =$ constant.

(342) $P_x = \frac{1}{2} m (v_0^2 - u_x^2) = \frac{1}{2} m a^2 \omega^2 [1 - \sin^2 \omega (t - \beta)]$
$= \frac{1}{2} m a^2 \omega^2 \cos^2 \omega (t - \beta) =$
$\frac{1}{2} m \omega^2 x^2$.

P_x is a maximum when $E_x = 0$; $P_x = 0$ when E_x is a maximum.

It is evident that the periodic oscillations along the free paths of the collisions will be sorted into groups according to the lengths of the free paths. If we identify E_x with the thermodynamic h it is possible to compute the correlative quantities in terms of the electromagnetic data. The interpenetration of the orbits of the negative electrons which are carried along with the positive nucleus, whose sphere of action is the limit of the free path, will produce subatomic disturbances in the configurations and temporary instability. Bohr conceives that the negative electron charges e pass suddenly from one stable configuration to another in order to effect generation of radiation. This would render the atoms excessively unstable, inasmuch as the number of collisions and wave frequency is an enormous numerical quantity per second. The subatomic agitations conform to the X-ray series of spectra, and the interatomic collisions, which are external, produce the visible series of spectra for a given element. This allows full scope for the development of both types of series in terms of the variables h, k.

Bohr's Theory of Non-Radiating Orbits in Atoms

Bohr's theory of successive non-radiating orbits in the structure of the atoms of the chemical elements, in order to account for the observed position of the lines of a series in the spectrum, has attained such success as to become the basis for further investigations. It assumes that the orbital structure of the electrons depends upon the internal constants, $h = 6.548 \times 10^{-27}$, $m = 8.845 \times 10^{-28}$, $e = 4.774 \times 10^{-10}$ *E. S. U*, or 1.5913×10^{-20} *E. M. U.* Thence, it follows that,

$$(343)\quad \nu = \frac{2\pi^2 m E^2 e^2}{h^3}\left(\frac{1}{\tau_2{}^2} - \frac{1}{\tau_1{}^2}\right),$$

τ being an integer characteristic for the orbits, 1, 2, 3, 4 . . . For $\tau = 1$, the inner orbit gives the convergence frequency.

The Series of Spectral Lines

The frequencies of the series of spectral lines can be expressed by Balmer's Formula,

$$(344)\quad \nu = \nu_0\left(1 - \frac{4}{m^2}\right)$$

where m takes successive integral values, or by Bohr's Formula,

$$(345)\quad \nu = \frac{2\pi^2 m e^4}{h^3}\left(\frac{1}{\tau_2{}^2} - \frac{1}{\tau_1{}^2}\right),$$

where τ_2 and τ_1 are whole numbers.

m = mass, e = charge, h = potential of the electron, such that

$$(346)\quad \frac{2\pi^2 m e^4}{h^3} = 3.235 \times 10^{15}, \text{ for } E = e.$$

Since the curves on Fig. 32 are functions of T and h, both being variable, we may note the following fact: If the h — ordinates be drawn for equal T — intervals, as $T_1 - T_0 = A$ degrees, they are spaced in positions very similar to those of the above formulas. It is supposed that this is in harmony with the computations which depend upon h and T, as they are related to the thermodynamic conditions of the gas. In this case the spacing depends upon the temperature of the live emissions in succession, and they become a method of determining the temperatures of the solar gases at different depths. Furthermore, variations in the positions of the lines of a series, as hydrogen, indicate temporary changes in the local temperatures of solar emission. The positions of the lines, as measured in the spectrum, may be used inversely to establish the position and the correct curve of the (h . T) function.

It is evident that h is a variable, and we shall show that the spectrum lines can be produced by the vibrations imposed upon the electronic orbits by reaction from the external collisions of the atoms and molecules.

Derivation of the Orbital Formulas

Assume the following relations:

a = the orbital radius.

ν = the orbital frequency = the vibration frequency.

$V = \frac{e}{r}$ = the potential at the distance r.

(347) $E = -\frac{\partial V}{\partial \nu} = \frac{e}{r^2}$ = the central force.

(348) $W = \frac{E\,e}{\nu} = V\,e$ = the work done between charges E, e.

(349) $\omega = \frac{2\pi}{\theta} = 2\pi\nu = 2\pi\frac{c}{\lambda}$ = the angular velocity.

(350) $\theta = \frac{2\pi}{\omega} = \frac{1}{\nu} = \frac{\lambda}{c}$ = the periodic time.

(351) $\nu = \frac{1}{\theta} = \frac{\omega}{2\pi} = \frac{c}{\lambda} = \frac{v}{2\pi a} = \frac{m\,o^2}{2h}$.

(352) $\varphi = \omega\,t = \frac{2\pi}{\theta}\,.\,t = 2\pi\nu\,.\,t = \frac{2\pi c}{\lambda}\,.\,t$, for the elapsed time t.

(353) $V = \frac{e}{\tau a} = \frac{E}{\tau n a}$, if the central charge is $E = n\,e$.

From the orbital velocity v, the constant kinetic energy in the orbit $\frac{1}{2}\,m\,u^2$, and the central acceleration f, have the following formulas:

(354) Velocity. $v = 2\pi a\,.\,\nu = a\,\omega = 2\pi a\,.\,\frac{c}{\lambda} = \frac{2\pi e^2}{\tau h}$

(355) Kinetic Energy. $\frac{1}{2}\,m\,v^2 = h\,\nu = V\,e = \frac{E\,e}{\tau n a} = \frac{e\,e}{\tau a} = \frac{g\,(z_1 - z_0)}{n_{10}\,(v_1 - v_0)_{10}}$.

(356) Acceleration to Center. $\frac{m\,v^2}{a} = \frac{2\,h\,\nu}{a} = \frac{2\,V\,e}{a} = \frac{2\,E\,e}{\tau n a^2} = \frac{2\,e\,e}{\tau a^2} = 2\,V^2\,\tau$.

(357) Radius of Orbit. $a = \frac{\tau\, h^2}{2\,\pi^2\, m\, e^2}$. Log. $2\,\pi^2\, m\, e^2 =$

$- 45.59979.$

(358) Velocity. $v = \frac{2\,\pi\, e^2}{\tau\, h}$. Log. $2\,\pi\, e^2 = - 18.15594.$

(359) Frequency. $\nu = \frac{2\,\pi^2\, m\, e^4}{\tau^2\, h^3}$ " $2\,\pi^2\, m\, e^4 = - 64.95755.$

(360) Angular Velocity. $\omega = \frac{4\,\pi^3\, m\, e^4}{\tau^2\, h^3}$. Log. $4\,\pi^3\, m\, e^4 =$

$- 63.75573.$

(361) Potential. $V = \frac{2\,\pi^2\, m\, e^3}{\tau^2\, h^2}$. Log. $2\,\pi^2\, m\, e^3 = - 54.27867.$

(362) Nuclear charge. $E = \frac{m\, v^2\, a\, n\, \tau}{2\, e}$

(363) Kinetic Energy. $\frac{1}{2}\, m\, v^2 = \frac{2\,\pi^2\, m\, e^4}{\tau^2\, h^2}$. Log. $2\,\pi^2\, m\, e^4 =$

$- 64.95755.$

(364) Acceleration to Center. $\frac{m\, v^2}{a} = \frac{8\,\pi^4\, m^2\, e^6}{\tau^3\, h^4}$.

Log. $8\,\pi^4\, m^2\, e^6 = - 108.85837.$

These can be used in computing the elements in the K-series, L-series, and the other constituents of the spectrum.

From the theory of electronic orbits we have directly the centrifugal force $= \frac{1}{2}\, m\, v^2 = h\, \nu = \frac{E\, e}{n\, a}$, where E is the charge on the positive nucleus, and n the number of electrons; hydrogen $n = 2$. For the velocity in the orbit, $v = 2\,\pi\, a.\, \nu$, where the angular frequency and the number of vibrations ν are assumed to be the same. Hence, we have,

(365) Frequency. $\nu = \frac{m\, v^2}{2\, h} = \frac{v}{2\,\pi\, a}$

(366) Radius. $a = \frac{h}{m\,\pi\, v}$

The central force of acceleration is,

(367) $$f = \frac{m v^2}{a} = \frac{2 E e}{n a^2} = \frac{E e}{a^2}, \text{ for hydrogen, } n = 2.$$

The charge on the central nucleus of positive electricity is,

$$\text{Charge } (+), E = \frac{m a v^2}{e} = \frac{m v^2}{e} \cdot \frac{h}{m v \pi} = \frac{\nu h}{e \pi} = e. \quad \text{Hence,}$$

(368) $$v^2 = \frac{2 h \nu}{m} = \frac{\pi^2 e^4}{h^2}. \quad \text{Thence,}$$

(369) $$\text{Frequency, } \nu = \frac{\pi^2 m e^4}{2 h^3} = \frac{2 \pi^2 m e^4}{h^3}\left(\frac{1}{n_2{}^2}\right), \text{ hydrogen.}$$

We compute the two formulas for ν and $\lambda = \frac{c}{\nu}$:

$\nu_m = \frac{g(z_1 - z_0)}{(v_1 - v_0)_{10}} \cdot \frac{1}{n_{10}} \cdot \frac{1}{h}$ Bigelow		$\nu = \frac{2 \pi^2 m e^4}{h^3}\left(\frac{1}{n^2}\right)$ Bohr		
Log. $g(z_1 - z_0)$	13.13805	Log. π^2	0.99430	
		Log. m	−28.94670	
		Log. e^4	−38.71552	Lyman as observed, also Millikan
			−64.65652	
Log. $(v_1 - v_0)_{10}$	5.15619			
Log. n_{10}	17.11295	Log. 2	0.30103	
Log. h	−25.96720	Log. h^3	−79.44770	*Science* April 6, 1917
	− 2.23634		−79.74873	
Log. ν_m	14.90171	Log. ν	14.90779	
Log. c	10.47712	Log. c	10.47712	
Log. λ_m	− 5.57541	Log. λ	− 5.56933	
Wave length (cm) λ_m	0.00003760	λ	0.00003710	0.00003650

The Bigelow formula seems to be equivalent to the Bohr formula for the case of the hydrogen series, since the convergence wave length is practically the same. It is evident that since in the Bigelow formula the h is an external potential, while in the Bohr formula h is a function of the kinetic energy,

$$h = \tfrac{1}{2} m v^2 / \nu,$$

it will be necessary to examine the formulas in their other relations.

Our thermodynamic data refer in all cases to the maximum ν_m, λ_m, in the Wien Displacement Law, $\lambda_m = \frac{.2891}{T_m}$, so that these results are practically identical. It should be noted that the Bigelow form uses the h_m of the external potential, while Bohr uses h = the constant or adiabatic value. We may, therefore, conclude that the Bohr computation refers to the adiabatic case, generally, and the Bigelow computation to the nonadiabatic case near the bottom of the solar reversing layer.

Moseley's equation for the relation between the frequency and the atomic numbers,

$$(370) \qquad \nu_N = A\,(N - b)^2 \text{ or } \nu_N = (a + b\,N)^2$$

or Uhler's more accurate equation, which is hyperbolic,

$$(371) \quad \nu_N = A + B\,N + \frac{e}{D - N} \text{ (\textit{Physical Review}, April, 1917),}$$

shows that the ultimate relations between the atomic numbers in the K_α, K_β, L_α, L_β, L_γ, types of the X-ray series are really very complicated. It remains to be seen what form further experiments will assign to these structural relations. We shall, therefore, reserve this subject for further examination, though it has seemed proper in this chapter to indicate some of the interesting developments which come from the non-adiabatic thermodynamics.

Sanford applies the Bohr theory to the K and L series with much success in *Physical Review*, May, 1917. These subjects will be resumed in a further publication on the structure of matter.

Moseley's Law

Moseley found an important relation between the atomic numbers of the elements in certain series, by assuming that the

addition of the b unit charges, $b \cdot e$, to the nucleus suffices to pass from one element N to the next in order $N+b$. For $\tau = 1$ (convergence).

(372) $$f = \frac{m v^2}{a} = \frac{m}{a} \cdot 4\pi^2 a^2 \nu^2 = 4\pi^2 m a \nu^2$$

(373) $$f_1 = \frac{2 E_1 e}{n a_1^2} = \frac{E_1 e}{a_1^2} \text{ and } f_2 = \frac{E_2 e}{a_2^2}, \text{ for } n = 2, \text{ hence,}$$

(374) $$f_1 a_1^2 = E_1 e = 4\pi^2 m \nu_1^2 a_1^3$$

(375) $$f_2 a_2^2 = E_2 e = 4\pi^2 m \nu_2^2 a_2^3$$

$$\left.\begin{matrix} \\ \end{matrix}\right\} \frac{E_1}{E_2} = \frac{\nu_1^2}{\nu_2^2} \cdot \frac{a_1^3}{a_2^3}$$

Moseley finds that the frequency is proportional to the square of the nuclear charge.

(376) $$\frac{\nu_1}{\nu_2} = \frac{E_1^2}{E_2^2} = \text{Moseley's Law. Hence,}$$

(377) $$\frac{E_1}{E_2} = \frac{E_1^4}{E_2^4} \cdot \frac{a_1^3}{a_2^3} \text{ and } E_1^3 a_1^3 = E_2^3 a_2^3, \text{ so that}$$

(378) (Millikan) $$\mathbf{a}_1 \mathbf{E}_1 = \mathbf{a}_2 \mathbf{E}_2.$$

This is derived from the central acceleration. On the other hand, from equation (330), we have, by assuming that the kinetic energy in the orbit is a constant, a different relation.

(379) (Bigelow) $$\mathbf{a}_1 \mathbf{E}_2 = \mathbf{a}_2 \mathbf{E}_1$$

There are, therefore, two distinct solutions by (346) and (347), the former depending upon assuming that the frequency in the orbits of the electrons is the same as the frequency in the spectral lines; the latter assumes that the kinetic energy in the orbit is a constant for all electrons, the radial distance conforming thereto. The former assumes that h is constant, and the latter that h is a variable; the former supposes that the fundamental relations are internal to the atom, the latter that the primary relations are external to the atom, and that the variations in the position of the lines of the spectrum are due to perturbations impressed by interpenetration of the active orbits of the electric charges during the collisions. The Bohr theory makes the variation of energy which causes radiation to depend

TABLE 112

Evaluation of $\left(\frac{1}{\tau_1^2} - \frac{1}{\tau_2^2}\right)$ in $\nu = \frac{2\pi^2 m e^4}{h^3}\left(\frac{1}{\tau_2^1} - \frac{1}{\tau_2^2}\right)$

		$\frac{1}{\tau_1^2} - \frac{1}{\tau_2^2}$	$\cos\phi$	ϕ
$\tau_1 = 1$	$\tau_2 = 2$	0.750	0.750	41° 25′
	3	0.889	0.889	27 15
	4	0.938	0.938	20 17
	5	0.960	0.960	16 16
	6	0.972	0.972	13 36
	7	0.980	0.980	11 15
	8	0.984	0.984	10 16
	9	0.988	0.988	8 52
	10	0.990	0.990	8 8
	11	0.992	0.992	7 15
Convergence ν		1.000		
$\tau_1 = 2$	$\tau_2 = 3$	0.139	0.556	56° 12′
	4	0.188	0.752	41 13
	5	0.210	0.840	32 51
	6	0.222	0.888	27 23
	7	0.230	0.920	23 5
	8	0.234	0.936	20 46
	9	0.238	0.952	17 50
	10	0.240	0.960	16 7
	11	0.242	0.968	14 31
Convergence ν		0.250		
$\tau_1 = 3$	$\tau_2 = 4$	0.049	0.441	65° 50′
	5	0.071	0.640	50 8
	6	0.083	0.748	41 35
	7	0.091	0.820	34 28
	8	0.096	0.865	30 8
	9	0.099	0.892	26 52
	10	0.101	0.910	24 30
	11	0.103	0.928	21 52
Convergence ν		0.111		

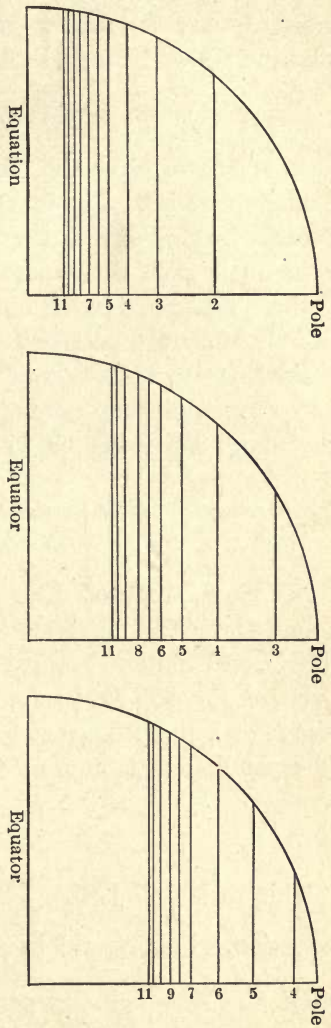


FIG. 38. Orbital Distributions.

upon the electrons jumping from one stable non-radiating orbit to another, but this would make the structure of atoms wholly unstable and precarious. The flat Saturnian systems of orbits should probably be superseded by a series of orbits which are arranged upon the surface of a sphere, in order to produce the polarizations and magnetizations that exist in molecules.

The evaluation of the term $\left(\frac{1}{\tau_1^2} - \frac{1}{\tau_2^2}\right)$ in the expression for the frequency ν gives the relative position of the spectrum lines in different series. The function $\cos \varphi$ is formed from successive divisions by the convergence value, computed for $\tau_2 = \infty$, and φ is the angle from the equator. If the electrons should revolve on these planes, as indicated, the frequencies would correspond with an Amperean polarized sphere. If the several atoms occur in different environments of h, depths, densities, temperatures, the corresponding radiations conform to those observed in the spectrum. This research will be continued

The Electronic Orbits in the K and L Series of Radiation Lines for h-Variable.

We have computed the values of the several terms according to formulas (322) to (339) for the K_α and L_α series, quoting Uhler's and Sanford's wave length values λ, as given in the *American Physical Review* for April and May, 1917. From these values of λ the frequency ν is computed, and with this ν the corresponding variable h by the formula,

$$h = \left(\frac{2\pi^2 m e^4}{\nu}\right)^{\frac{1}{3}}. \tag{380}$$

This variable h is then applied to a, v, V, $\frac{1}{2} m v^2$, $\frac{m v^2}{a}$, E, in succession. Finally, we have,

$$A = \frac{\sqrt{\nu}}{E} = \text{a constant by Moseley's Law.} \tag{381}$$

It is noted that in the K_α radiation the value of A is somewhat smaller in the middle than at the ends of the series; the same is true of the L_α series. *In each case the divergence is small, and*

TABLE 113

$K\alpha$ — RADIATION. $E = (N-1)\,e$. $n = (N-1)$.

$$\tfrac{1}{2} m v^2 = h\nu = Ve = \frac{Ee}{\tau n a}. \qquad \frac{m v^2}{a} = \frac{2h\nu}{a} = \frac{2Ve}{a} = \frac{2Ee}{\tau n a^2}.$$

	N Atomic Number	λ Ångstrom	$h = \left(\frac{2\pi^2 m e^4}{\nu}\right)^{\frac{1}{3}}$	$a = \frac{h^2}{2\pi^2 m e^4}$	$v = \frac{2\pi e^2}{h}$	$V = \frac{2\pi^2 m e^3}{h^2}$	$\frac{1}{2} mv^2 = \frac{2\pi^2 m e^4}{h^2}$	$\frac{m v^2}{a} = \frac{8\pi^4 m e^5}{\tau^3 h^4}$	$\sqrt{\nu} = \left(\frac{c}{\lambda}\right)^{\frac{1}{2}}$	$E = (N-1)\,e$	$A = \sqrt{\nu}$
			Log	Log	Log	Log	Log	Log	Log	Log	Log
H	1	914.000	−27.81379	− 8.02779	8.34215	−2.65109	−11.32997	−3.60321	7.75809	−10.67888	17.07921
Na	11	11.951	−27.18595	−10.77211	8.96999	−1.90677	−10.58565	0.11457	8.69986	−9.67888	17.02098
Mg	12	9.915	−27.15557	−10.71135	9.00037	−1.96753	−10.64641	0.23609	8.74542	−9.72027	17.02515
Al	13	8.361	−27.13423	−10.66867	9.02171	0.01021	−10.68909	0.32145	8.77743	−9.75806	17.01937
Si	14	7.134	−27.11025	−10.62071	9.04569	0.05817	−10.73705	0.41737	8.81340	−9.79282	17.02058
P	15	6.168	−27.09019	−10.58059	9.06575	0.09829	−10.77717	0.49761	8.84349	−9.82501	17.01848
S	16	5.360	−27.06986	−10.53993	9.08608	0.13895	−10.81783	0.57893	8.87398	−9.85497	17.01901
Cl	17	4.721	−27.05149	−10.50319	9.10445	0.17569	−10.85457	0.65241	8.90155	−9.88300	17.01855
K	19	3.742	−27.01784	−10.43589	9.13801	0.24299	−10.92187	0.78701	8.95201	−9.93415	17.01786
Ca	20	3.360	−27.00226	−10.40473	9.15368	0.27415	−10.95303	0.84933	8.97539	−9.95763	17.01776
Se	21	3.030	−28.98729	−10.37479	9.16865	0.30409	−10.98297	0.90921	8.99784	−9.97991	17.01793
Ti	22	2.748	−28.97315	−10.34651	9.18279	0.33237	−9.01125	0.96577	9.01905	−8.00110	17.01795
V	23	2.505	−28.96041	−10.32103	9.19553	0.35785	−9.03673	1.01673	9.03816	−8.02130	17.01686
Cr	24	2.290	−28.94676	−10.29373	9.20918	0.38515	−9.06403	1.07133	9.05864	−8.04061	17.01803
Mn	25	2.099	−28.93415	−10.26851	9.22179	0.41037	−9.08925	1.12177	9.07756	−8.05909	17.01847
Fe	26	1.934	−28.92230	−10.24481	9.23364	0.43407	−9.11295	1.16917	9.09533	−8.07682	17.01851
Co	27	1.790	−28.91109	−10.22239	9.24485	0.45649	−9.13537	1.21401	9.11214	−8.09385	17.01829
Ni	28	1.658	−28.90000	−10.20021	9.25594	0.47867	−9.15755	1.25837	9.12877	−8.16024	17.01853
Ca	29	1.545	−28.88979	−10.17979	9.26615	0.49909	−9.17797	1.29921	9.14411	−8.12604	17.01806
Zn	30	1.440	−28.87960	−10.15941	9.27634	0.51947	−9.19835	1.33997	9.15938	−8.14128	17.01810
Ge	32	1.259	−28.86015	−10.12051	9.29579	0.55837	−9.23725	1.41777	9.18855	−8.17024	17.01831
As	33	1.172	−28.84979	−10.09979	9.30615	0.57909	−9.25297	1.45921	9.20410	−8.18403	17.02007
Se	34	1.107	−28.84153	−10.08327	9.31441	0.59561	−9.27449	1.49225	9.21649	−8.19739	17.01910
Br	35	1.038	−28.83221	−10.06463	9.32373	0.61425	−9.29315	1.52953	9.23046	−8.21036	17.02010
Rb	37	0.924	−28.81537	−10.03093	9.34057	0.64793	−9.32681	1.59689	9.25573	−8.23518	17.02055
Sr	38	0.874	−28.80731	−10.01483	9.34863	0.66405	−9.34293	1.62913	9.26781	−8.24708	17.02073

The wave lengths are taken from the *American Physical Review*, April and May, 1917, from articles by Uhler and Sanford.

TABLE 113—*Continued.*

K_α – RADIATION. $E = (N-1)\,e$. $n = (N-1)$.

$$\tfrac{1}{2} m v^2 = h\nu = V e = \frac{E e}{\tau n a}. \qquad \frac{m v^2}{a} = \frac{2 h \nu}{a} = \frac{2 V e}{a} = \frac{2 E e}{\tau n a^2}.$$

	N Atomic Number	λ Ångstrom	$h = \left(\frac{2\pi^2 m e^4}{\nu}\right)^{\frac{1}{3}}$	$a = \frac{h^2}{2\pi^2 m e^4}$	$v = \frac{2\pi e^2}{h}$	$V = \frac{2\pi^2 m e^3}{h^2}$	$\frac{1}{2}\frac{mv^2}{a} = \frac{2\pi^2 m e^4}{h^2}$	$\frac{\frac{m v^2}{a}}{} = \frac{8\pi^4 m e^6}{\tau^3 h^4}$	$\sqrt{\nu} = \left(\frac{c}{\lambda}\right)^{\frac{1}{2}}$	$E = (N-1)\,e$	$A = \frac{\sqrt{\nu}}{E}$
			Log	Log	Log	Log	Log	Log	Log	Log	Log
X	39	0.840	−28.79557	−10.00135	9.36037	0.68753	−9.36641	1.67609	9.28642	−8.25866	17.02776
Zr	40	0.795	−28.79360	−11.98741	9.36231	0.69147	−9.37035	1.68397	9.28838	−8.26994	17.01844
Nb	41	0.751	−28.78555	−11.97131	9.37039	0.70757	−9.38645	1.71617	9.30045	−8.28094	17.01951
Mo	42	0.714	−28.77804	−11.95629	9.37790	0.72259	−9.40147	1.74621	9.31171	−8.29166	17.02005
Ru	44	0.644	−28.76311	−11.92643	9.39283	0.75245	−9.43133	1.80593	9.33412	−8.31235	17.02177
Rh	45	0.617	−28.75691	−11.91403	9.39903	0.76485	−9.44373	1.83073	9.34342	−8.32233	17.02109
Pd	46	0.588	−28.74994	−11.90009	9.40600	0.77879	−9.45767	1.85861	9.35387	−8.33209	17.02178
Ag	47	0.564	−28.74390	−11.88801	9.41204	0.79087	−9.46975	1.88267	9.36292	−8.34164	17.02128
Cd	48	0.538	−28.73707	−11.87435	9.41887	0.80453	−9.48341	1.91009	9.37317	−8.35098	17.02219
In	49	0.512	−28.72990	−11.86001	9.42604	0.81887	−9.49775	1.93877	9.38393	−8.36012	17.02381
Su	50	0.487	−28.72265	−11.84551	9.43329	0.83337	−9.51225	1.96777	9.39480	−8.36908	17.02572
Sb	51	0.470	−28.71751	−11.83523	9.43843	0.84365	−9.52253	1.98833	9.40251	−8.37785	17.02466
Te	52	0.458	−28.71377	−11.82775	9.44217	0.85113	−9.53001	2.00329	9.40813	−8.38645	17.02168
I	53	0.439	−28.70763	−11.81547	9.44831	0.86341	−9.54229	2.02885	9.41733	−8.39488	17.02245
Cs	55	0.400	−28.69416	−11.78853	9.46178	0.89035	−9.56923	2.08173	9.43753	−8.41127	17.02626
Ba	56	0.387	−28.68938	−11.77897	9.46656	0.89991	−9.57879	2.10085	9.44471	−8.41924	17.02547
La	57	0.374	−28.68443	−11.76907	9.47151	0.90981	−9.58869	2.12065	9.45213	−8.42707	17.02606
Ce	58	0.358	−28.67810	−11.75641	9.47784	0.92247	−9.60135	2.14597	9.46162	−8.43475	17.02687
Pr	59	0.345	−28.67275	−11.74571	9.48319	0.93317	−9.61205	2.16737	9.46965	−8.44231	17.02734
Nd	60	0.333	−28.66762	−11.73543	9.48832	0.94345	−9.62233	2.18793	9.47734	−8.44973	17.02761
W	74	0.210	−28.60088	−11.60197	9.55506	1.07691	−9.75579	2.45485	9.57745	−8.54220	17.03525

The wave lengths are taken from the *American Physical Review*, April and May, 1917, from articles by Uhler and Sanford.

TABLE 114

L_α – RADIATION. $E = (N-7)\,e$. $n = (N-7)$.

$$\tfrac{1}{2} m v^2 = h\nu = Ve = \frac{Ee}{\tau n a}. \qquad \frac{m v^2}{a} = \frac{2 h \nu}{a} = \frac{2 Ve}{a} = \frac{2 Ee}{\tau n a^2}.$$

	N Atomic Number	λ Ångstrom	$h = \left(\frac{2\pi^2 m e^4}{\nu}\right)^{\frac{1}{3}}$	$a = \frac{h^2}{2\pi^2 m e^4}$	$u = \frac{2\pi e^2}{h}$	$V = \frac{2\pi^2 m e^3}{h^2}$	$\frac{1}{2} m v^2 = \frac{2\pi^2 m e^4}{h^2}$	$\frac{m v^2}{a} = \frac{8\pi^4 m^2 e^6}{\tau^3 h^4}$	$\sqrt{\nu} = \left(\frac{c}{\lambda}\right)^{\frac{1}{2}}$	$E = (N-7)\,e$	$A = \frac{\sqrt{\nu}}{E}$
			Log	Log	Log	Log	Log	Log	Log	Log	Log
Zr	40	6.091	−27.08837	−10.57695	9.06757	0.10193	−10.78081	0.50489	8.84622	−9.19739	17.64883
Nb	41	5.749	−27.08001	−10.56023	9.07593	0.11865	−10.79753	0.53833	8.85877	−9.21036	17.64841
Mo	42	5.432	−27.07180	−10.54381	9.08414	0.13507	−10.81395	0.57117	8.87108	−9.22295	17.64813
Ru	44	4.861	−27.05572	−10.51165	9.10022	0.16723	−10.84611	0.63549	8.89520	−9.24708	17.64812
Rh	45	4.662	−27.04967	−10.49955	9.10627	0.17933	−10.85821	0.65969	8.90428	−9.25866	17.64562
Pd	46	4.385	−27.04080	−10.48181	9.11514	0.19707	−10.87595	0.69517	8.91758	−9.26994	17.64764
Ag	47	4.170	−27.03352	−10.46725	9.12242	0.21163	−10.89051	0.72429	8.92849	−9.28094	17.64755
Su	50	3.619	−27.01301	−10.42623	9.14293	0.25265	−10.93153	0.80633	8.95927	−9.31235	17.64682
Sb	51	3.458	−27.00642	−10.41305	9.14952	0.26583	−10.94471	0.83269	8.96915	−9.34233	17.64682
La	57	2.676	−28.96931	−10.33883	9.18663	0.34005	−9.01893	0.98113	9.02482	−9.37785	17.64697
Ce	58	2.567	−28.96330	−10.32681	9.19264	0.33027	−9.03095	1.00517	9.03383	−9.38645	17.64738
Pr	59	2.471	−28.95777	−10.31575	9.19817	0.36313	−9.04201	1.02729	9.04213	−9.39488	17.64725
Nd	60	2.382	−28.95246	−10.30513	9.20349	0.37375	−9.05263	1.04953	9.05009	−9.40316	17.64693
Sa	62	2.208	−28.94138	−10.28297	9.21456	0.39591	−9.07479	1.09285	9.06656	−9.41924	17.64732
Eu	63	2.130	−28.93627	−10.27275	9.21967	0.40617	−9.08501	1.11329	9.07437	−9.42707	17.64730
Gd	64	2.057	−28.93122	−10.26265	9.22474	0.41623	−9.09511	1.13349	9.08195	−9.43475	17.64720
Ho	66	1.914	−28.92079	−10.24179	9.23515	0.43709	−9.11597	1.17521	9.09759	−9.44973	17.64786
Er	68	1.790	−28.91109	−10.22239	9.24485	0.45649	−9.13537	1.21401	9.11214	−9.46421	17.64793
Ta	73	1.525	−28.88690	−10.17401	9.26904	0.50487	−9.18375	1.31077	9.14843	−9.49842	17.65003
W	74	1.486	−28.88415	−10.16851	9.27179	0.51037	−9.18925	1.32177	9.15255	−9.50495	17.64760
Os	76	1.397	−28.87521	−10.15063	9.28073	0.52815	−9.20713	1.35753	9.16596	−9.51773	17.64826
Ir	77	1.354	−28.87068	−10.14157	9.28526	0.53731	−9.21619	1.37565	9.17275	−9.52398	17.64877
Pt	78	1.316	−28.86656	−10.13333	9.28938	0.54555	−9.22443	1.39213	9.17893	−9.53014	17.64879
Au	79	1.284	−28.86300	−10.12621	9.29294	0.55267	−9.23155	1.40637	9.18428	−9.53621	17.64807
Hg	80	1.243	−28.85830	−10.11681	9.29764	0.56207	−9.24095	1.42517	9.19136	−9.54220	17.64916
Tb	81	1.209	−28.85429	−10.10879	9.30165	0.57009	−9.24897	1.44121	9.19735	−9.54811	17.64924
Pb	82	1.179	−28.85065	−10.10151	9.30529	0.57737	−9.25625	1.45577	9.20281	−9.55394	17.64887
Bi	83	1.144	−28.84629	−10.09279	9.30965	0.58609	−9.26497	1.47321	9.20935	−9.55969	17.64966
Th	90	0.971	−28.82255	−10.04531	9.33339	0.63357	−9.31245	1.56817	9.24495	−9.59796	17.64699
U	92	0.919	−28.81458	−10.02937	9.34136	0.64951	−9.32839	1.60007	9.25690	−9.60830	17.64860

Moseley's formula $\sqrt{\nu} = A\,(N-7)\,e$. Mean A = . . . 17.64767

The wave lengths are taken from the *American Physical Review*, April and May, 1917, from articles by Uhler and Sanford.

it proves that Moseley's Law conforms to the variable h in the series. Since the potential energy h represents the resistance to the kinetic energy in the medium, because we have assumed the kinetic energy constant, and the central force variable from one element to another, it is seen that the theory of orbits takes on very much greater flexibility, and really represents the complex of the thermodynamic conditions which produce radiation. These vary with the depth in gases, temperature, density, pressure,

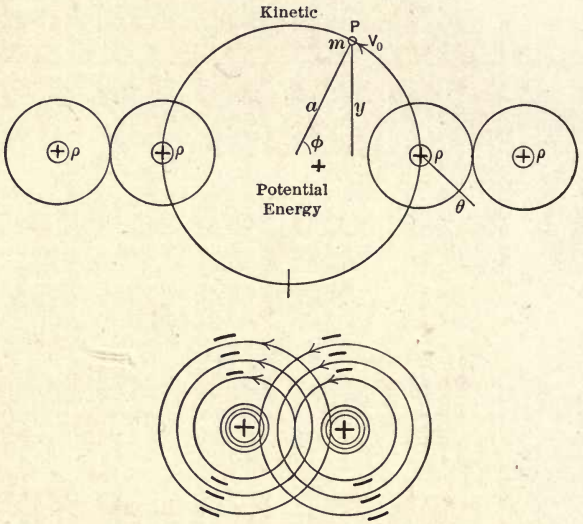


FIG. 39. Interpenetration of the Orbits of the Electrons in the Atoms During Collisions.

gas efficiency, surrounding each atom as represented by n and N. Hence, different subordinate series of the same element originate in different environments on the *outside*, while the several chemical elements are derived from the different structural configurations of the electrons on the *spherical surface* of the atoms.

The Interpenetration of the Electron Orbits at the Contact of Collision

The complex systems of atoms in collision must cause interpenetration of the individual orbits of the electrons, and this

must communicate a series of internal vibrations to the radiating particles or electric charges.

The potential energy h in the radiation function is related to the kinetic energy k in much the same way that has just been described. If kT is a constant then $h\nu$ is a variable, and a very complex variable whose mean values alone can appear in the thermodynamic formulas. The important point to recognize is that the electrons may remain on their stationary orbits according to the structure of the chemical elements, and acquire the perturbations producing the radiation during the confusion of interpenetration. This avoids the difficulty in Bohr's theory which requires the electrons to pass from one orbit to another during radiation. The amperean spherical distribution of the interacting elementary charges, E at the center, and e in the several orbits, probably present the basis for the structural permanences which are inherent in the chemical elements. Fig. 39 represents the collision of two polarized atoms whose orbits are projected on the equatorial plane.

The Electromagnetic Waves Due to the Sudden Motion and Stoppage of an Electric Charge in Collisions

Following Heaviside's exposition of the effect of suddenly starting or stopping the motion of an electric charge, as in collisions, we have the distribution of the electric disturbance D and the magnetic induction B, in producing the plane electromagnetic waves of Fig. 40.

Let ρ take on suddenly the velocity v, so that in the time t it reaches the polar position of the sphere $+\rho$. The outside radial displacement changes into a current along the meridians from the positive to the negative pole, with its magnetic induction on the parallels in the vector sense, so that the polar field becomes the plane wave $[D \,.\, B]$, as the sphere enlarges. Let ρ be suddenly stopped, then the displacement reverses from the negative to the positive pole along the meridians, with induction on the parallels in the opposite direction, and renewed inner radial displacement to the center. This, also, releases a plane

wave $[D \,.\, B]$ at the positive pole. Similarly, during collisions of two orbit-atoms, there are sudden motions and stoppages, with reversed displacements and inductions, sending plane

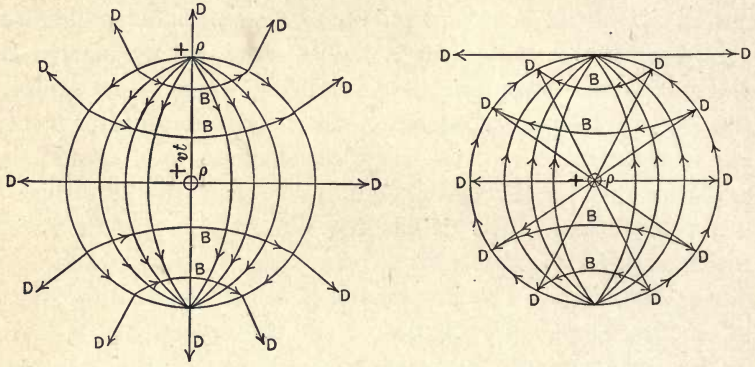


FIG. 40. The Sudden Motion and Stoppage of an Electric Charge ρ.

electromagnetic waves into space. When the orbits of atoms in collision interpenetrate, these waves are complicated in their frequencies, in accordance with the results seen in their characteristic spectrum lines.

Furthermore, during successive collisions at the end of the free path C, there exist the potential and the kinetic energies, which may be analyzed as potential along the free path with h as the average value, and $2\,h$ as the maximum value, the time of one period being $\frac{1}{\nu}$ second of time, while the kinetic energy is expressed by the motion in the circle, such that $k\,T = \frac{1}{2}\,m\,v^2$.

$$(382)\quad div\, W = -\,e_0\,J - h_0\,G + Q + \dot{\jmath} + H$$

$$(383)\qquad W = V\,(E - e_0)\,(H - h_0) = V\,E_1\,H_1 = v\,(\dot{\jmath} + \dot{H}).$$

All these forces, expressed and implied, are in action during the collisions of complex atoms and molecules.

The Variable Intensity of the Sun's Radiation in the 26.68-Day Period of the Synodic Rotation

Besides showing that the intensity of the solar radiation is variable in the 11-year period to the amount of 1% to 2%, it appears that a similar variation occurs as the sun turns on its axis. Fig. 41 gives the normal direct curves of the solar variation, as registered in the terrestrial magnetic field and the meteorological elements, according to the author's papers of 1893, 1895, 1898, and the Meteorological Treatise, 1915. The Cordoba-Pilar curve of the pyrheliometric mean intensities, 1912–1916, results in a nearly identical curve, and this proves that the solar radiation is variable in solar longitude and affects all the terrestrial elements in the 26.68-day period. Similarly, the La Quiaca pyrheliometric data produce the small curve in the

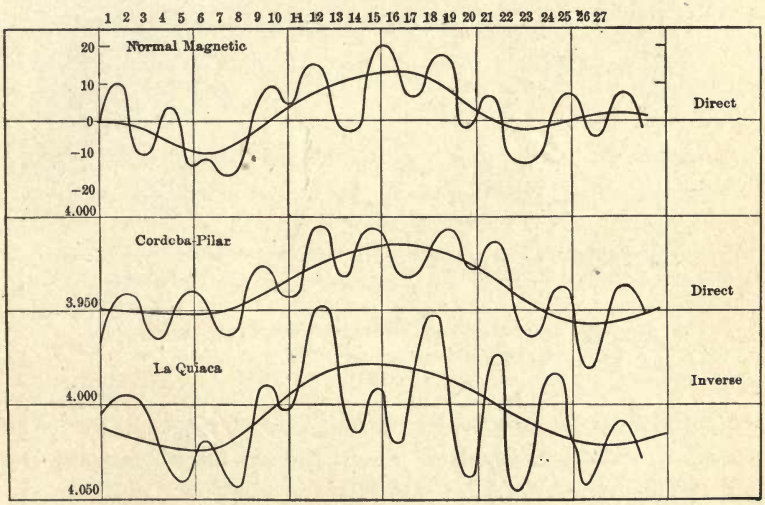


FIG. 41.—The variable intensity of the solar radiation in the 26.68-day synodic period.

inverse form, the amplitudes being much larger. It seems that there are inversion and damping of the variations in the intensity of the radiation in the lower atmosphere, a subject of importance for further research.

International Character Numbers

The character numbers of the amplitude of the disturbance of the magnetic field, as published by the International Commission for 35 stations during 1915, produces nearly the same inverse curve. This curve distributes its minor crests on four

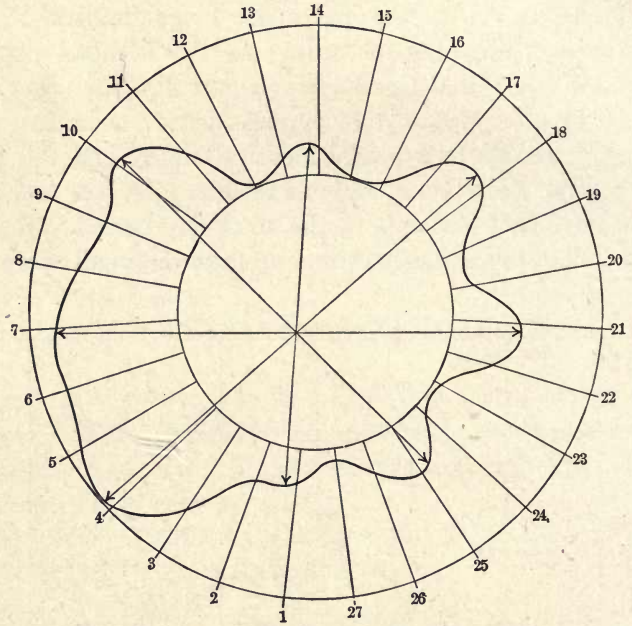


FIG. 42 —Magnetic Character Numbers, 1915.
International Commission, 35 Stations.

axes, whose center is eccentric to the axis of rotation, as if the center of radiation revolves about the center of mass of the sun. This eccentricity of radiation is evidently sufficient to form the basis of weather forecasts, because these maximum points have been proved to persist through 70 years, 1840–1916. It will be necessary to abandon the practise of publishing magnetic and meteorological data on the calendar months, which has no scientific meaning, and substitute the 26.68-day period with epoch, June 13.72, 1887. Compare the Meteorological Treatise, p. 334.

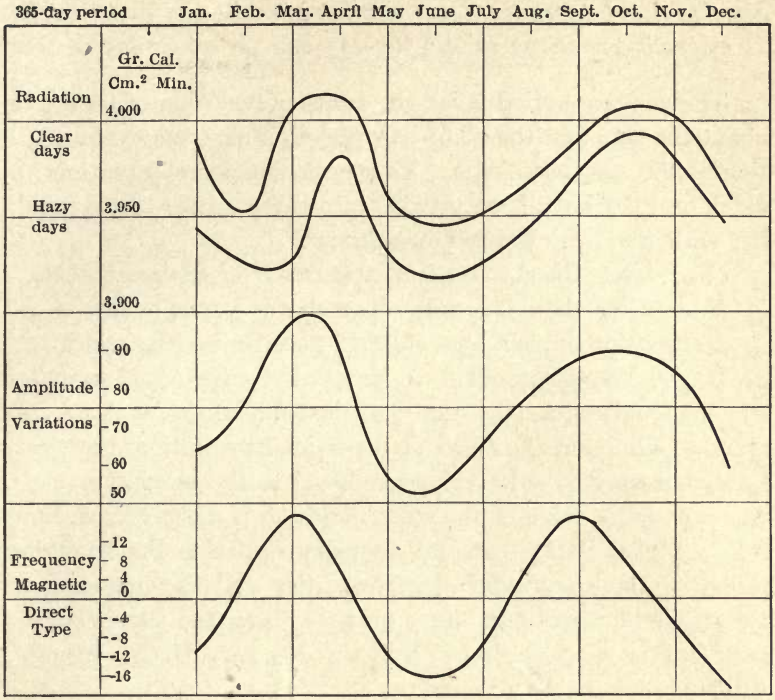


FIG. 43.—Synchronous variations of the radiation and the magnetic field in the 365-day period.

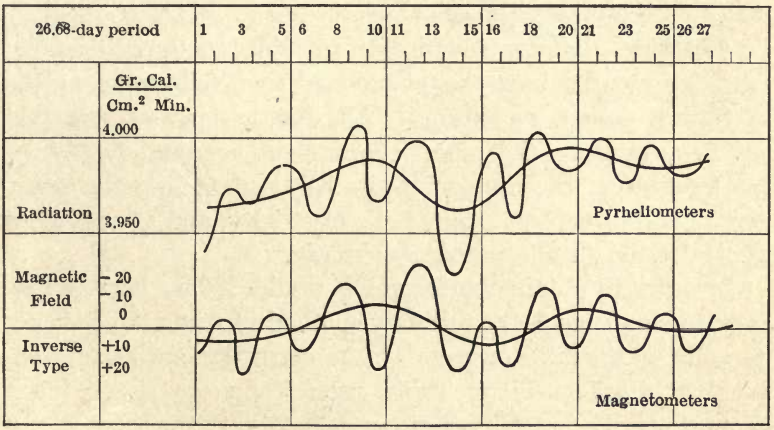


FIG. 44.—Synchronous variation of the radiation and the magnetic field in the 26.68-day period.

Synchronism between the solar radiation intensity and the terrestrial magnetic variations in the 365-day and the 26.68-day periods

The 365-day period is for the annual revolution of the earth about the sun, and the 26.68-day period is for the synodic rotation of the sun on its axis. The pyrheliometer observations in Cordoba-Pilar have been divided into three classes, corresponding with the local weather conditions:

Clear, when the sky is apparently free from haze and dust.

Haze, when there is a milky haze due to vapor or ice.

Dust, when the low level dust has been raised by wind.

It was found practical to determine correcting ordinates which should reduce the haze and dust days to clear days. In order to eliminate the effect of the annual revolution, the clear, haze, and dust days have been reduced to the *mean clear days*, whose average value of the solar radiation is 3.980 gr. cal./cm.^{2} min. Fig. 43 shows that the clear-day radiation, the hazy-day radiation, the amplitudes of irregularities, and the frequency of the magnetic direct type have maxima when the sun is on the earth's equator. Compare Bulletin No. 21, U. S. Weather Bureau, 1898, pages 100–108.

Fig. 44 shows that the solar radiation and the magnetic field synchronize in the 26.68-day period, when the radiation is reduced to the mean clear standard.

It follows that the compilation of tables in the unscientific calendar months must be abandoned and collections in the 26.68-day period substituted. All observations of radiation must be freed from the imperfections due to haze and dust in the low levels before they are compiled. All the data of meteorology, terrestrial magnetism, atmospheric electricity, and solar physics generally, must conform to these criteria.

In order to obtain the foregoing results it has been necessary to supplement the ordinary method of using the Bouguer formula by the table on page 370 of correction-ordinates compiled at Cordoba-Pilar, Table 115. They are the result of separating the 5-years' record, 1912–1916, into the three groups indicated. Other stations must prepare similar tables representing their climatic conditions.

The Effect of Dust in the Lower Strata

The results of observations with pyrheliometers, to determine the intensity of the solar radiation, and the amount of its variations in different intervals, have been always unsatisfactory, because they were defective in two particulars: (1) The potential energy of the radiation is omitted entirely in the computations; (2) the effect of the dust and impurities in the lower strata has not been eliminated. Unfortunately, the records as usually published give insufficient data from which either of these terms can be fully computed. It is necessary to know the intensity at various zenith distances, and the coefficient of transmission; also, the temperature, vapor pressure and barometric pressure, for computing the potential terms; the state of the sky, clear, hazy, dusty, force of the wind, for determining the dust effect on the scattering. Neither the Smithsonian Astrophysical Observatory, nor the U. S. Weather Bureau, publish their intensities with enough auxiliary data to make it practical to include their numerous observations in a definitive reduction, such as Cordoba-Pilar now possesses.

In Argentina the five-year records at Cordoba-Pilar have furnished the following system of monthly corrections, which, added to the zenith value of the intensity, produce results such as are illustrated in the preceding diagrams. The necessity for these corrections is evident from the fact that the normal intensity of radiation is *always lessened* by atmospheric impurities, the changes being in one direction. Hence, any mean value of a series depends upon the number of imperfect days which have been incorporated, so that the resulting mean values merely reflect the effects of the admixture of such defective conditions. The following tables indicate the corrections referred to, with examples of the results. It is certainly improper to ascribe to the radiation of the sun itself those terms which depend upon the local atmospheric conditions.

TABLE 115

CORDOBA-PILAR, 1912–1917

Table of Reduction to the Mean Clear System

Month	Clear	Haze	Dust
January	−0.010	+0.040	+0.170
February	+0.030	+0.050	+0.220
March	−0.030	+0.060	+0.150
April	−0.030	+0.060	+0.150
May	+0.020	+0.050	+0.200
June	+0.030	+0.060	+0.210
July	+0.020	+0.050	+0.200
August	+0.010	+0.020	+0.190
September	−0.010	0.000	+0.170
October	−0.020	−0.010	+0.160
November	−0.030	0.000	+0.150
December	+0.030	+0.020	+0.210

TABLE 116

DATA REDUCED TO THE MEAN CLEAR DAYS

Months	1912	1913	1914	1915	1916	1917
Jan., Feb.	4.059	3.946	4.001	4.012	3.888	3.960
Mar., April	4.013	4.031	3.921	3.949	3.876	3.957
May, June	3.988	4.007	4.022	4.033	3.920	3.956
July, Aug.	4.037	3.995	3.979	3.952	3.974	3.952
Sept., Oct.	3.988	4.003	4.030	3.959	3.886	.....
Nov., Dec.	3.934	4.076	4.029	4.023	3.973	.....
Means	4.004	4.014	3.997	3.988	3.913	3.956

It is seen, among other things, that the year 1912 is *near the maximum*, while it was recorded and discussed in the Northern Hemisphere—Mt. Wilson, Mt. Weather, Washington, Bassour—*as a minimum*, the latter being due simply to the dust in the atmosphere from the volcano Katmai. In Pilar, during dry weather, a wind storm fills the lower strata with dust, and the observed intensity drops to 3.70, 3.60, or even 3.40 calories. Hence, the inclusion of such dust effects in any mean values simply vitiates them, and the results are useless. Until these defects can be fully eliminated from the observed radiation

intensities, the data can not be utilized in any of the solar radiation problems.

The Systems of Units Employed in Meteorology

In the United States the public use of the British system of measures made it necessary to employ some of them in the Government Weather Service. These are the pressure in inches of mercury, the temperature in degrees Fahrenheit, the wind velocity in miles per hour. The density has been always erroneously computed on the adiabatic system, the gas efficiency has never been other than a constant, and all the dependent thermodynamic terms are consequently in error. These weather-map data are, therefore, inconsistent, and can be reduced to the M. K. S. or the C. G. S. systems only by laborious transformations, which it is impractical to apply to so large a mass of records as has been compiled in the United States since 1871. These data are, therefore, only remotely accessible to scientific studies. In order to supplement this public system the Weather Bureau has employed a different but mixed system, compiled from several sources. Similarly, the International system is not self-consistent, and is likewise inconvenient in scientific researches. Furthermore, these imperfect data are assembled in such a way as further to embarrass their value.

1. The Washington 8 A.M. and 8 P.M. hours are made the times for simultaneous observations throughout the United States, but they become 7 A.M., 7 P.M. at Chicago, 6 A.M., 6 P.M. at Omaha, and 5 A.M., 5 P.M. at San Francisco, so that the daily means are always incomplete and generally erroneous.

2. These daily means of inconsistent data are compiled in monthly tables, according to the calendar months, but these have no proper connection with solar recurrent phenomena. It would be difficult to devise a program of work more competent than that one to obscure the fact of solar synchronism in the terrestrial weather conditions. Table 117 contains an example of these systems, based upon the balloon ascension at Drexel, Nebraska, October 31, 1915, which illustrates the divergent and inconsistent methods in use.

TABLE 117

EXAMPLES OF THE METEOROLOGICAL DATA IN FIVE SYSTEMS. BALLOON ASCENSION, DREXEL, NEB., OCT. 31, 1915

Term	Sign	Dimension	M. K. S. System	C. G. S. System	U. S. W. B. System	British System	International System
Height.....	Z	L	1000 meters	100000 cm.	1000 meters	3281 feet	1000 meters
Pressure.....	P	$\frac{M}{L\,T^2}$	89420 $\frac{\text{Joules}}{m^3}$	894200 $\frac{\text{ergs}}{\text{cm.}^3}$	894.2 mmbar	26.405 inches	670.7 millim.
Temperature....	T	Deg.	284.°9 absolute	284.°9 absolute	11.°9 centigrade	53.°4 Fahrenheit	11.°9 centigrade
Vapor Pressure..	e	$\frac{M}{L\,T^2}$	428.8 $\frac{\text{Joules}}{m^3}$	4288 $\frac{\text{ergs}}{\text{cm.}^3}$	4.288 mmbar	0.1266 inches	3.216 millim.
Velocity.....	q	$\frac{L}{T}$	20.4 $\frac{\text{meters}}{\text{sec.}}$	2040 cm./sec.	20.4 $\frac{\text{meters}}{\text{sec.}}$	45.6 $\frac{\text{miles}}{\text{hour}}$	73.4 $\frac{\text{kilometers}}{\text{hour}}$
Gravity.....	g	$\frac{L}{T^2}$	9.806 met./sec.^{2}	980.6 cm./sec.^{2}	980.6 cm./sec.^{2}	32.173 feet/sec.^{2}	9.806 met./sec.^{2}
Stratum.....	(Z_1-Z_0)	L	1000 meters	100000 cm.	100000 cm.	3281 feet	1000 meters
Potential.....	$g(Z_1-Z_0)$	$\frac{L^2}{T^2}$	9806 $\left(\frac{\text{met.}}{\text{sec.}}\right)^2$	98060000 $\left(\frac{\text{cm.}}{\text{sec.}}\right)^2$	980.6×10^5 ergs	105560 $\left(\frac{\text{feet}}{\text{sec.}}\right)^2$	9.806 $\left(\frac{\text{met.}}{\text{sec.}}\right)^2$
Density.....	ρ	$\frac{M}{L^3}$	1.2110 $\frac{\text{kilog.}}{m^3}$	0.001211 $\frac{\text{gr.}}{\text{cm.}^3}$	.....	.....	.....
Efficiency.....	R	$\frac{L^2}{T^2\,\text{deg.}}$	259.18 $\frac{m^2}{\text{sec.}^2\,\text{deg.}}$	2591800 $\frac{\text{cm.}^2}{\text{sec.}^2\text{deg.}}$	.....	.....	.....
Kinetic Energy..	$Pv=RT$	$\frac{L^2}{T^2}$	73840 $\frac{m^2}{\text{sec.}^2}$	738400000 $\frac{\text{cm.}^2}{\text{sec.}^2}$	.....	.....	.....
Volume—n.....	n	$\frac{1}{L^3}$	2.4073×10^{25}	2.4073×10^{19}	.....	.....	.....
Mass—N.....	N	$\frac{1}{M}$	5.7019×10^{26}	5.7019×10^{23}	.....	.....	.....

The correct values of ρ, R, $P\,v = R\,T$, n, N are lacking from all current Meteorologies. The computations of thermodynamic data are practicable only in (M. K. S.) or (C. G. S.) systems.

The Bar as a Practical Unit for Absolute Pressure

An effort has been made to introduce a new practical unit of absolute pressure into meteorology, depending upon 1,000,000 C. G. S. units of pressure, as follows:

C. G. S.	McAdie	von Bjerknes	Bigelow
1000000.....	1 megabar	1 bar	1 kilobar
1000.....	1 kilobar	1 millibar	1 bar
1.....	1 bar	1 microbar	1 millibar

McAdie* begins with 1 bar = 1 C. G. S. unit or dyne; and proceeds through 1 kilobar = 1,000 C. G. S., to 1 megabar = 1,000,000 C. G. S. On the other hand, von Bjerknes proceeds in the opposite direction, from 1 bar = 1,000,000 C. G. S., through 1 millibar = 1,000 C. G. S. to 1 microbar = 1 C. G. S. unit. But it is evident that, in practise, *the natural unit of pressure is* 1,000 *C. G. S.*, and it becomes 1 kilobar = 1,000,000 C. G. S. and 1 millibar = 1 C. G. S. Pressure would then be described in convenient language, for example:

C. G. S.	M. K. S.	Bars
1013.220	1013.22	"
955.180	955.18	"
900.100	900.10	"
848.140	848.14	"
798.930	798.93	"

This conforms closely to the customary nomenclature of millimeters of mercury in its outward form, and they become nearly identical using the factor of multiplication 0.75. Compare "Meteorological Treatise," p. 4. The introduction of practical units obscures the true significance of the data in higher meteorology, and in computations of the thermodynamic and radiation formulas only the simple M. K. S. and C. G. S. systems can be utilized. In solar thermodynamics these simplifications are wholly impossible. The M. K. S. system is convenient for

* "The Principles of Aerography," p. 30, 1917.

General Meteorology, and the C. G. S. system is better for atomic physics. No mixed systems of any sort are permissible.

The Long Range Forecasts of Weather Conditions

In view of the immense possibilities of this branch of solar physics, which is still undeveloped, and the fact that there is a persistent synchronism between solar and terrestrial atmospheric phenomena, already traced for more than half a century, it is desirable that meteorologists should generally arrange their data in a form for immediate and convenient use. At present the observations are made solely for the construction of synchronous weather charts, in relative systems of pressure, temperature, and wind velocity, but without regard to the general requirements of science. The mass of such observations has become so enormous that it is impractical to continue to make the necessary transformations. The following reforms are recommended to all meteorological services:

1. Adoption of the (M. K. S.) absolute system for pressure, temperature, and wind vectors.

2. Observations at the local hours primarily, but reduced to the synoptic chart hours for weather forecasts.

3. The assembling of observations in the 26.68-day solar period and the abandonment of the calendar-month system.

4. The closest alliance between astrophysicists and meteorologists.

5. Extension of the non-adiabatic thermodynamics to all branches of atmospheric physics.

The current methods of operation will continue to conceal and distort all kinds of solar effects in the terrestrial processes merely in consequence of the bad workmanship. Until these steps in reform have been taken, all critical speculations and skeptical remarks based upon inaccurate data and methods are useless, as they hinder the progress of science.

APPENDIX

The Pyrheliometer and the Poynting Equation

Heaviside remarks that the Poynting equation constitutes the *first proof* of the truth of Maxwell's electromagnetic theory of radiation, and it follows that every apparatus intended to measure the intensity of the solar radiation—pyrheliometer, bolometer—must conform in its theory to that equation. It is written:

$$\frac{1}{4\pi}\int\int [E\,H]_n\,dS = \frac{1}{4\pi}\frac{\partial}{\partial t}\int\int\int \left(\tfrac{1}{2}H^2 + \tfrac{1}{2}E^2\right) d\tau + A,$$

and it means that during unsteady temperatures the surface flux of the radiation, as received on the instrument, is equivalent to the volume density absorbed within the apparatus, plus the free heat or waste due to lack of equilibrium while the temperature is adjusting itself to that of the corresponding black radiation. During such an adjustment there is a transformation of the molecular and atomic forces within the apparatus, by means of the kinetic and the potential energies through complex interchanges which are difficult at present to follow. The bolometer is not an absolute instrument, but merely measures *relative ordinates in the spectrum*, which indicate the efficient temperature of the radiation indirectly. The pyrheliometer measures only the rate of change in a temperature thermometer, such as the varying A produces, but it does not distinguish clearly either the kinetic energy, the potential energy, or the free heat, from one another. The kinetic energy, the potential energy, the work of expansion energy, and the free heat, are all involved in the right side of the Poynting equation, after the electromagnetic energy has been transformed into the equivalent thermodynamic energies. Since the pyrheliometer measures

only the relative changes of temperature in the unit time at different zenith distances, and not the absolute volume density in the final state at any time, it follows that the Langley-Abbot method of discussion, which seeks to substitute the Bouguer formula of depletion for the Poynting equation of equilibrium, is wholly misleading. When the spectrum of 6,000° is arbitrarily selected as that to which the pyrheliometer is to be referred, this is contradicted by the bolometer ordinates; when the extrapolation is made from the surface to the vanishing plane, from sec. $z = 1$ to sec. $z = 0$, the Bouguer formula is abandoned; when the relative intensity of the observed radiation is assumed to be complete, or absolute in amount, it is evident that this is falsely identified with the free heat implied in A, and that the potential energy is specifically neglected. The following three pieces of apparatus have done great injury to meteorology: the mercurial barometer, by substituting some arbitrary scale value of pressure for the absolute units of force; the evaporation pan, by its erroneous distortion of the amount of water lost as compared with a large free water surface; and the pyrheliometer, by its incorrect interpretation of the solar constant of radiation, as compared with the complete theory of the Poynting equation, which is fully confirmed by the results of the thermodynamics of the terrestrial atmosphere, of the solar atmosphere and of the bolometric spectra. There is probably no apparatus more difficult to interpret correctly than is the pyrheliometer, because it demands a full knowledge of the behavior of radiation in gases, in glass, in mercury, in metals, during variable transformations, in which the kinetic, potential, expansion and free heat energies are all undergoing mutual readjustments. To compare it with other similar apparatus is to obtain only relative values of radiation, and to ascribe these to the absolute solar intensities is erroneous.

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